# Some developments in the physics of the light quarks

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Symposium in honour of Jan Stern, Orsay, June 3, 2008

# Part I

# Reminiscences

# *Once upon a time, when Jan and I studied physics . . .*

- *in 1962, when I completed my studies at the University of Bern, I had heard about QED, the Fermi theory of $\beta$ decay and nonrelativistic potential models for the nuclear forces, but as far as particle physics is concerned, that was about it*

  *there was nothing to miss. The theory of the strong interaction was a genuine mess: elementary fields for baryons and mesons, Yukawa interaction for the strong forces, perturbation theory with coupling constants of order 1, nuclear democracy, bootstrap . . . absolutely nothing worked even half ways, beyond general principles like Lorentz invariance, causality, unitarity, crossing, dispersion relations*

- *Jan was more lucky in this regard: started a little later and studied at the Charles University in Prague, where there was an active group in particle physics: Votruba, Vancura, Havlícek, Formanék, . . .*

$\Rightarrow$ *during his studies, he must have encountered some of the new ideas which paved the way to an understanding of the strong interaction*

# The new ideas

- the strong interaction has a hidden, approximate symmetry Nambu 1960
- eightfold way Gell-Mann, Ne'eman 1961
- pattern of symmetry breaking, $\Omega^-$ Gell-Mann, Okubo 1961/1962
- quark model Gell-Mann, Zweig 1962
- puzzle: why are the symmetries not exact ?
  exact consequences of approximate properties ?
  charges & currents form an exact algebra
  even if they do not commute with the Hamiltonian Gell-Mann 1964
- test of current algebra: size of $\langle N|A^\mu|N\rangle \sim g_A$
  Adler 1965, Weisberger 1966
- prediction from current algebra: $\pi\pi$ scattering lengths Weinberg 1966

# Mass of the pion

- formula for pion mass *Gell-Mann, Oakes & Renner 1968*

$$M_\pi^2 = \underbrace{(m_u + m_d)}_{\Uparrow\ \text{explicit}} \times |\langle 0|\, \underbrace{\bar{q}\, q}_{\Uparrow\ \text{spontaneous}} \,|0\rangle| \times \frac{1}{F_\pi^2}$$

  - at that time, the existence of quarks was still questionable
  - quarks were treated like the veal used to prepare a pheasant in the royal French cuisine

  $\Rightarrow$ formula does not appear like this in the paper

- the GMOR relation explains why the energy gap of the strong interaction is so small . . . in terms of a new mystery: why are $m_u$ and $m_d$ so small ?

# *Quark masses*

- *even before the discovery of QCD, attempts at estimating the masses of the quarks were made*
- *bound state models for mesons and baryons*
  $m_u + m_u + m_d \simeq M_p \qquad m_u \simeq m_d$

$\Rightarrow$ $m_u \simeq m_d \simeq$ *300 MeV* *"constituent masses"*

- *remarkably simple and successful picture*
  *explains the pattern of energy levels without QCD*
- *model for spontaneous symmetry breakdown*
  *requires much smaller fermion masses* *Nambu & Jona-Lasinio 1961*
- *same conclusion from sum rules for currents* *Okubo 1969*

$\Rightarrow$ *conceptual basis of royal French cuisine ?*

- *smart people considered Regge theory very promising*
  *Veneziano model 1968*

*This was the state of our art when I met Jan*

# Where I presumably met Jan for the first time

IL NUOVO CIMENTO VOL. LIX A, N. 3 1° Febbraio 1969

## Algebra of Currents on the Light Cone.

J. JERSÁK and J. STERN
*Charles University - Prague* (*)

(ricevuto il 18 Giugno 1968)

**Summary.** — On the basis of observation that the fixed-mass sum rules are Fourier transforms of the commutators of currents on a lightlike hyperplane a new treatment of the algebra of currents is proposed. It consists in use of the currents integrated over the lightlike hyperplane (« lightlike » charges) and thus includes the commutators of currents on the whole light cone. An algebra of the lightlike charges is a sufficient assumption for the derivation of the fixed-mass sum rules and may be represented in the one-particle space with any momenta. Hence this extension of the usual equal-time current algebra allows us to abandon the methods like $p \to \infty$ without losing any of its advanteges.

# Abstract for those who prefer it in Russian

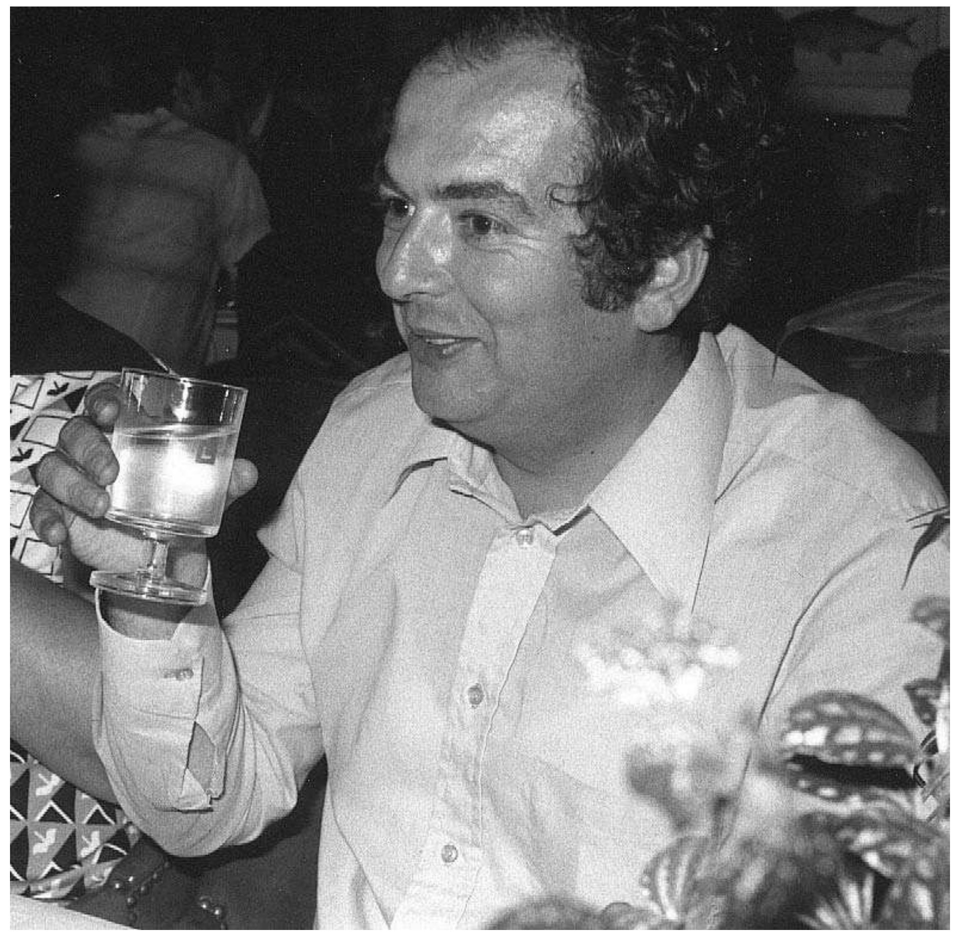
(*) *Traduzione a cura della Redazione.*

**Алгебра токов в световом конусе.**

**Резюме** (*). — Исходя из наблюдения, что правила сумм с фиксированной массой представляют преобразования Фурье коммутаторов тока на свето-подобной гиперплоскости, предлагается новая трактовка алгебры токов. Трактовка состоит в использовании токов, проинтегрированных по свето-подобной гиперплоскости («свето-подобные» заряды), и тем самым включает коммутаторы токов на всем световом конусе. Алгебра свето-подобных зарядов является достаточным предположением для вывода правил сумм с фиксированной массой и может быть представлена в одно-частичном пространстве с любыми импульсами. Следовательно, это расширение обычной алгебры токов при равных временах позволяет нам отказаться от методов, подобных $p \to \infty$, без потери каких-либо преимуществ этих методов.

(*) *Переведено редакцией.*

# *In natura*



# First paper we wrote together: February 1970

Volume 31B, number 7 PHYSICS LETTERS 30 March 1970

## NEW DISPERSION SUM RULES FOR INELASTIC e-p SCATTERING

H. LEUTWYLER *
*CERN, Geneva, Switzerland*

and

J. STERN **
*Institut de Physique Nucléaire, Division de Physique Théorique, Orsay, France*

Received 23 February 1970

Causality combined with scaling implies a set of sum rules for the electromagnetic structure functions. We propose a local interpolation between the Regge and Bjorken limits to convert these relations into finite energy sum rules relating the deep inelastic and resonance regions.

In the present paper we exploit two properties of the electromagnetic structure functions

$$\frac{1}{2\pi}\int \mathrm{d}x \exp(\mathrm{i}qx)\langle \mathrm{p}|[j_\mu(x), j_\nu(0)]|\mathrm{p}\rangle_\mathrm{c} =$$

$$= \{q_\mu q_\nu - g_{\mu\nu}q^2\} V_1 +$$

$$+ \frac{1}{m^2}(p_\mu q_\nu + p_\nu q_\mu)\nu m - p_\mu p_\nu q^2 - g_{\mu\nu}\nu^2 m^2\} V_2$$

which determine the inelastic e-p cross sections:

causality, which requires the commutator to vanish for $x^2$ 0;

the fact that $V_1$ and $V_2$ appear to obey the scaling laws

The scaling laws (1) therefore guarantee the convergence of the integral and the no subtraction assumption is justified. In terms of the invariants $\nu, q^2$ the dispersion relation may be rewritten

$$V_i^{\mathrm{ret}}(\nu, a\nu - b) = \frac{1}{2\pi\mathrm{i}}\int \frac{\mathrm{d}\nu'}{\nu' - \nu - \mathrm{i}\epsilon} V_i(\nu', a\nu' - b) \qquad (3)$$

where $a$ and $b$ are arbitrary parameters varying in the range ‡ $-\infty < a < \infty$, $b \geq \frac{1}{4}a^2$. With this representation one immediately verifies that $V^{\mathrm{ret}}$ must scale like $(2\nu m^2)^{-1} F_{-1}{}^{\mathrm{ret}}(\xi)$ in the Bjorken limit, where

$$F_1^{\mathrm{ret}}(\xi) = \frac{1}{2\pi\mathrm{i}}\int \frac{\mathrm{d}\xi'}{\xi - \xi' - \mathrm{i}\epsilon} \frac{F_l(\xi')}{\xi'} \qquad (4)$$

# Extended version: March 1970

7.A.2 Nuclear Physics B20 (1970) 77-101. North-Holland Publishing Company

# SINGULARITIES OF CURRENT COMMUTATORS ON THE LIGHT CONE

H. LEUTWYLER *
*CERN, Geneva*

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Received 23 March 1970

Abstract: We consider the forward proton matrix element of the electromagnetic current commutator and express its singularities on the light cone in terms of the scaling functions. This relation between the light cone and the deep inelastic region implies a family of sum rules for the electroproduction structure functions and connects the equal time commutators with the scaling functions. In particular the operator part of the Schwinger term and the related divergences of the electromagnetic mass of the proton are expressed in terms of measurable quantities.

