Recent developments in the physics of the light quarks

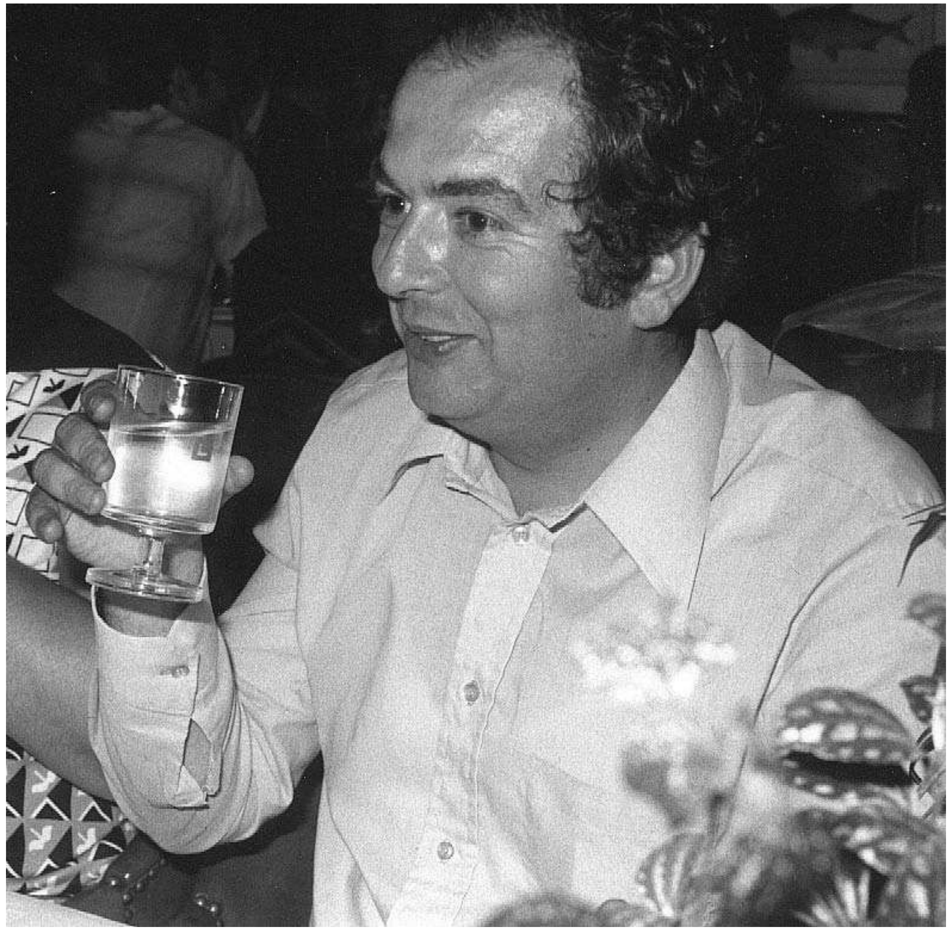
H. Leutwyler

University of Bern

QCD08

Montpellier, July 10, 2008

In memorian Jan Stern



29. 6. 1942 – 2. 7. 2008

Plan of talk

1. *Lattice results for effective coupling constants of SU(2)×SU(2)*
2. *Comparison with experiment:* K_{e4}, $K_{3\pi}$*, pionic atoms*
3. *Lattice results concerning SU(3)×SU(3), Zweig rule*
4. *Exact formula for resonances, illustration:* M_σ, Γ_σ
5. *Puzzling results for* $K_{\mu 3}$ *decay: clash with Callan-Treiman*