# QCD

- QCD was discovered in 1973
  - many considered this a wild speculation
  - for all quantum field theories encountered in nature before, the spectrum of physical states was the same as the one of the kinetic part of the Lagrangian *Pauli*
  - also true of the electroweak theory

    *Glashow 1961, Weinberg 1967, Salam 1968*
- only gradually, particle physicists abandoned their outposts in no man's and no woman's land, returned to the quantum fields and resumed discussion in the good old Gasthaus zu Lagrange *Jost*

$\Rightarrow$ Standard Model, clarified the picture enormously

# Standard Model

*Standard Model appeared like a miracle:*

- *weak, e.m. and strong interactions are very different*
  *nevertheless, they are all generated by gauge fields*

  *IG Physik, Gesellschaft mit besonderer Haftung, advertisement ca. 1973*

  **Im Falle eines Falles**
  **klebt ein EICHFELD**
  **wirklich alles !**

  *Bezugsquellennachweis*

  *H. Weyl, Z. Phys. 56 (1929) 330, C. N. Yang and R. Mills, Phys. Rev. 96 (1954) 191*

- *gauge fields are renormalizable in* $d = 4$
- *miracle leads to new puzzle: SM cannot be the full truth*
  *must be an effective theory adequate at low energies*
  *no reason for an effective theory to be renormalizable*

$\Rightarrow$ *why is the SM renormalizable ?*

# Quantum theory on null planes

- *for three years, Jan was engaged as a guest professor at the University of Bern: 1977/78 and 1982/84*
- *I had a wonderful time discussing physics with him my prejudices were not always identical with his, but this only animated the discussions. . .*
- *he, Jiři and I vigorously pursued the idea that formulating the quantum theory on null planes might give a coherent description of mesons and baryons in terms of quarks*
- *our efforts led to a very thorough paper in Annals of Physics, which you might take pleasure reading, even now, 30 years later . . .*

ANNALS OF PHYSICS **112**, 94–164 (1978)

# Relativistic Dynamics on a Null Plane*

H. LEUTWYLER

*Institut fur Theoretische Physik, der Universität Bern, Sidlerstrasse 5, 3012, Bern, Switzerland*

AND

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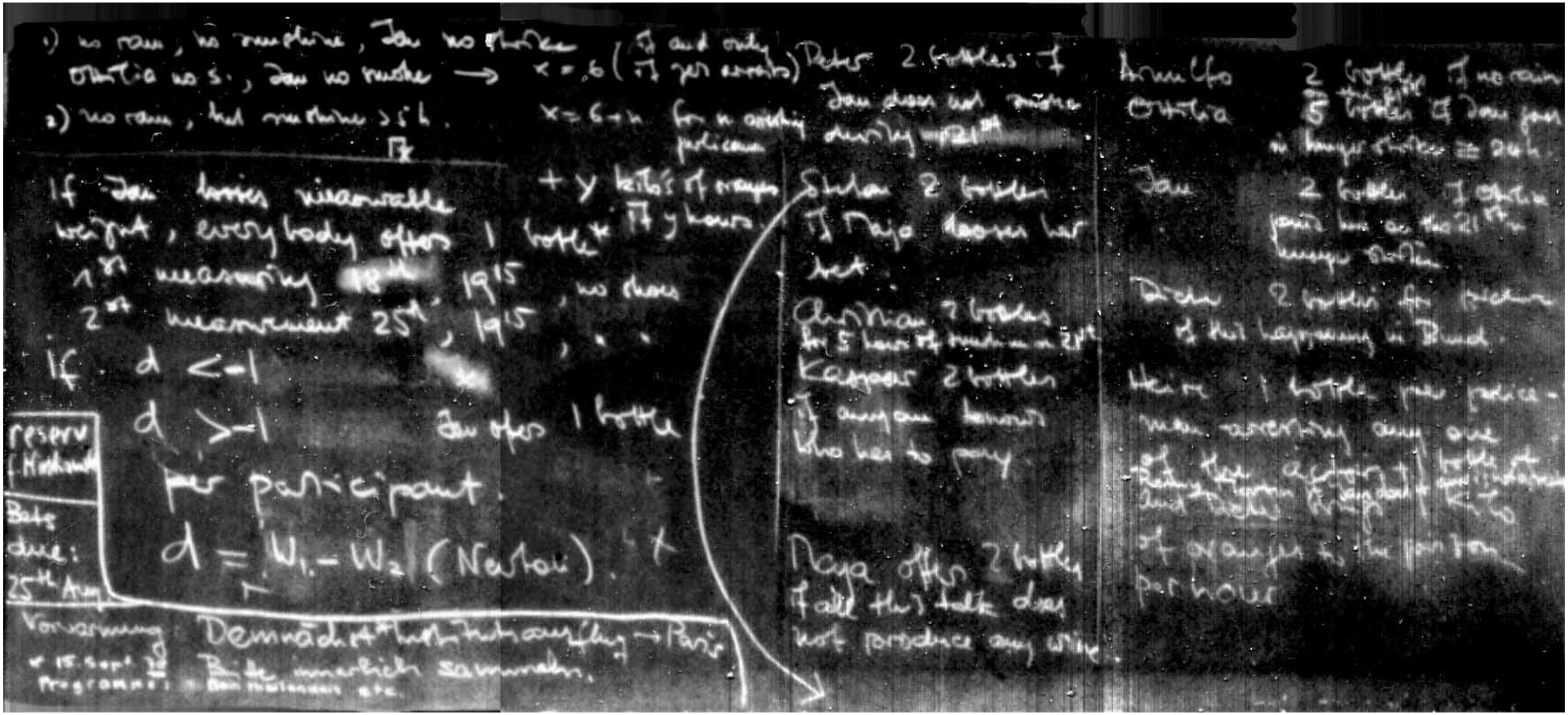
Received June 8, 1977

In view of possible applications to the quark model and to hadron spectroscopy, we investigate relativistic Hamiltonian quantum theories of finitely many degrees of freedom. We make use of the fact that if null planes are used as initial surfaces, the structure of the theory closely resembles nonrelativistic quantum mechanics: the inner variables that describe the structure of the system uncouple from the motion of the system as a whole.

# Suivez le guide !

# Making bets in the coffee room, on a Friday evening

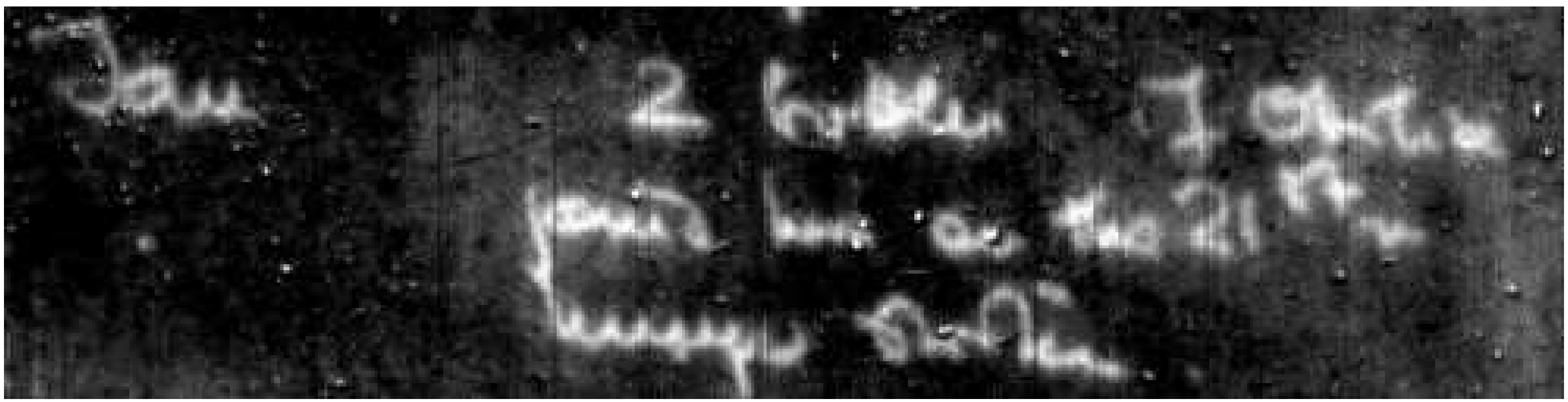



I thank Ottilia Haenni for the picture and Hans Bebie
for managing to make the scribbling legible

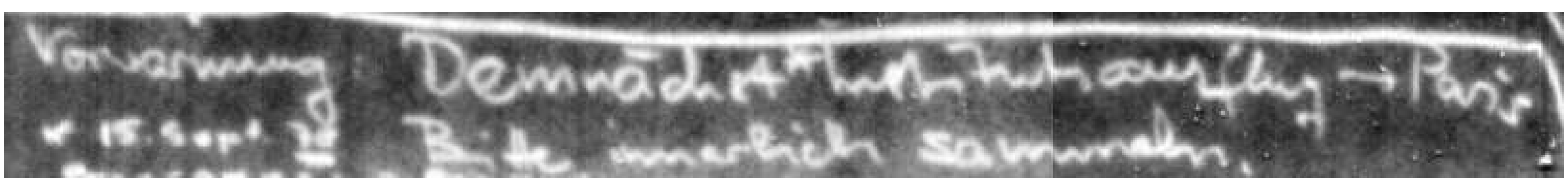
# *Exerpts*

Ottilia's offer: 5 bottles if Jan goes on hungerstrike for $\geq$ 24 hours

Jan's answer: 2 bottles if Ottilia joins him on the $21^{st}$ hour



Vorwarnung: Demnächst* Institutsausflug $\longrightarrow$ Paris
$*$ 15. Sept. 78 Bitte innerlich sammeln



# Part II

# Recent developments in the physics of the light quarks

# *How well do we understand the strong interaction today ?*

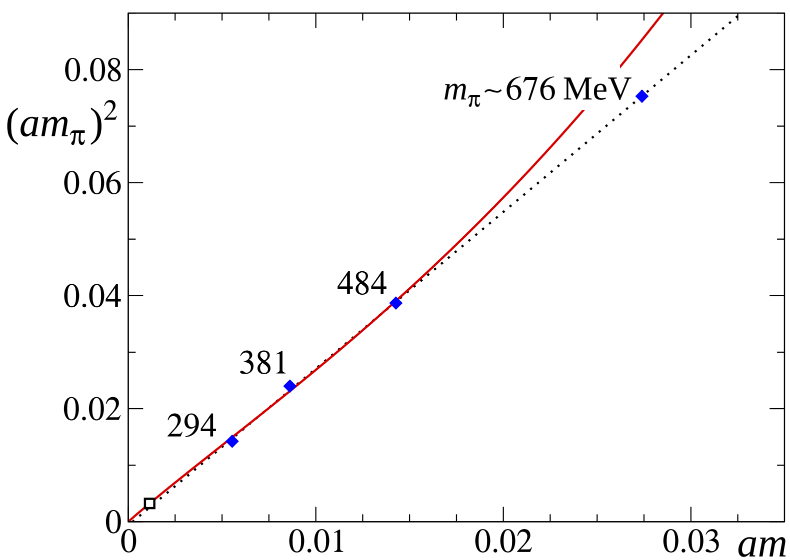
- *QCD with massless quarks is the ideal of a theory: no dimensionless free parameters*
- *high energy side looks like what we are used to: relevant degrees of freedom are visible in the Lagrangian, can treat the interaction as a perturbation*
- *gives rise to a rich structure at low energies*
- *low energies are out of reach of perturbation theory*

$\Rightarrow$ *progress in understanding is slow*

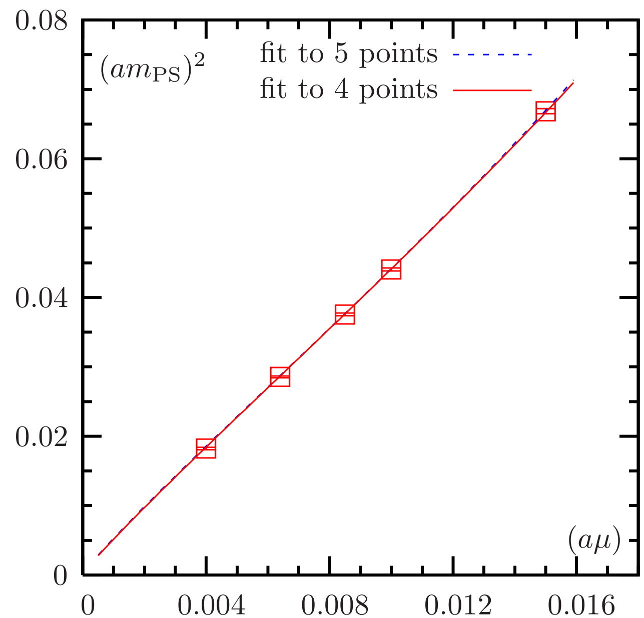
- $\exists$ *many models that resemble QCD: instantons, monopoles, bags, superconductivity, gluonic strings, linear $\sigma$ model, hidden gauge, NJL, AdS/CFT, but . . .*

# Lattice results for $M_\pi$

- *Gell-Mann-Oakes-Renner formula can now be checked: can determine $M_\pi$ as a function of $m_u = m_d = m$*




*Lüscher, Lattice conference 2005*




*ETM collaboration, hep-lat/0701012*

- *no quenching, quark masses are sufficiently light*

$\Rightarrow$ *legitimate to use $\chi$PT for the extrapolation to the physical values of $m_u, m_d$*

# *Lattice*

- *quality of data is impressive*
- *proportionality of* $M_\pi^2$ *to the quark mass appears to hold out to values of* $m_u, m_d$ *that are an order of magnitude larger than in nature*
- *main limitation: systematic uncertainties*
  *in particular:* $N_f = 2 \longrightarrow N_f = 3$

# *Chiral perturbation theory*

- *consequences of hidden, approximate symmetry can be worked out by means of an effective field theory* *Weinberg 1979*
- *hidden symmetry controls energy gap of QCD*

$\Rightarrow$ *can calculate how gap grows as the symmetry breaking parameters* $m_u, m_d$ *are turned on*

- *hidden symmetry also determines the interaction of the Goldstone bosons at low energies, among themselves, as well as with other hadrons*

# *Expansion of $M_\pi^2$ in powers of the quark mass*

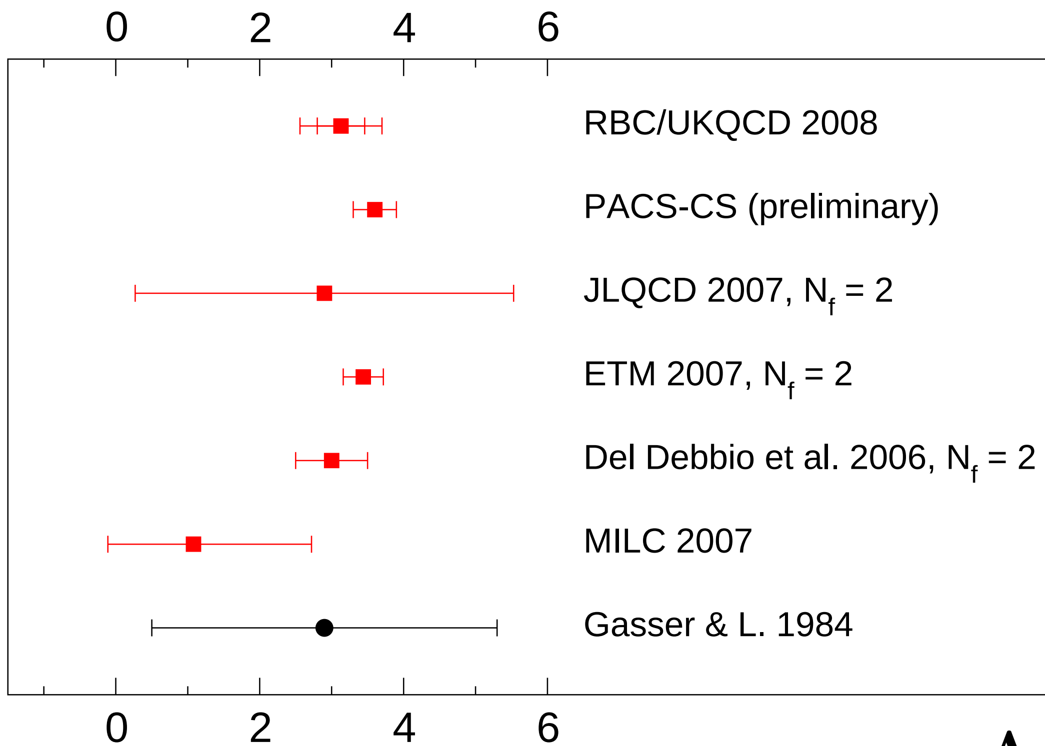
- *Gell-Mann-Oakes-Renner formula represents leading term of $\chi$PT*
- *disregard from isospin breaking, set $m_u = m_d = m$*
- *at NLO, the expansion contains a logarithm* Langacker & Pagels 1973

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

$$M^2 \equiv 2Bm$$

- *coefficient is determined by pion decay constant*
  *symmetry does not determine the scale $\Lambda_3$*
- *crude result, based on SU(3)$\times$SU(3):*
  $0.2$ GeV $\lesssim \Lambda_3 \lesssim 2$ GeV Gasser & L. 1984

## *Lattice allows more accurate determination of* $\Lambda_3$




*horizontal axis shows the value of* $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

*range for* $\Lambda_3$ *obtained in 1984 corresponds to* $\bar{\ell}_3 = 2.9 \pm 2.4$

*result of RBC/UKQCD 2008:* $\bar{\ell}_3 = 3.13 \underset{stat}{\pm 0.33} \underset{syst}{\pm 0.24}$

# *Expansion of $F_\pi$ in powers of the quark mass*

- *also contains a logarithm at NLO:*

$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

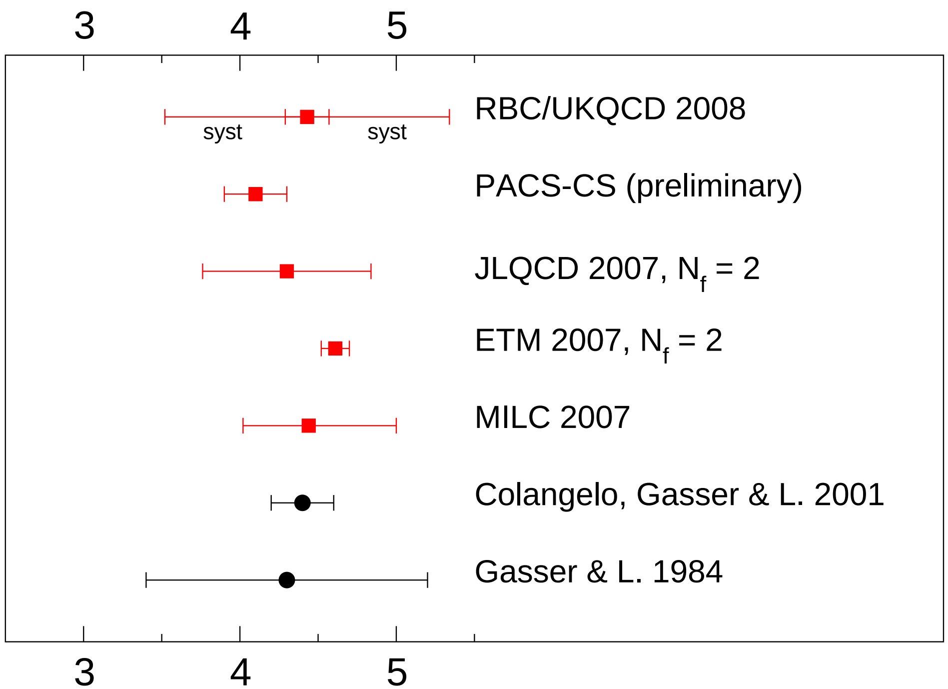
$F$ *is value of pion decay constant in limit* $m_u, m_d \to 0$

- *structure is the same, coefficients and scale of logarithm are different*
- *quark mass dependence of* $F_\pi$ *can also be measured on the lattice*

$\Rightarrow$ *measurement of* $\Lambda_4$

- *alternative method: determine the scalar form factor of the pion, radius* $\langle r^2\rangle_s \Leftrightarrow \bar{\ell}_4$

# Lattice results for $\Lambda_4$



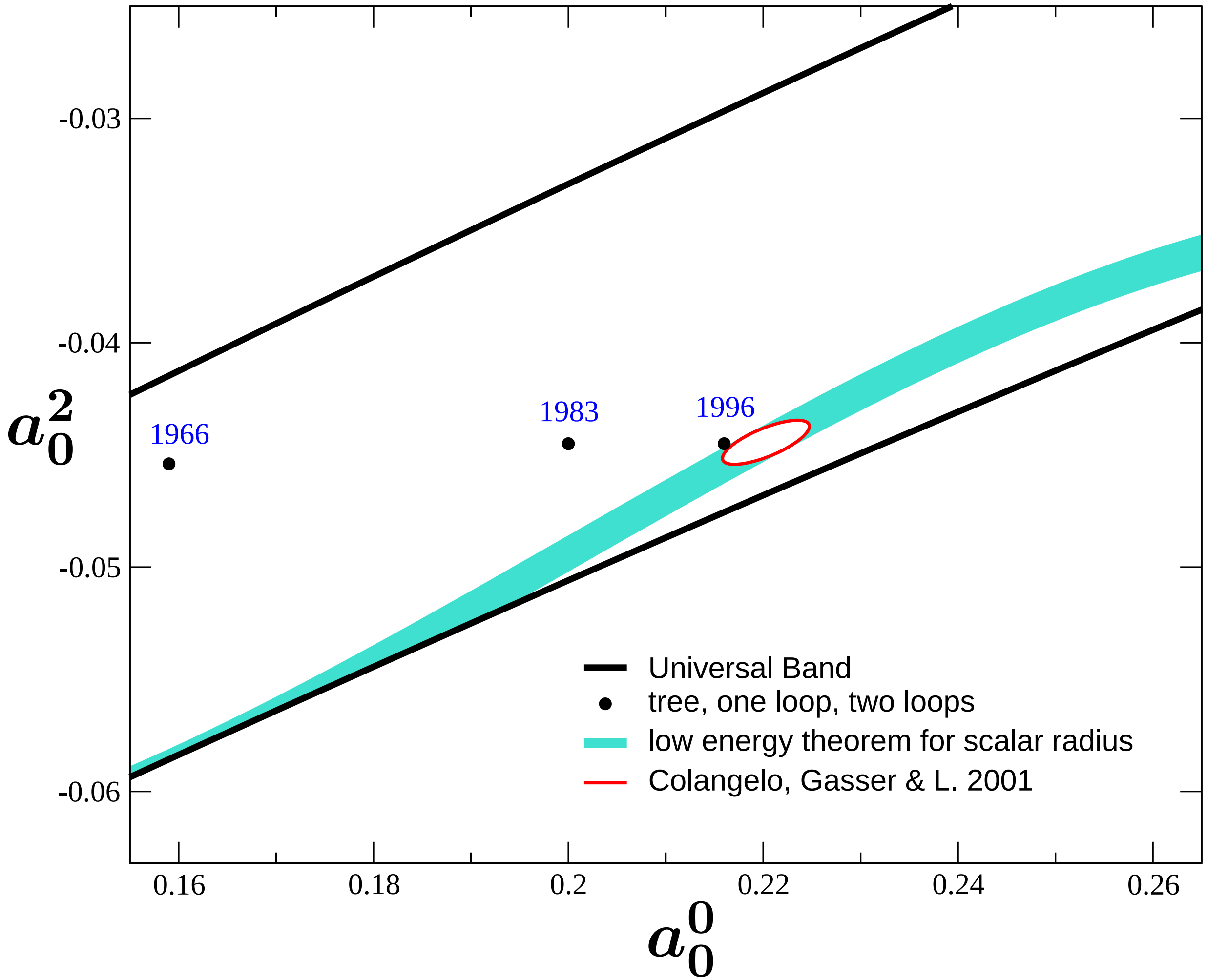

$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

- *lattice results beautifully confirm the prediction for the sensitivity of $F_\pi$ to $m_u, m_d$:*