Energy gap of QCD

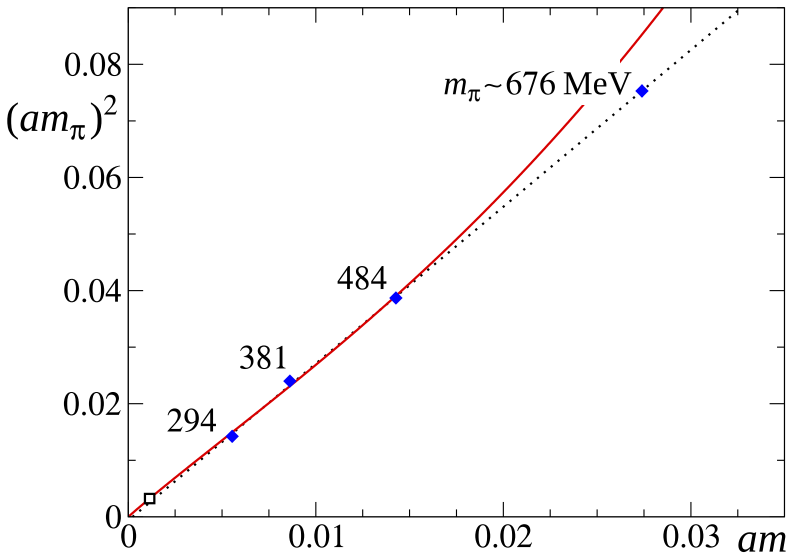
- *Main characteristic of QCD at low energies: energy gap is very small,* $M_\pi \simeq$ *140 MeV*
- *Nambu found out why this is so: the strong interaction has a hidden, approximate symmetry* *Nambu 1960*
- *Gap is determined by the masses of the two lightest quarks*

Gell-Mann, Oakes & Renner 1968

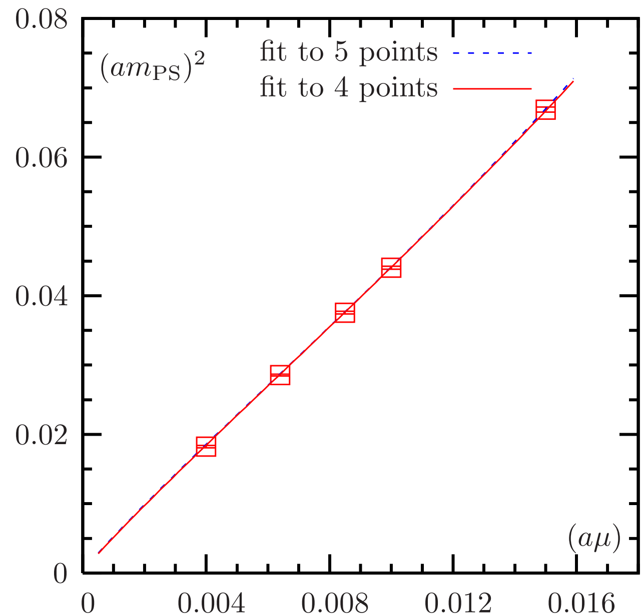
$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \text{explicit}}}{+} m_d) \times |\langle 0|\, \underset{\substack{\Uparrow \\ \text{spontaneous}}}{\bar{u}u}\, |0\rangle| \times \frac{1}{F_\pi^2}$$

Checking the GMOR formula on the lattice

- *Can determine* M_π *as a function of* $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- *No quenching, quark masses are sufficiently light*

$\Rightarrow$ *Legitimate to use* χ*PT for the extrapolation to the physical values of* m_u, m_d

Expansion of M_π^2 in powers of the quark masses

- *Gell-Mann-Oakes-Renner formula represents leading term of the chiral perturbation series*
- *Disregard isospin breaking, set $m_u = m_d = m$*
- *Expand in powers of m, keeping m_s fixed*
- *At NLO, the expansion contains a logarithm*