$$\frac{F_\pi}{F} = 1.072(7)$$

*Colangelo and Dürr 2004*

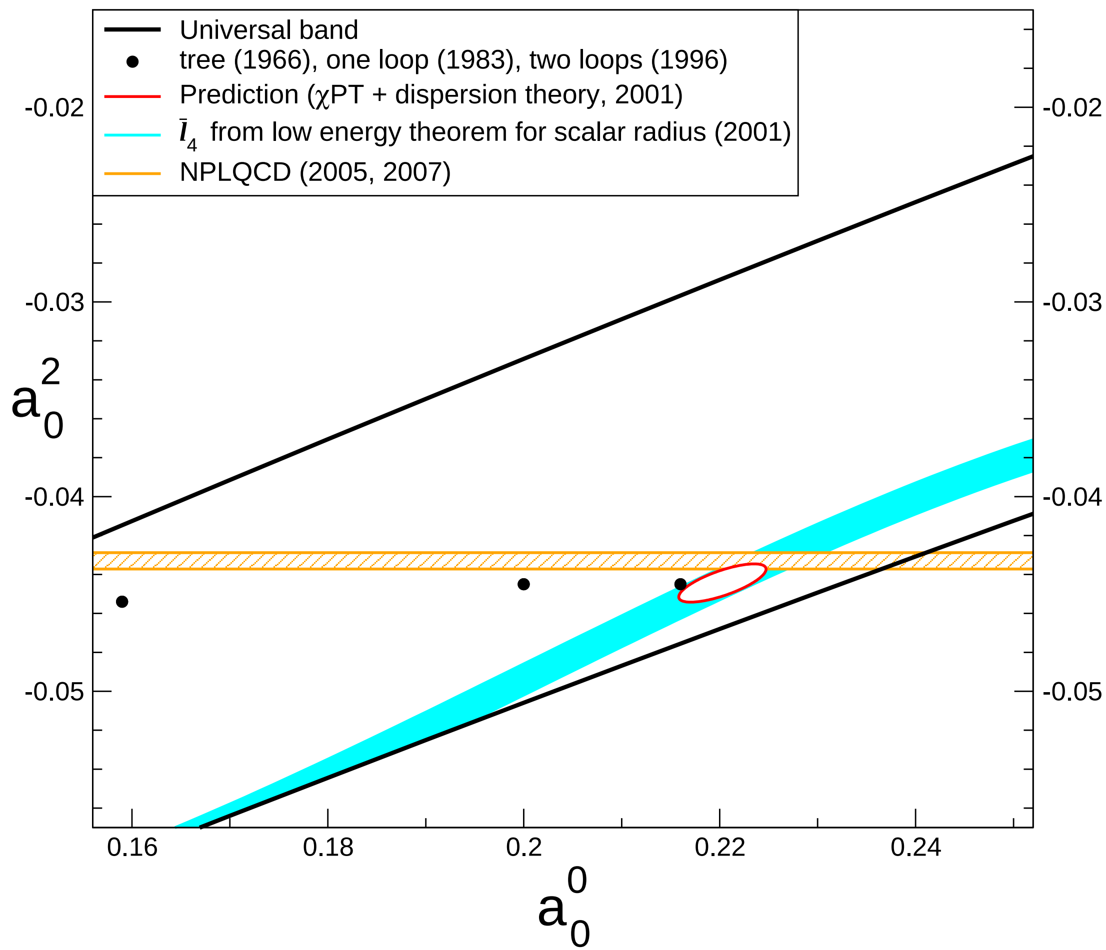
# *Predictions for the S-wave $\pi\pi$ scattering lengths*




*sizable corrections in $a_0^0$, while $a_0^2$ nearly stays put*

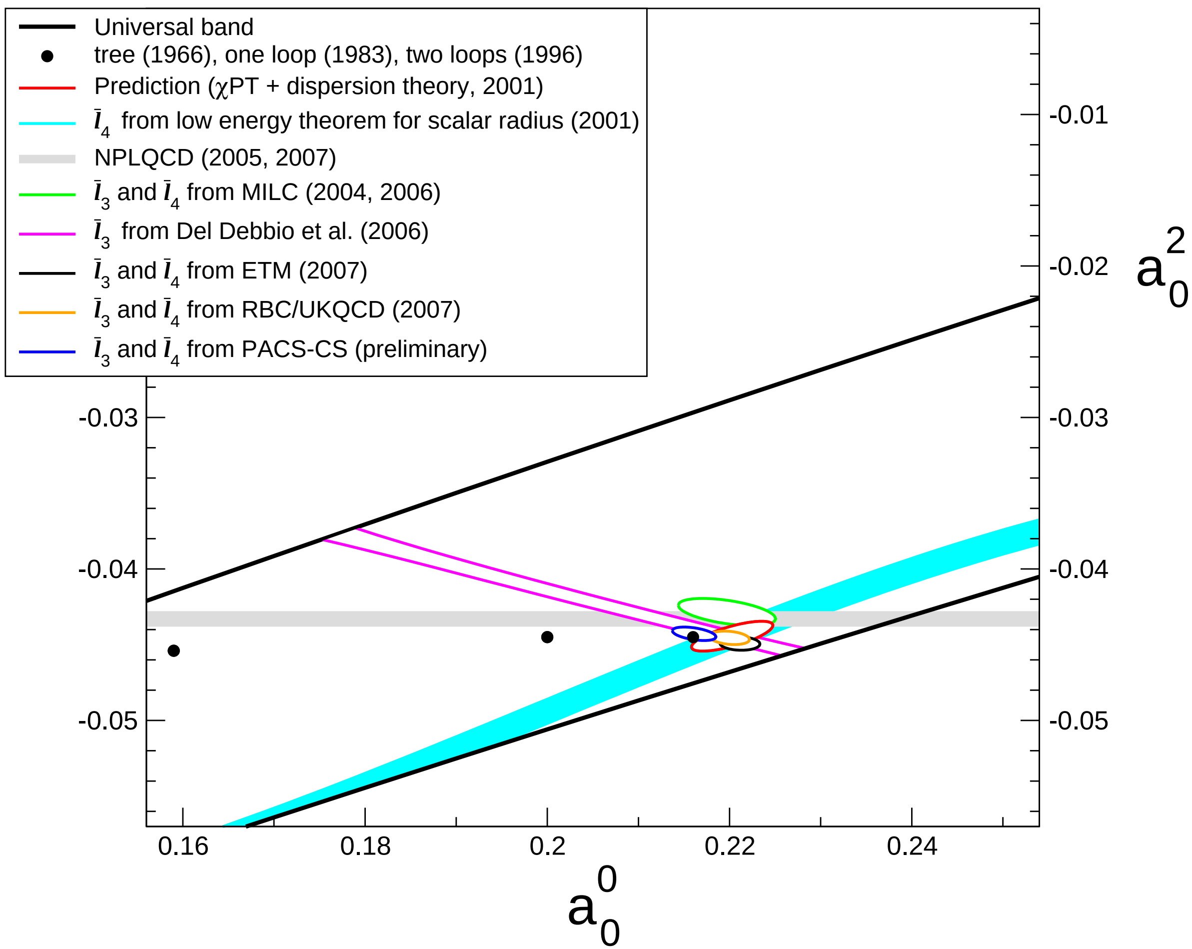
# *Lattice result for $a_0^2$*

- *lattice allows direct measurement of $a_0^2$ via volume dependence of energy levels*



# Consequence of lattice results for $\ell_3$, $\ell_4$

- *uncertainty in prediction for $a_0^0$, $a_0^2$ is dominated by the uncertainty in the effective coupling constants $\ell_3$, $\ell_4$*
- *can make use of the lattice results for these*



## Experiments on light flavours at low energy

- production experiments $\pi N \longrightarrow \pi\pi N$, $\psi \longrightarrow \pi\pi\omega \ldots$

  problem: pions are not produced in vacuo

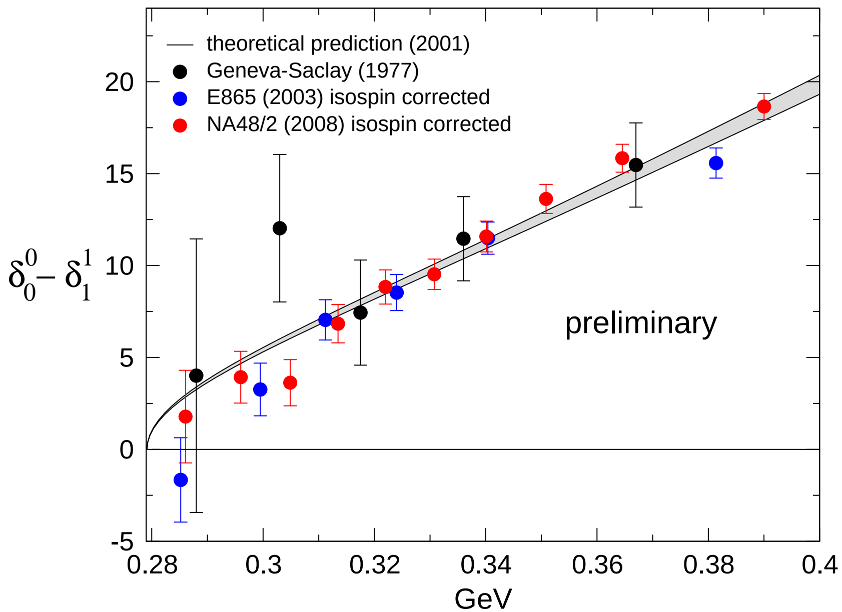
$\Rightarrow$ extraction of $\pi\pi$ scattering amplitude not simple

  accuracy rather limited

- $K^{\pm} \longrightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ data: CERN-Saclay, E865, NA48/2
- $K^{\pm} \longrightarrow \pi^{0}\pi^{0}\pi^{\pm}$ cusp near threshold: NA48/2
- $\pi^{+}\pi^{-}$ atoms, DIRAC

# $K_{e4}$ decay

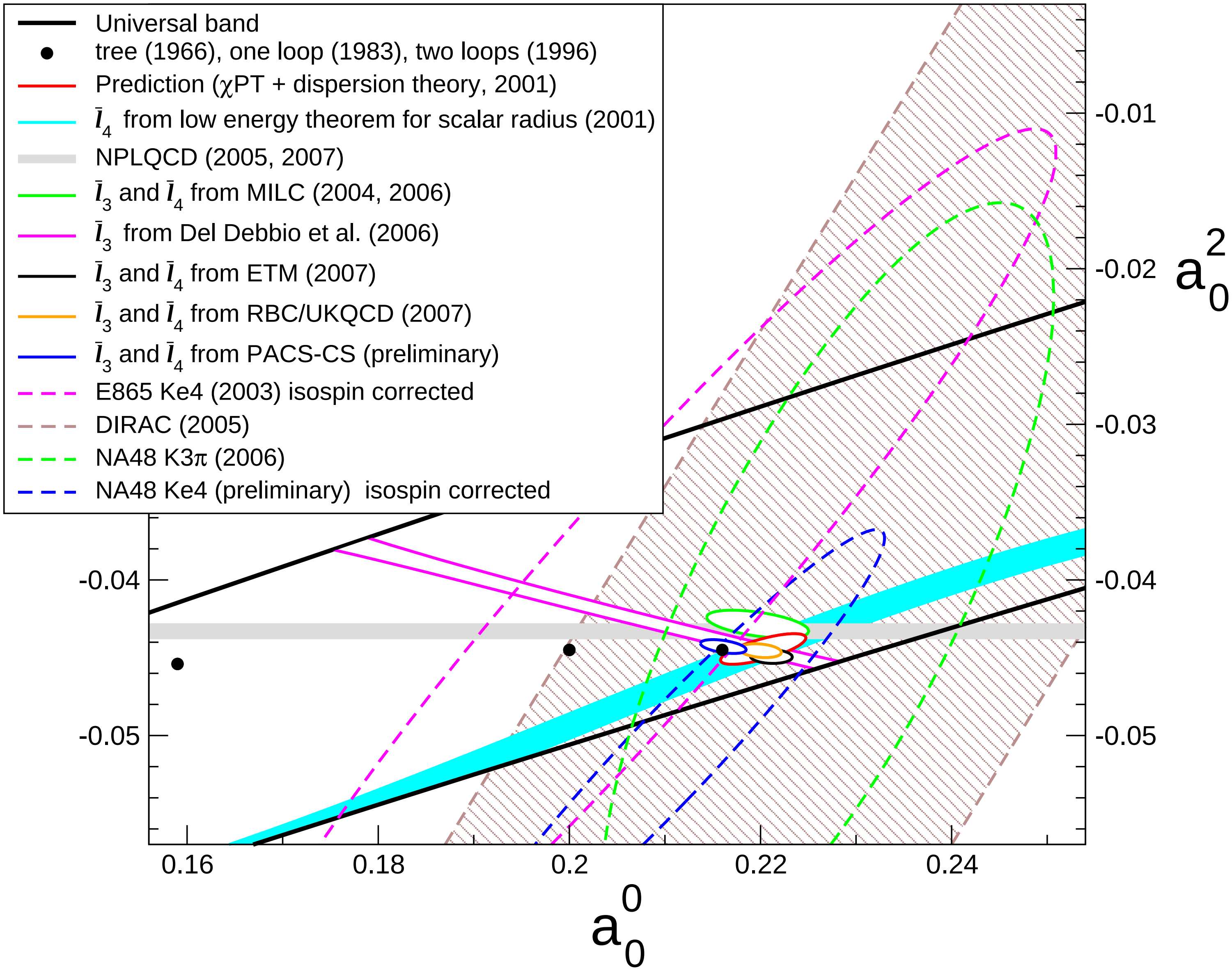
- $K \to \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- theory predicts $\delta_0^0 - \delta_1^1$ as function of energy