Langacker & Pagels 1973, Gasser and Zepeda 1980, Gasser 1981

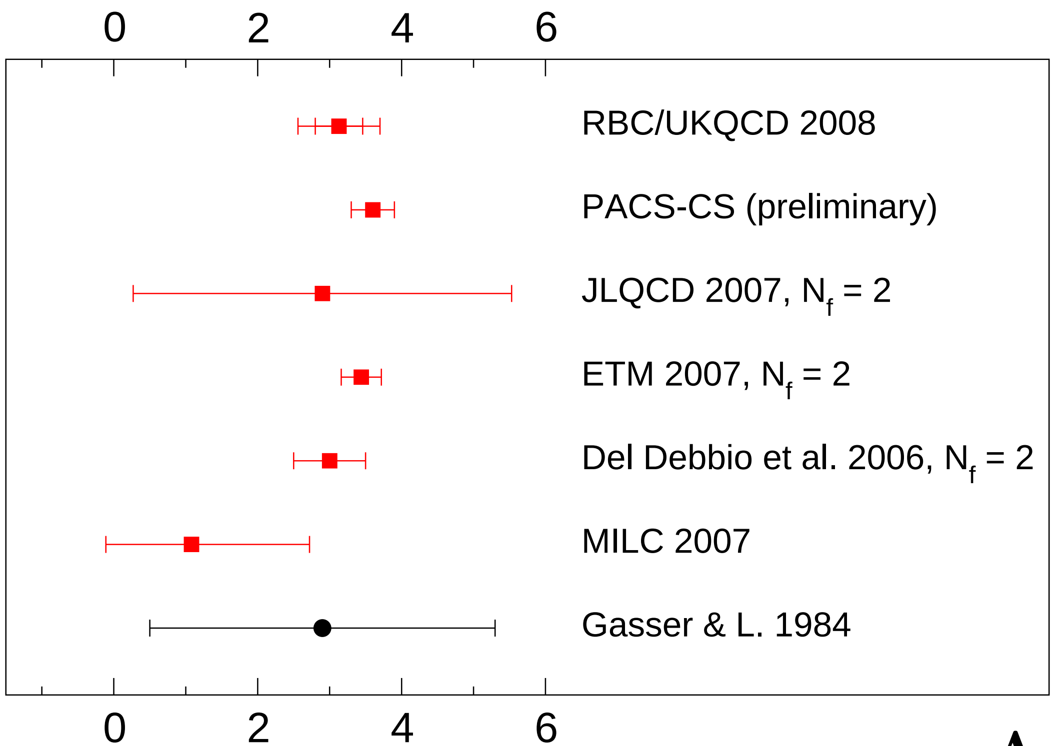
$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

$$M^2 \equiv 2Bm$$

- *Coefficient is determined by the pion decay constant*
  Symmetry does not determine the scale Λ_3
- *Crude result, based on SU(3)×SU(3):*
  0.2 *GeV* $\lesssim \Lambda_3 \lesssim 2$ *GeV*

Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



Horizontal axis shows the value of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

Range for Λ_3 *obtained in 1984 corresponds to* $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of RBC/UKQCD 2008: $\bar{\ell}_3 = 3.13 \underset{stat}{\pm 0.33} \underset{syst}{\pm 0.24}$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

F is value of pion decay constant in limit $m_u, m_d \to 0$

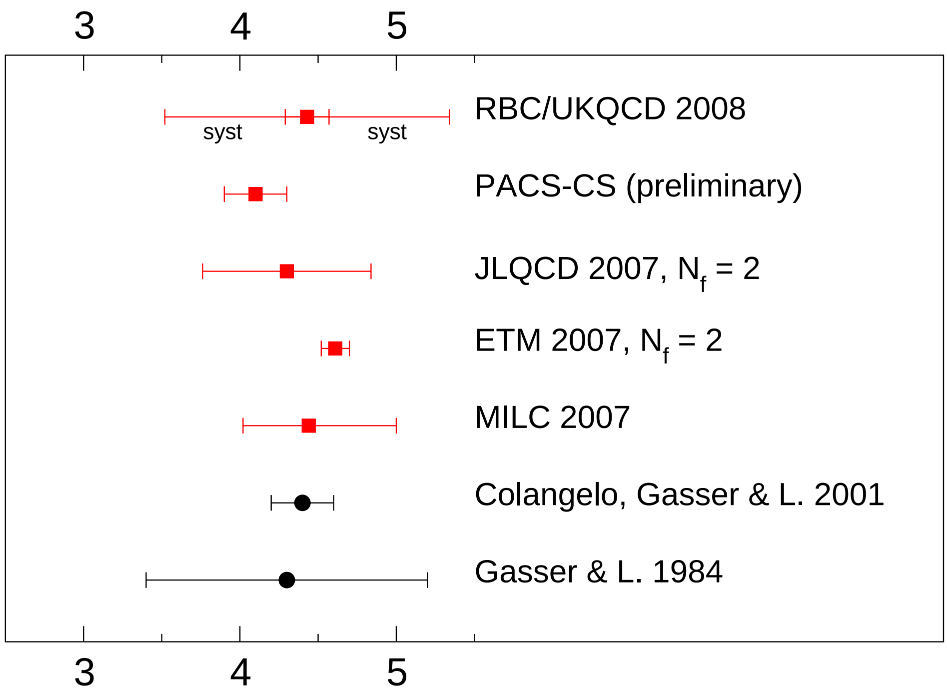
- Structure is the same, coefficients and scale of logarithm are different

- Quark mass dependence of F_π can also be measured on the lattice

$\Rightarrow$ measurement of Λ_4

- Alternative method: determine the scalar form factor of the pion, radius $\langle r^2\rangle_s \Leftrightarrow \bar{\ell}_4$

Lattice results for Λ_4



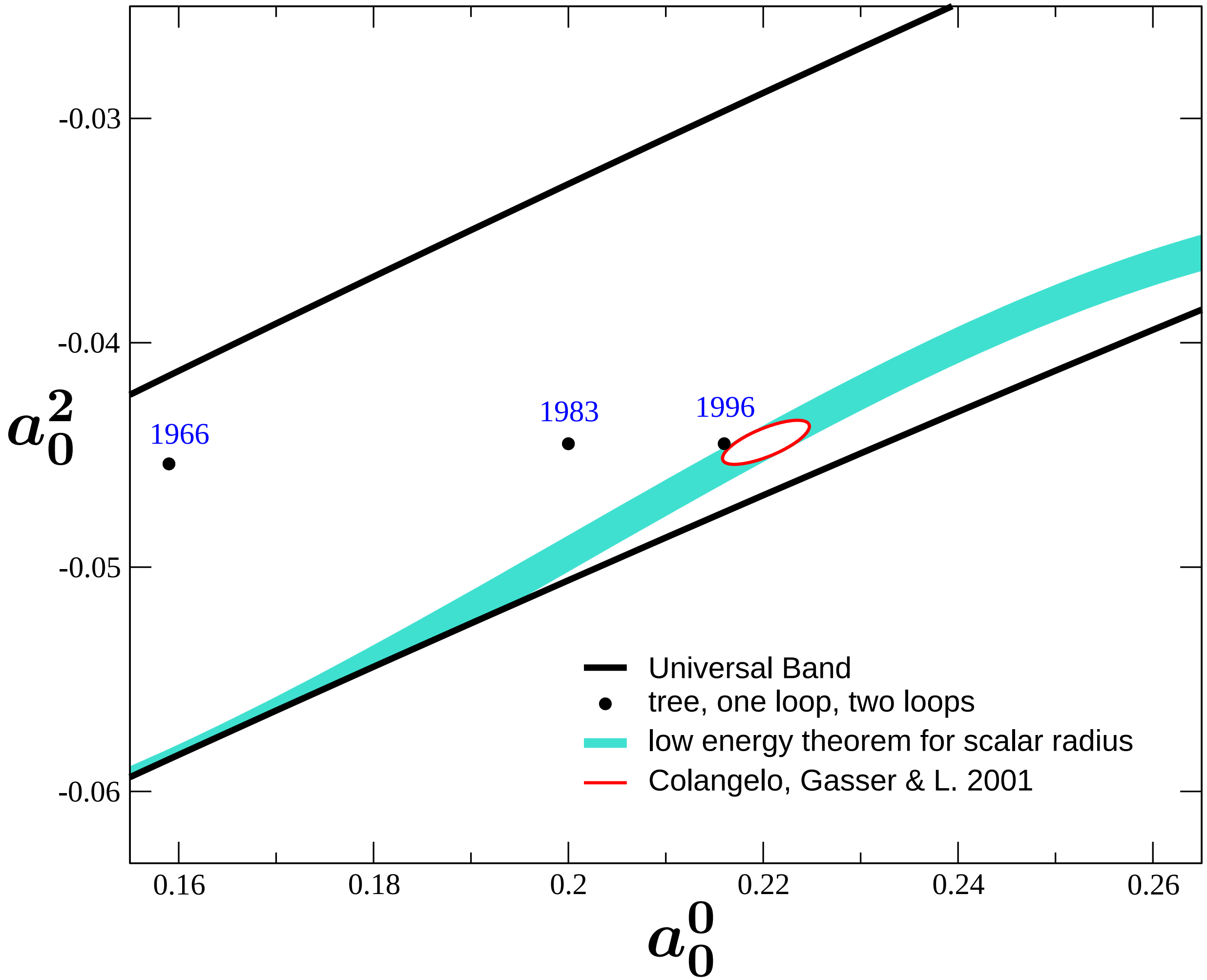
$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