- there was a discrepancy here, because a pronounced isospin breaking effect from $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

*Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007*

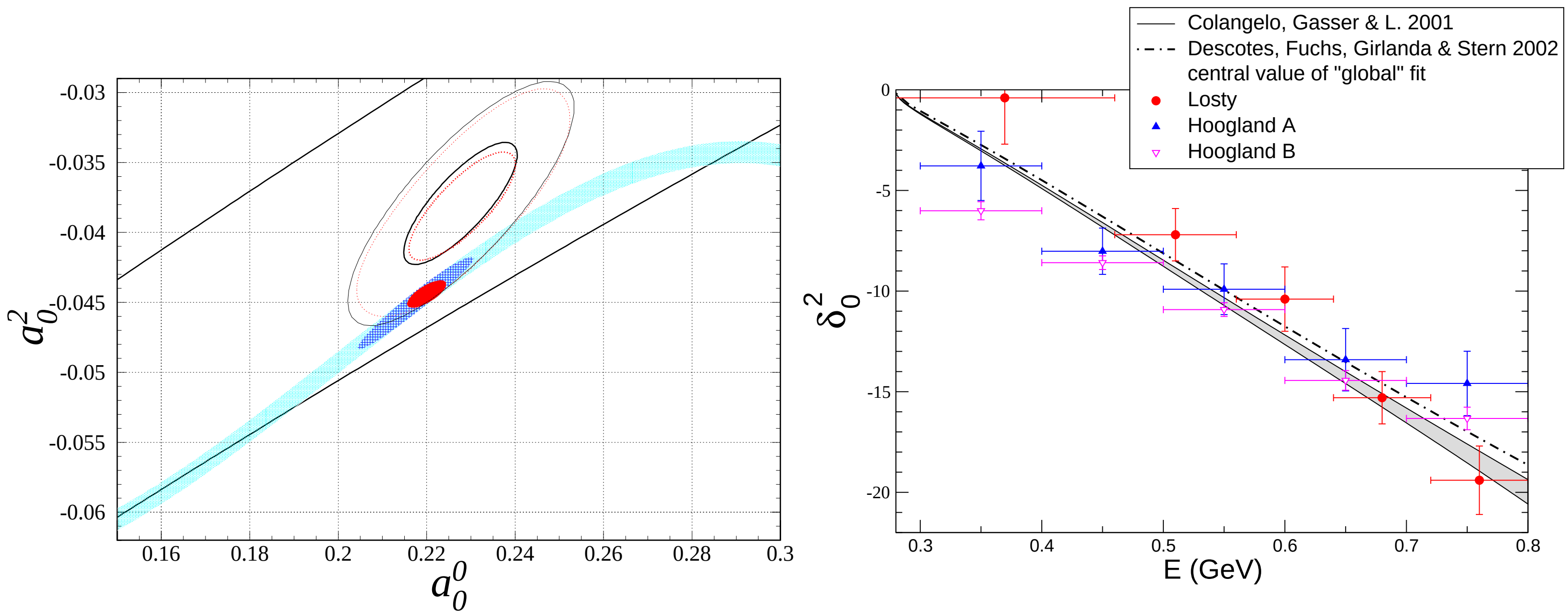
# $a_0^0, a_0^2$ : prediction, lattice & experiment



# Conflict with $\pi\pi$ data on exotic S-wave

- *fit to the data on $\pi\pi$ scattering (Losty, Hoogland A) disagrees with the theoretical prediction by about $2\,\sigma$*

*Descotes-Genon, Fuchs, Girlanda & Stern 2002*




- *if the DFGS analysis is applied to the new precision data from NA48/2, the disagreement may well persist: $K$-decay and the lifetime of pionium give a strong constraint only on $a_0^0 - a_0^2$*

# Conclusions for SU(2)×SU(2)

- *expansion in powers of $m_u, m_d$ yields a very accurate low energy representation of QCD*
- *lattice results confirm the GMOR relation*

$\Rightarrow$ $M_\pi$ *is dominated by the contribution from the quark condensate*

$\Rightarrow$ *energy gap of QCD is understood very well*

- *lattice approach allows an accurate measurement of the effective coupling constant $\ell_3$ already now*
- *even for $\ell_4$, the lattice starts becoming competitive with dispersion theory*
- *expect significant results for effective coupling constants of NNLO in SU(2)×SU(2) very soon*
- *despite the beautiful new experimental results in low energy pion physics, theory is still ahead, particularly concerning $a_0^2$ ...*

# Expansion in powers of $m_s$

- *theoretical reasoning:*
  - *the eightfold way is an approximate symmetry*
  - *the only coherent way to understand this within QCD:* $m_s - m_d$, $m_d - m_u$ *can be treated as perturbations*
  - *since* $m_u, m_d \ll m_s$

  $\Rightarrow$ $m_s$ *can be treated as a perturbation*

  $\Rightarrow$ *expect expansion in powers of* $m_s$ *to work, but convergence to be comparatively slow*

- *in principle, this can now also be checked on the lattice*
  - *those effective coupling constants that are relevant for* $M_\pi$, $M_K$, $F_\pi$, $F_K$ *have been determined on the lattice, both for SU(2)×SU(2) and SU(3)×SU(3)*

  LO: $B, F, B_0, F_0$ NLO: $\ell_3, \ell_4, L_4, L_5, L_6, L_8$

## Paramagnetic inequalities

- consider the limit $m_u, m_d \longrightarrow 0$, $m_s$ physical
  - $F$ is value of $F_\pi$ in this limit
  - $\Sigma$ is value of $|\langle 0|\, \bar{u}u |\, |0\rangle$ in this limit
  - $B$ is value of $M_\pi^2/(m_u + m_d)$ in this limit
- exact relation: $\Sigma = F^2 B$
- inequalities set up by Jan and collaborators:
  both $F$ and $\Sigma$ should decrease if $m_s$ is taken smaller

$$F > F_0\,,\ \Sigma > \Sigma_0$$

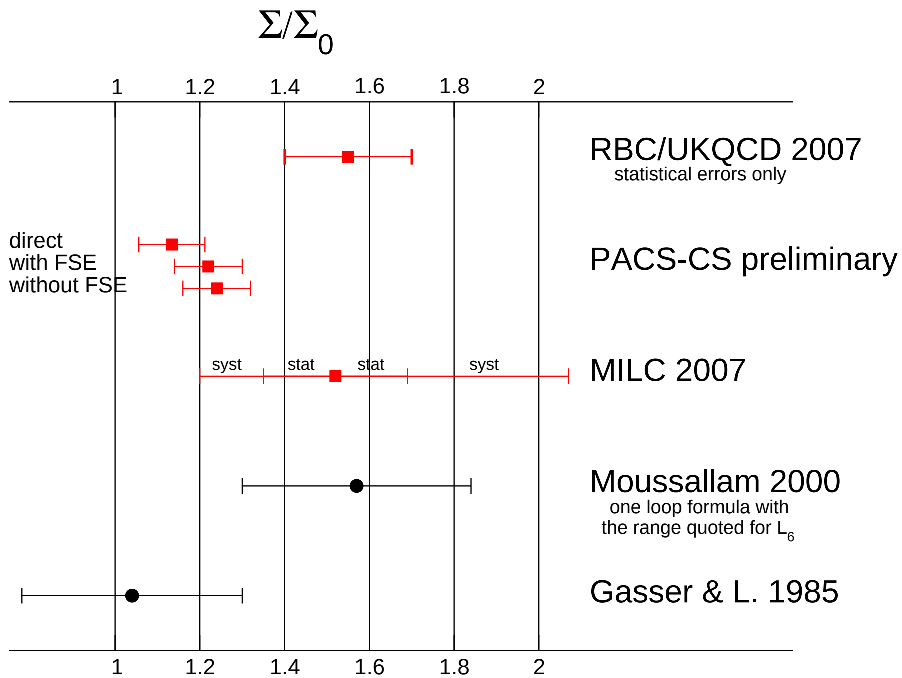
for a recent discussion, see S. Descotes-Genon at Lattice 2007

- if $N_c$ is taken large, $F, \Sigma$ and $B$ become independent of $m_s$

$\Rightarrow$ $(F/F_0 - 1), (\Sigma/\Sigma_0 - 1), (B/B_0 - 1)$ violate the OZI rule

- the lattice results confirm the parametric inequalities, but do not yet allow to draw conclusions about the size of the OZI-violations

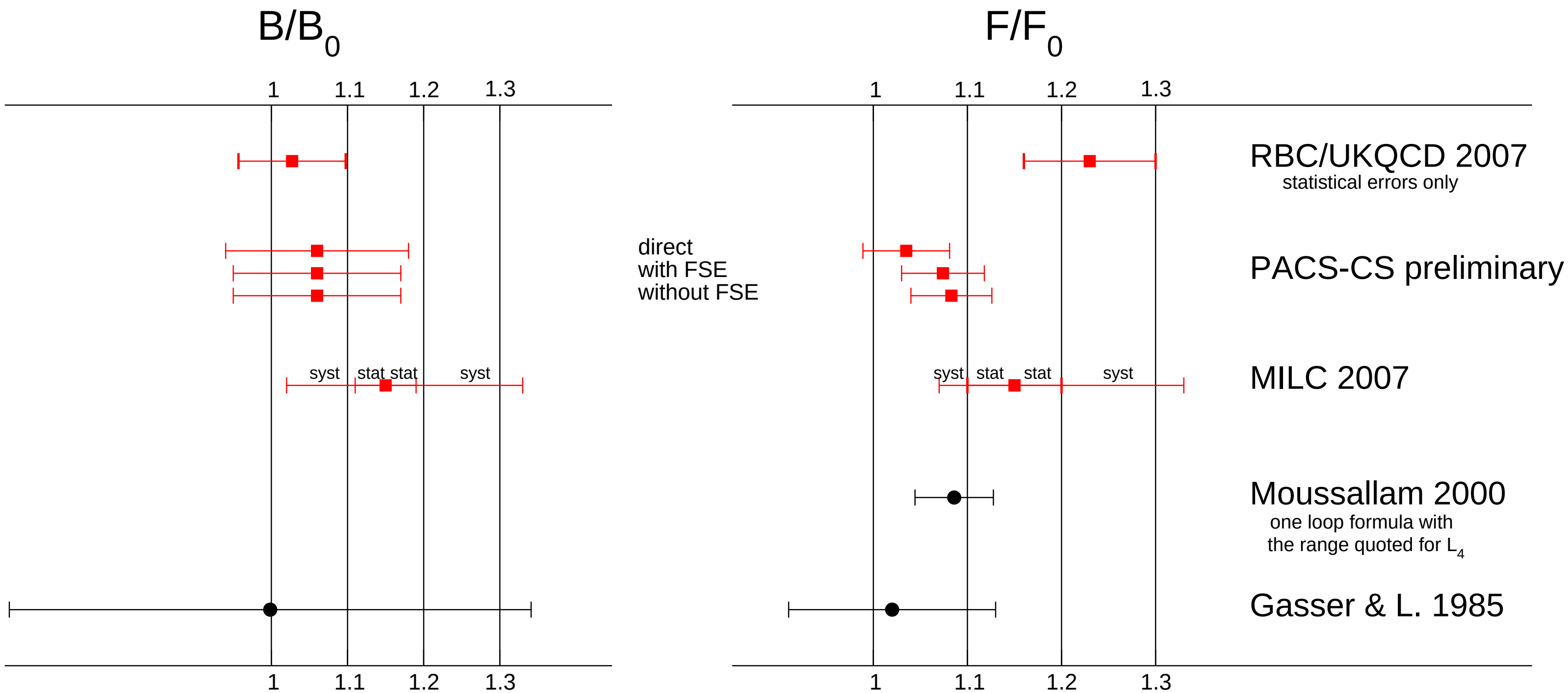
# Condensate




- *central values of RBC/UKQCD and PACS-CS for $\Sigma/\Sigma_0$ lead to qualitatively different conclusions concerning OZI-violations*