- *Lattice results beautifully confirm the prediction for the sensitivity of F_π to m_u, m_d:*

$$\frac{F_\pi}{F} = 1.072 \pm 0.007$$

Colangelo and Dürr 2004

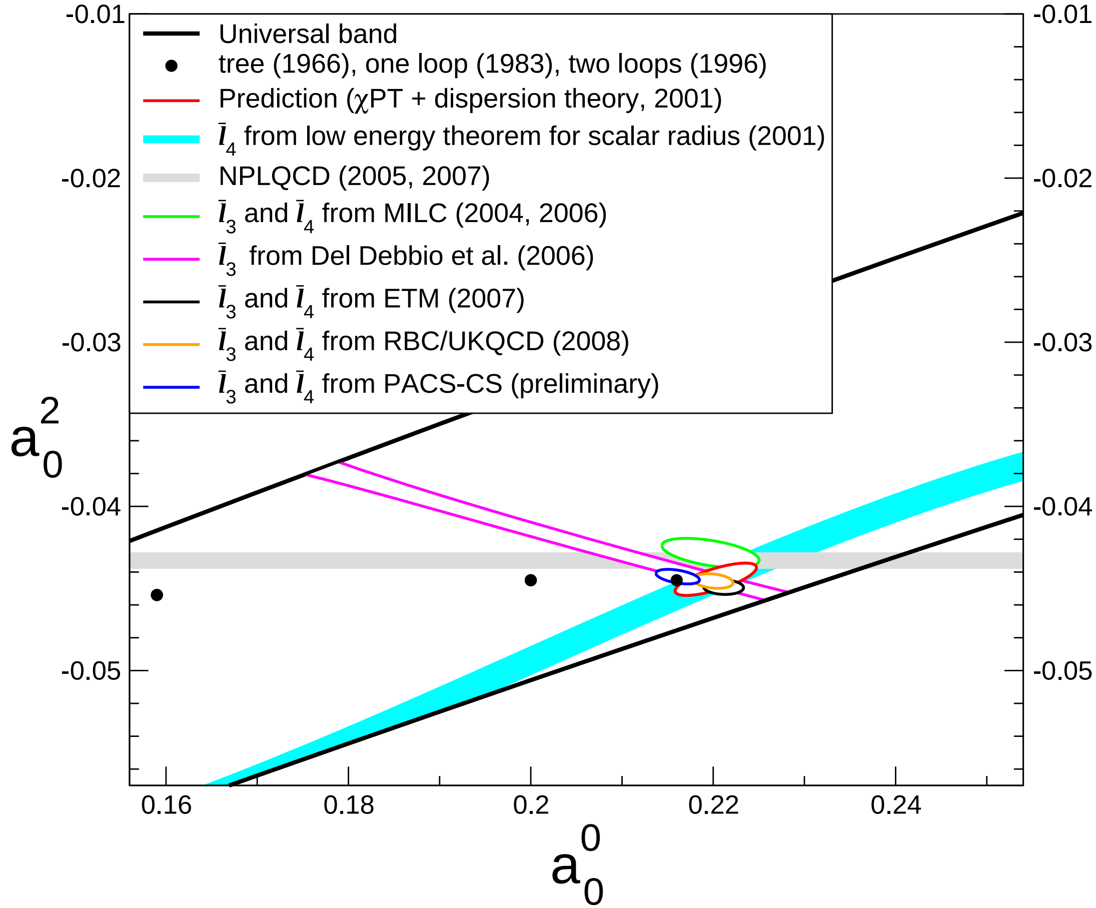
Predictions for the S-wave $\pi\pi$ scattering lengths