$\Rightarrow$ *discrepancy indicates large systematic errors*

# Results for $B$, $F$




- *results for $B$ are coherent, indicate small OZI-violations in $B$*

$\Rightarrow$ $F/F_0$ *is the crucial factor in* $\Sigma = F^2 B$

# Expansion in powers of $m_s$

- $$B = B_0(1 + m_s\, r_1^B + m_s^2\, r_2^B + \ldots)$$
  $$F = F_0(1 + m_s\, r_1^F + m_s^2\, r_2^F + \ldots)$$ *modulo logarithms*
  $$\Sigma = \Sigma_0(1 + m_s\, r_1^\Sigma + m_s^2\, r_2^\Sigma + \ldots)$$
- *expansion only useful if* $1 \gg m_s r_1 \gg m_s^2\, r_2$
- *values of RBC/UKQCD for* $L_4$ *and* $L_6$ *imply*

$$m_s r_1^B = 0.043 \pm 0.033$$
$$m_s r_1^F = 0.20 \pm 0.04$$ *purely statistical errors*
$$m_s r_1^\Sigma = 0.43 \pm 0.09$$

$\Rightarrow$ *for the central values of RBC/UKQCD, the expansion looks like*

$$B = B_0(1 + 0.043 - 0.013)$$
$$F = F_0(1 + 0.20 + 0.03)$$
$$\Sigma = \Sigma_0(1 + 0.43 + 0.12)$$

$\Rightarrow$ *large OZI-violations, but expansion in powers of* $m_s$ *looks OK*

# Conclusions for SU(3)$\times$SU(3)

- *the available lattice data allow for very juicy OZI-violations, but are also consistent with* $B/B_0 \simeq F/F_0 \simeq \Sigma/\Sigma_0 \simeq 1$
- *if the central value of RBC/UKQCD for* $F/F_0$ *were confirmed within small uncertainties, we would be faced with a qualitative puzzle:*
  - $F_\pi$ *is the pion wave function at the origin*
  - $F_K$ *is a little larger because one of the two valence quarks moves more slowly* $\longrightarrow$ *wave function more narrow* $\longrightarrow$ *higher at the origin:* $F_K/F_\pi \simeq 1.19$
  - $F/F_0 = 1.23$ *indicates that the wave function is more sensitive to the mass of the sea quarks than to the mass of the valence quarks . . . very strange* $\longrightarrow$ *extraordinarily interesting*
- *note: most of the numbers quoted are preliminary, errors purely statistical, continuum limit, finite size effects, . . .*

# *Puzzling results on* $K_L \to \pi\mu\nu$

- *hadronic matrix element of weak current:*

$$\langle K^0|\bar{u}\gamma^\mu s|\pi^-\rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

- *scalar form factor* $\sim \langle K^0|\partial_\mu(\bar{u}\gamma^\mu s)|\pi^-\rangle$

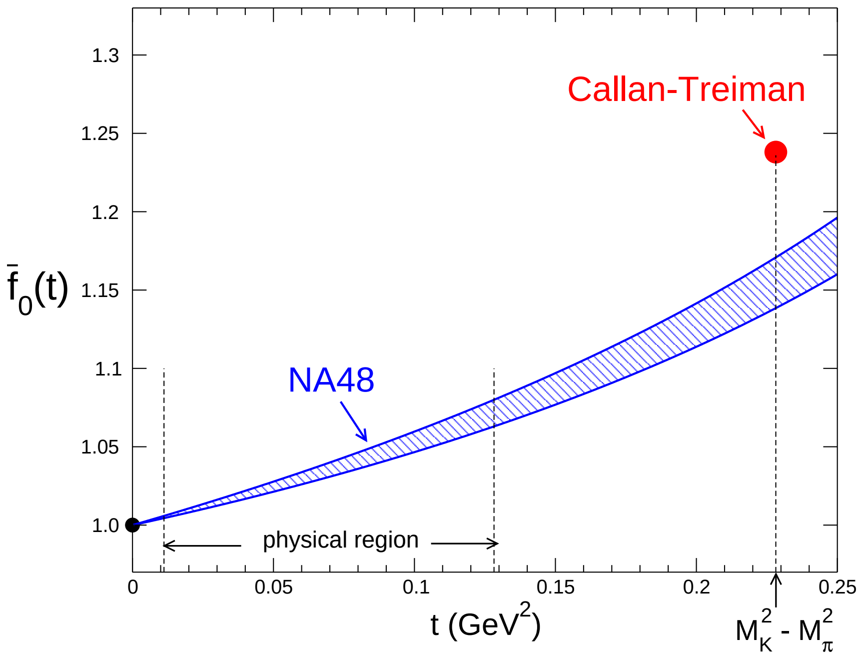
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- *low energy theorem of Callan and Treiman (1966):*

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi}\left\{1 + O(m_u, m_d)\right\} \simeq 1.19$$

$f_0(0) = f_+(0) \simeq 0.96$ *relevant for determination of* $V_{us}$

# *Comparison with experiment*




*NA48, Phys. Lett. B647 (2007) 341*

*141 authors,* $2.3 \times 10^6$ *events*

*plot shows normalized scalar form factor*

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- *Callan-Treiman relation in this normalization:*

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)}$$

- *experimental value:* $\dfrac{F_K}{F_\pi f_+(0)} = 1.2446 \pm 0.0041$

*Bernard and Passemar 2008*

# Corrections, extrapolation

- *Callan-Treiman-relation is exact only for* $m_u, m_d \to 0$

  *corrections of NLO were worked out long ago, are tiny* *Gasser & L. 1985*

  *form factor now known to NNLO* *Post & Schilcher 2002,*

  *Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005*

  *including the uncertainties from* $m_u, m_d \neq 0$:

  $$\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$$ *Bernard & Passemar 2008*

$\Rightarrow$ *cannot blame the discrepancy on the prediction*

- *CT–point is not in physical region, extrapolation needed*

  *curvature can be calculated with dispersion theory*

  *Jamin, Oller & Pich 2004, Bernard, Oertel, Passemar & Stern 2006*

$\Rightarrow$ *cannot blame the discrepancy on the extrapolation*

# Slope of the scalar form factor

- *definition of the slope* $$\bar{f}_0(t) = 1 + \frac{\lambda_0\, t}{M_{\pi^+}^2} + O(t^2)$$
- *Callan-Treiman-relation implies sharp prediction:*

$$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$

*Jamin, Oller & Pich 2004*

- *Update with current experimental information*

$$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

*Bernard, Oertel, Passemar & Stern, preliminary*

- *To be compared with the result of NA48:*

$$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$

*Fit with dispersive representation of BOPS*

# Implications

- NA48 data on $K_L \longrightarrow \pi\mu\nu$ disagree with SM

  if confirmed, the implications are dramatic:

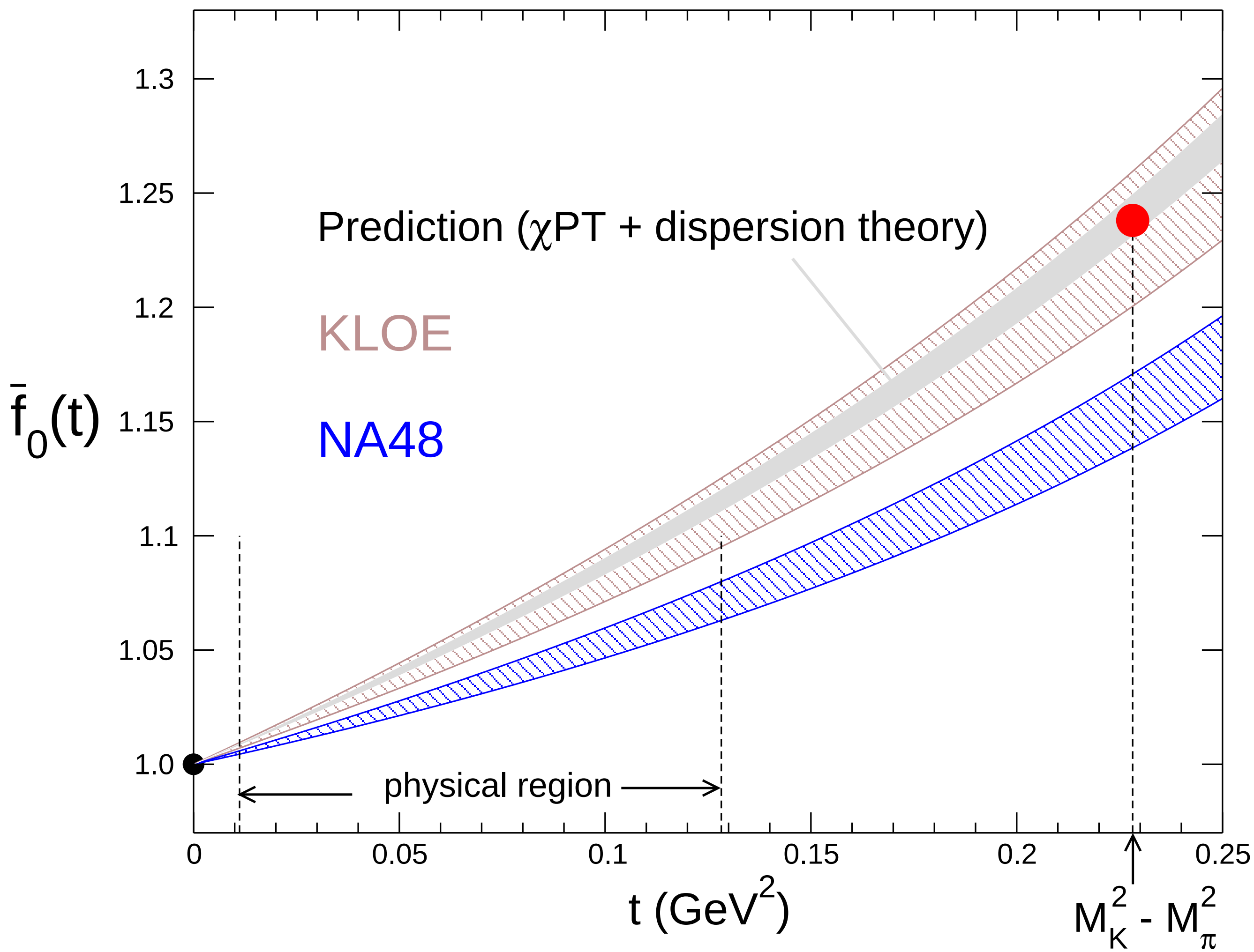
$\Rightarrow$ right-handed currents ? Bernard, Oertel, Passemar & Stern 2006