Sizable corrections in a_0^0, while a_0^2 nearly stays put

Consequence of lattice results for ℓ_3, ℓ_4

- Uncertainty in prediction for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4
- Can make use of the lattice results for these



Experiments on light flavours at low energy

- *Production experiments* $\pi N \longrightarrow \pi\pi N$, $\psi \longrightarrow \pi\pi\omega \ldots$

  Problem: pions are not produced in vacuo

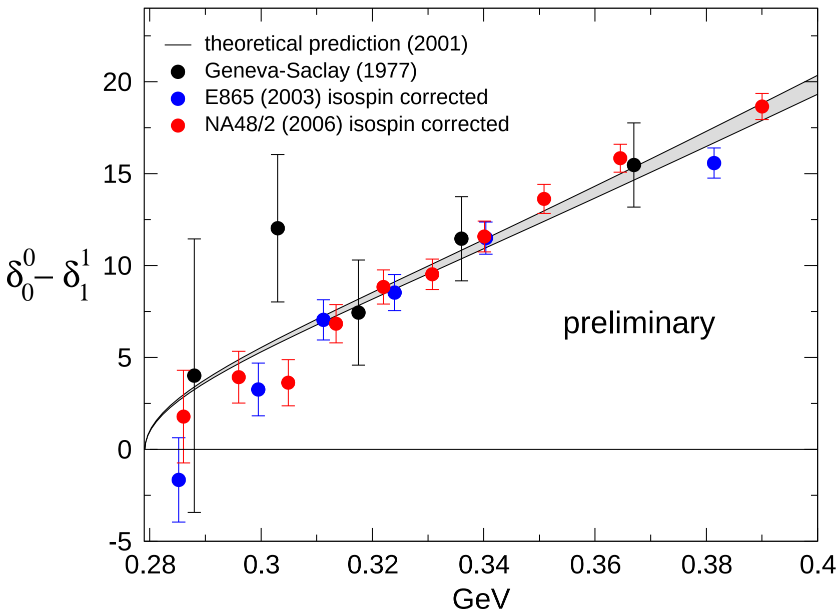
  $\Rightarrow$ *Extraction of* $\pi\pi$ *scattering amplitude not simple*

  Accuracy rather limited

- $K^{\pm} \longrightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ *data: CERN-Saclay, E865, NA48/2*
- $K^{\pm} \longrightarrow \pi^{0}\pi^{0}\pi^{\pm}$ *cusp near threshold: NA48/2*
- $\pi^{+}\pi^{-}$ *atoms, DIRAC*