there are not many places where the SM disagrees with observation

need to investigate these carefully

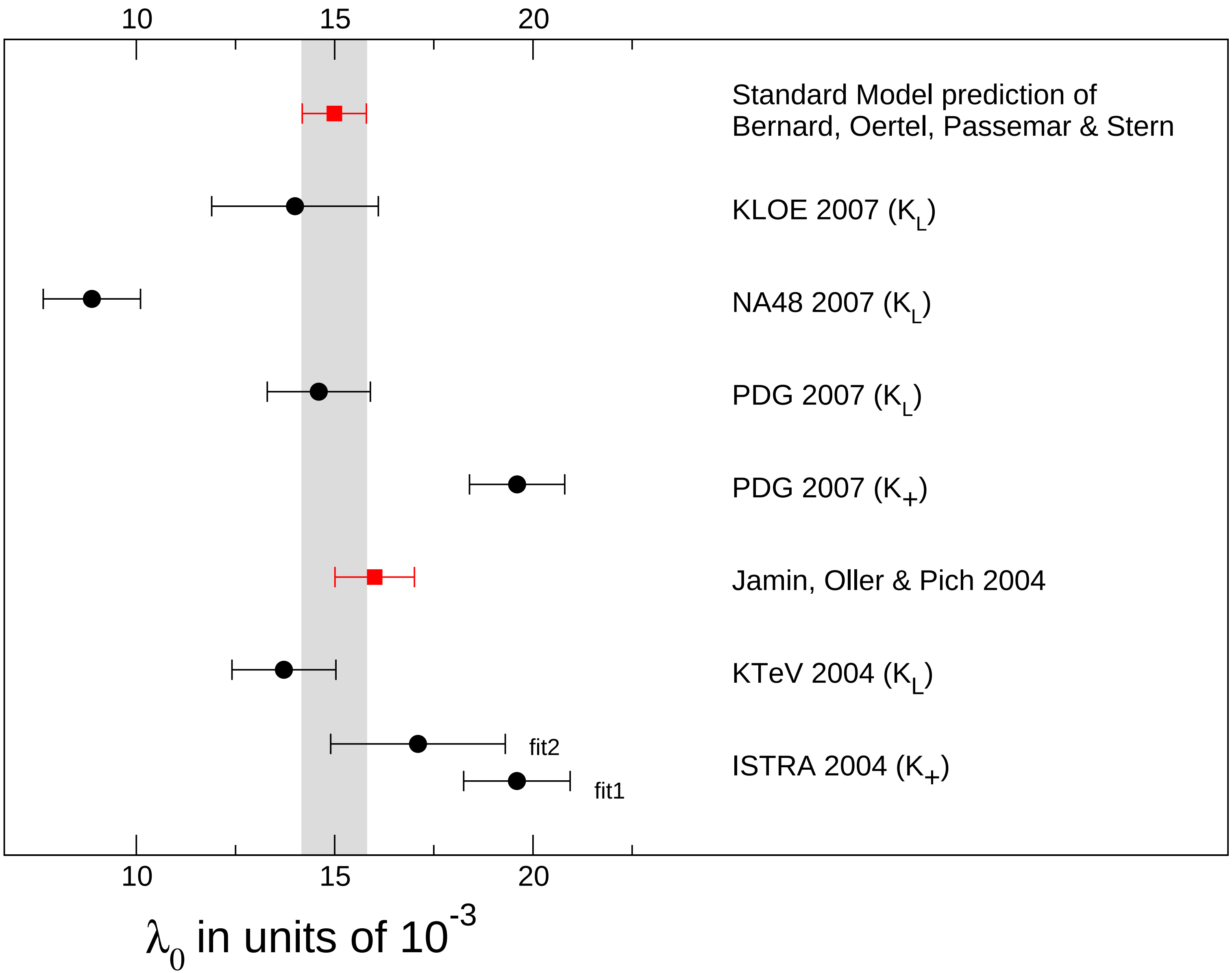
at low energies, high precision is required

# New data from KLOE

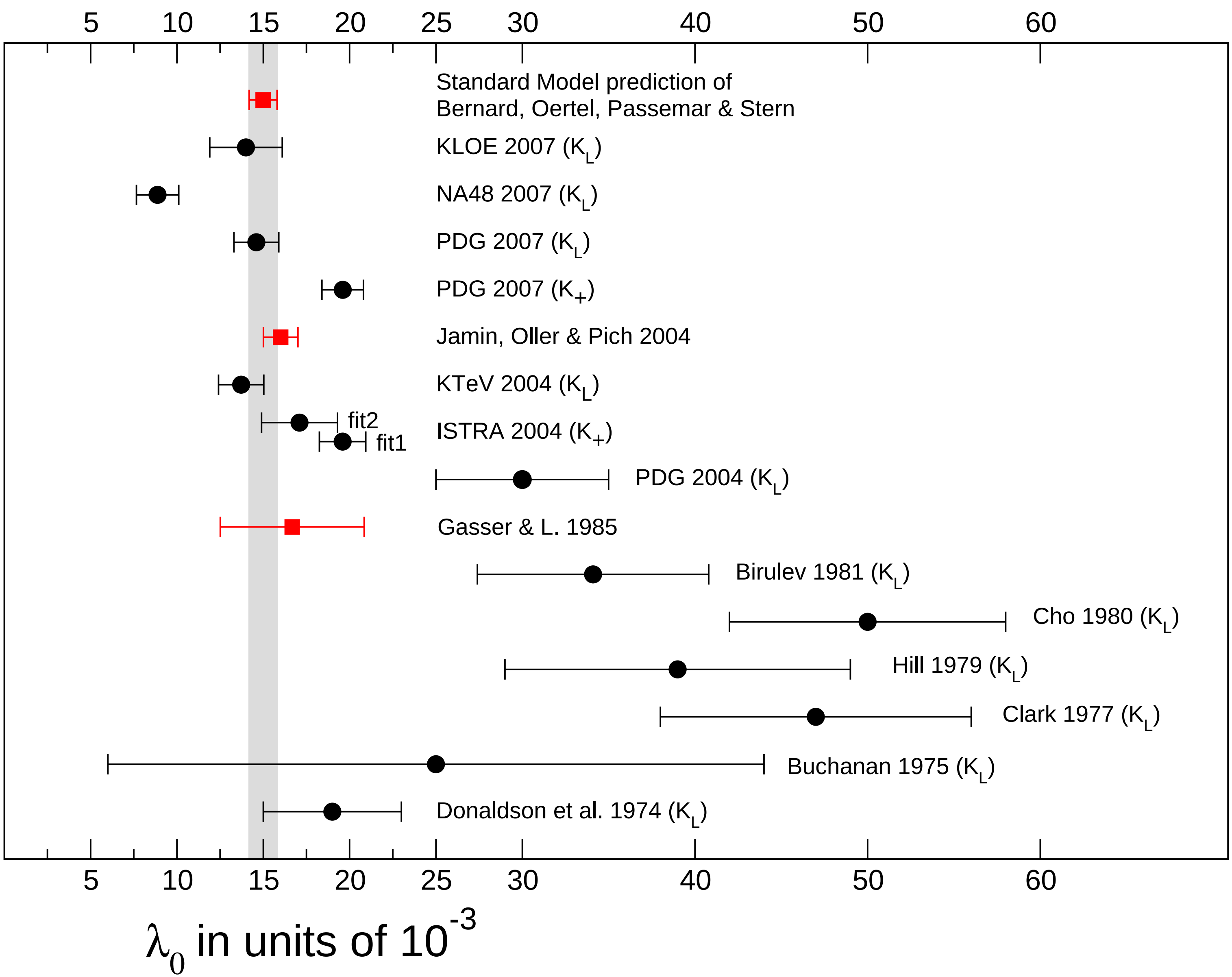



*I thank Emilie Passemar for some of the material shown in this figure*

# *Comparison of results for the slope*



# Older measurements



# Conclusions for $K \to \mu\nu\pi$

- *experiment is difficult, discrepancies need to be resolved*

  *Donaldson 1974:* $\mathbf{1.6 \times 10^6}$ *events*

  *ISTRA 2004:* $\mathbf{0.54 \times 10^6}$ *events*

  *KTeV 2004:* $\mathbf{1.5 \times 10^6}$ *events*

  *NA48 2007:* $\mathbf{2.3 \times 10^6}$ *events*

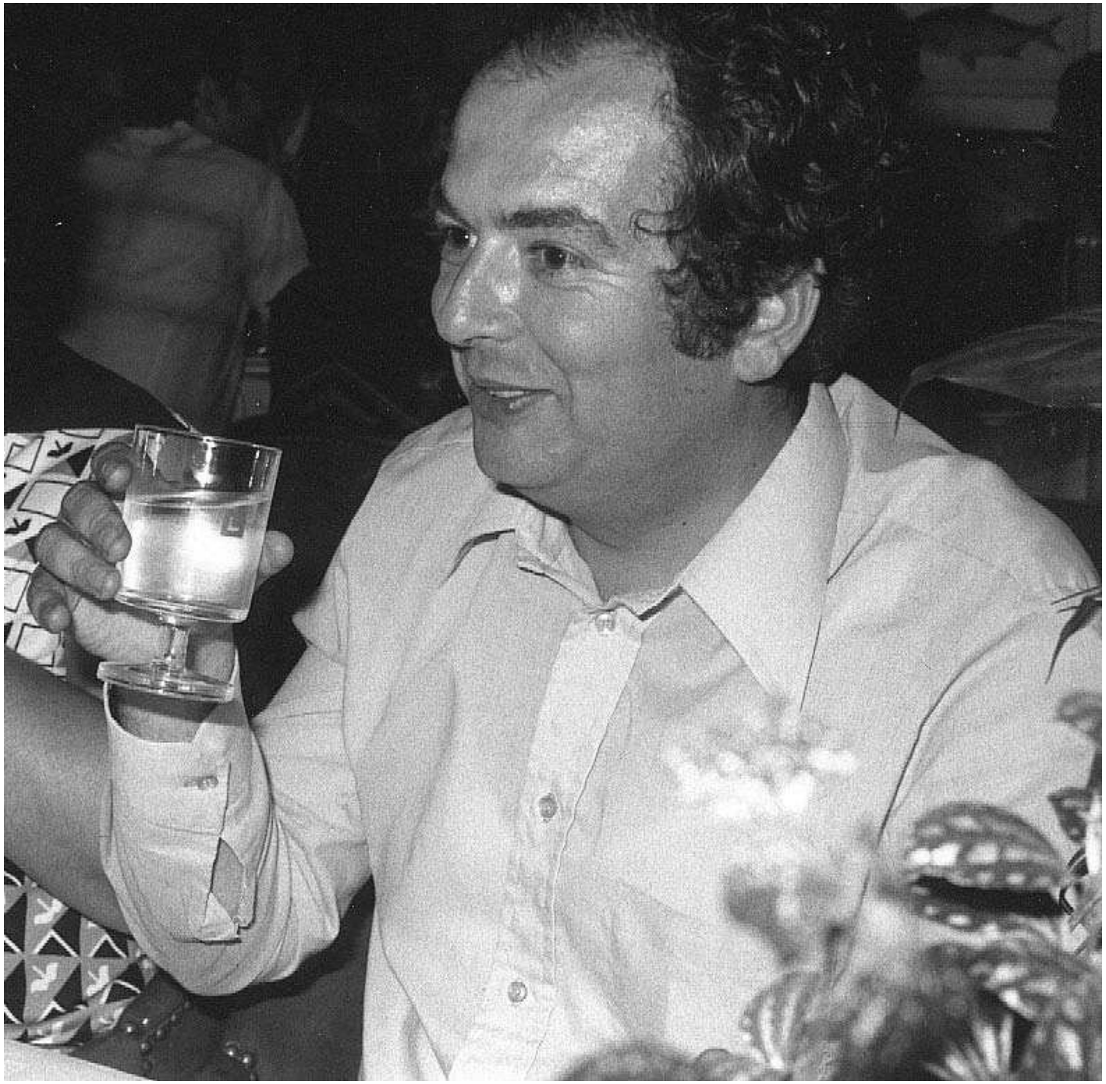
- *dispersion theory fixes the shape of the form factors*
  *publishing linear fits is nonsensical*

- *Jan and collaborators worked hard to improve the analysis of the KTeV collaboration, paper to appear soon*

- *NA48 should improve their data analysis as well . . . and extend it to charged kaons (isospin breaking)*

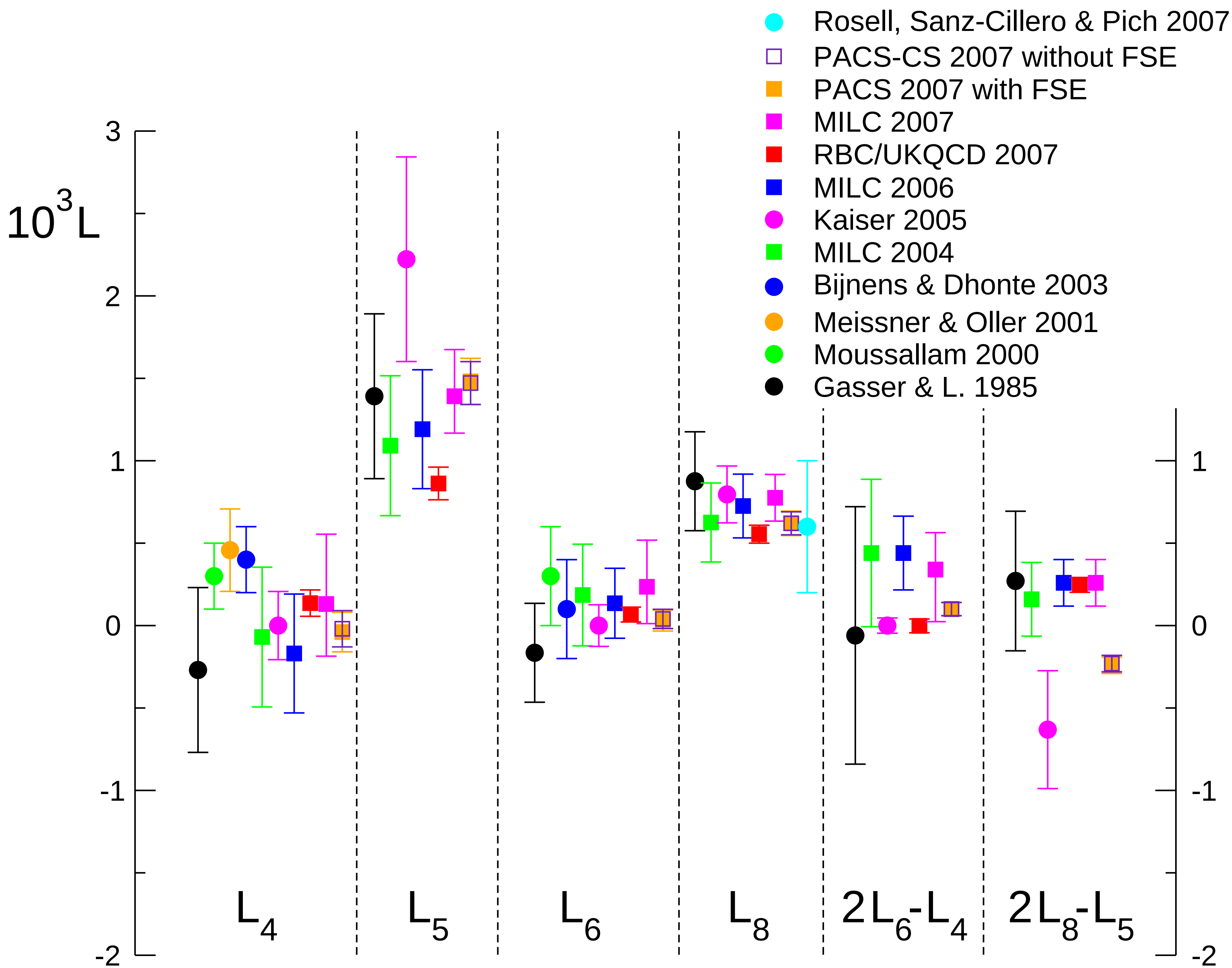
$\Rightarrow$ *plenty of work ahead for all of us !*

# na zdraví !

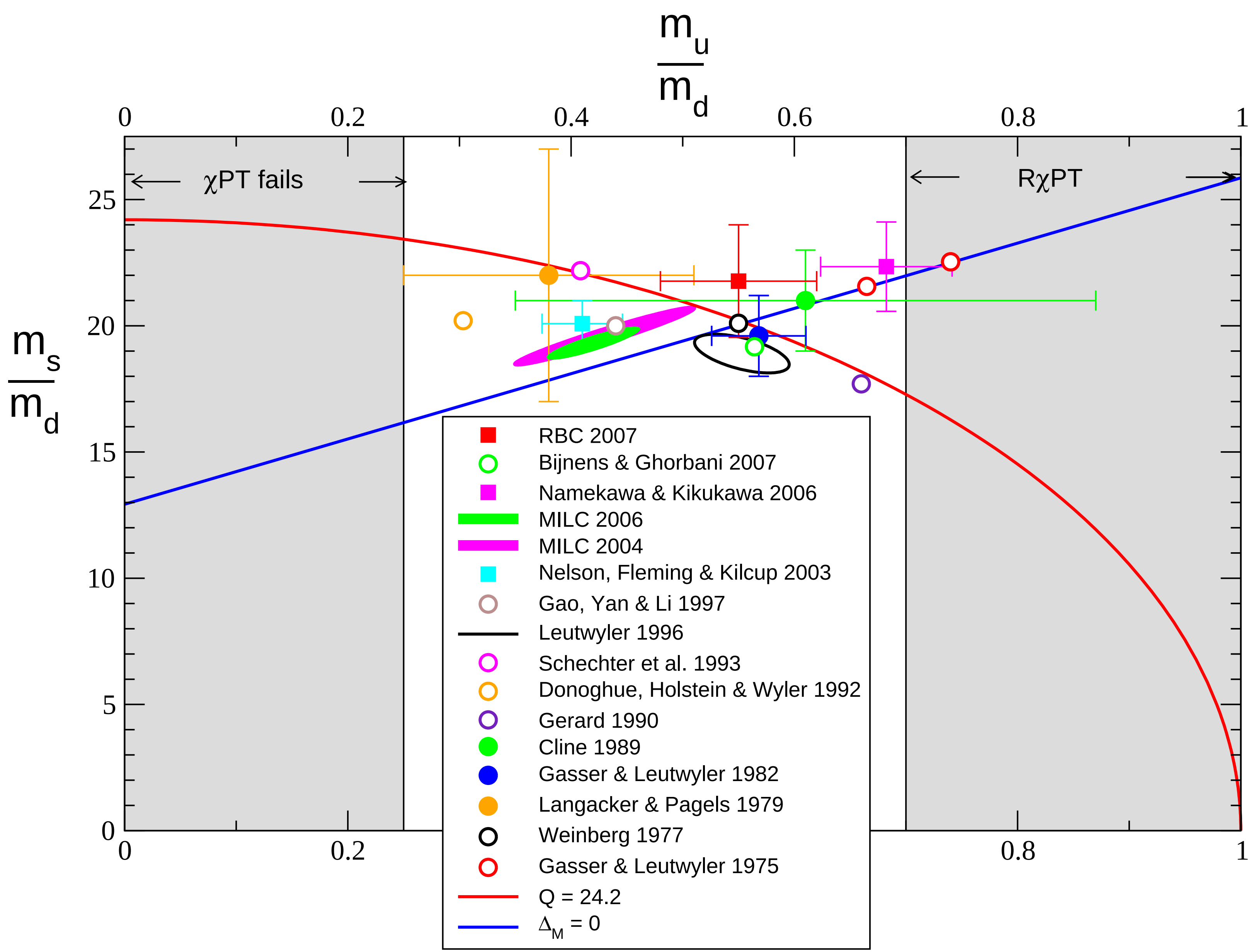


# SPARES

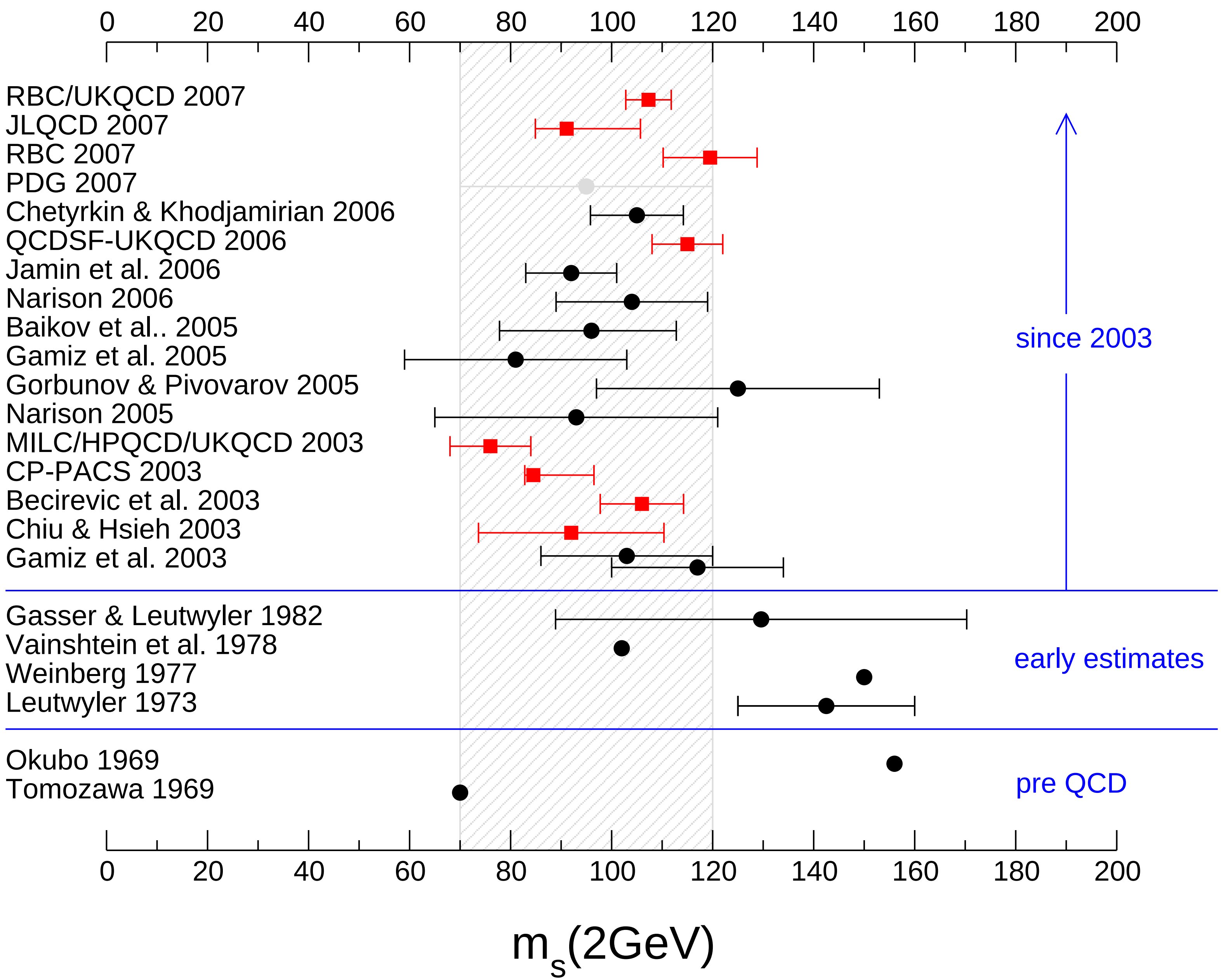
# *Effective coupling constants of SU(3)×SU(3)*



# *Quark mass ratios*



# *Value of $m_s$*


0 20 40 60 80 100 120 140 160 180 200
RBC/UKQCD 2007
JLQCD 2007
RBC 2007
PDG 2007
Chetyrkin & Khodjamirian 2006
QCDSF-UKQCD 2006
Jamin et al. 2006
Narison 2006
Baikov et al.. 2005
Gamiz et al. 2005
Gorbunov & Pivovarov 2005
Narison 2005
MILC/HPQCD/UKQCD 2003
CP-PACS 2003
Becirevic et al. 2003
Chiu & Hsieh 2003
Gamiz et al. 2003
since 2003
Gasser & Leutwyler 1982
Vainshtein et al. 1978
Weinberg 1977
Leutwyler 1973
early estimates
Okubo 1969
Tomozawa 1969
pre QCD
0 20 40 60 80 100 120 140 160 180 200
$m_s$(2GeV)