K_{e4} decay

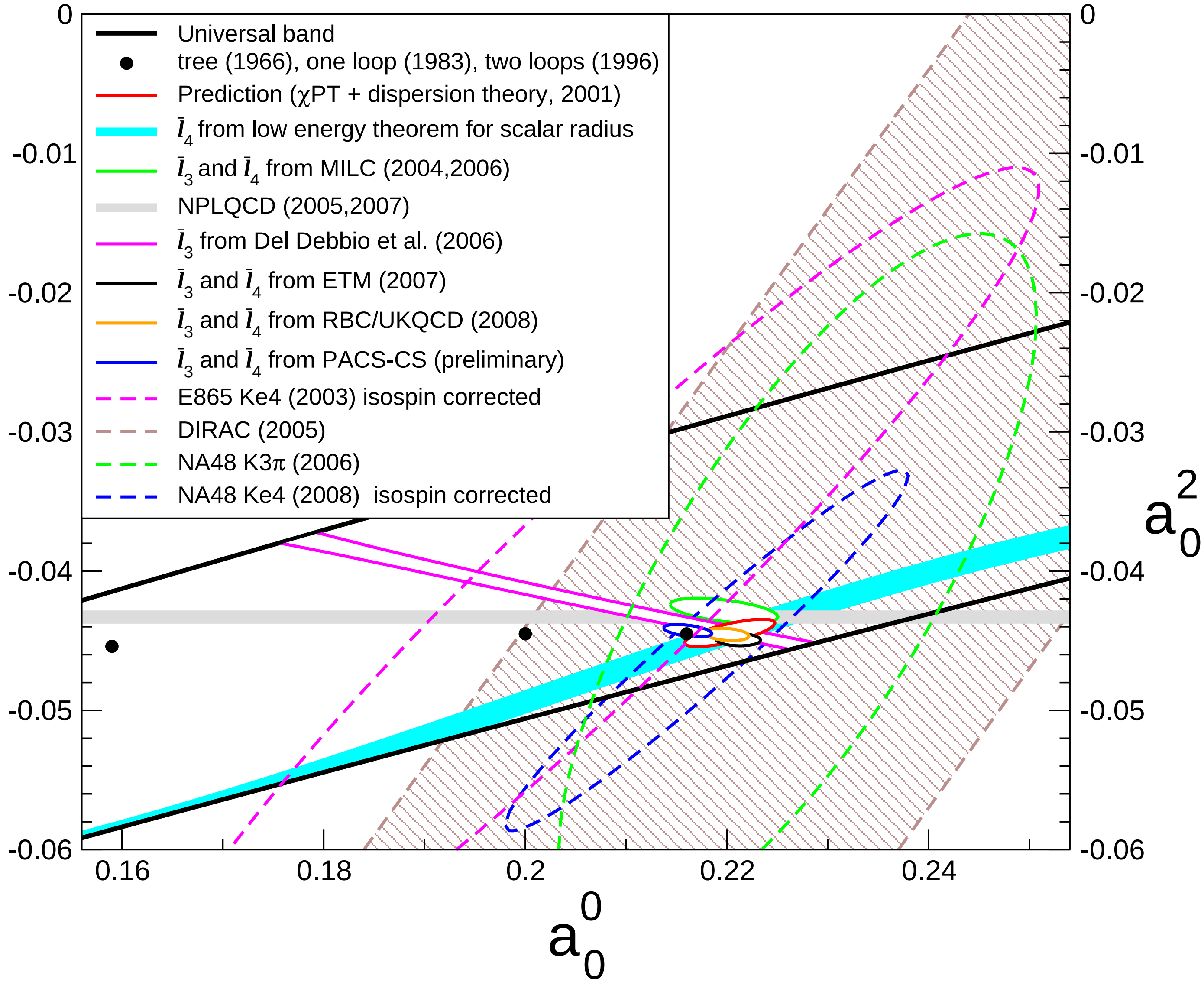
- $K \to \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007

a_0^0, a_0^2 : prediction, lattice & experiment



Conclusions for SU(2)×SU(2)

- *Expansion in powers of m_u, m_d yields a very accurate low energy representation of QCD*
- *Lattice results confirm the GMOR relation*

$\Rightarrow$ M_π *is dominated by the contribution from the quark condensate*

$\Rightarrow$ *Energy gap of QCD is understood very well*

- *Lattice approach allows an accurate measurement of the effective coupling constant ℓ_3 already now*
- *Even for ℓ_4, the lattice starts becoming competitive with dispersion theory*

Expansion in powers of m_s

- *Theoretical reasoning:*
  - *The eightfold way is an approximate symmetry*
  - *The only coherent way to understand this within QCD:* $m_s - m_d$, $m_d - m_u$ *can be treated as perturbations*
  - *Since* $m_u, m_d \ll m_s$

$\Rightarrow$ m_s *can be treated as a perturbation*

$\Rightarrow$ *Expect expansion in powers of* m_s *to work,*
but convergence to be comparatively slow

- *In principle, this can now also be checked on the lattice*

Paramagnetic inequalities

- *Consider the limit* $m_u, m_d \longrightarrow 0$, m_s *physical*
  - F *is value of* F_π *in this limit*
  - Σ *is value of* $|\langle 0|\,\bar{u}u|\,|0\rangle$ *in this limit*
  - B *is value of* $M_\pi^2/(m_u + m_d)$ *in this limit*
- *Exact relation:* $\Sigma = F^2 B$
- F_0, B_0, Σ_0*: values for* $m_u = m_d = m_s = 0$
- *Inequalities set up by Jan Stern and collaborators:*
  both F *and* Σ *should decrease if* m_s *is taken smaller*

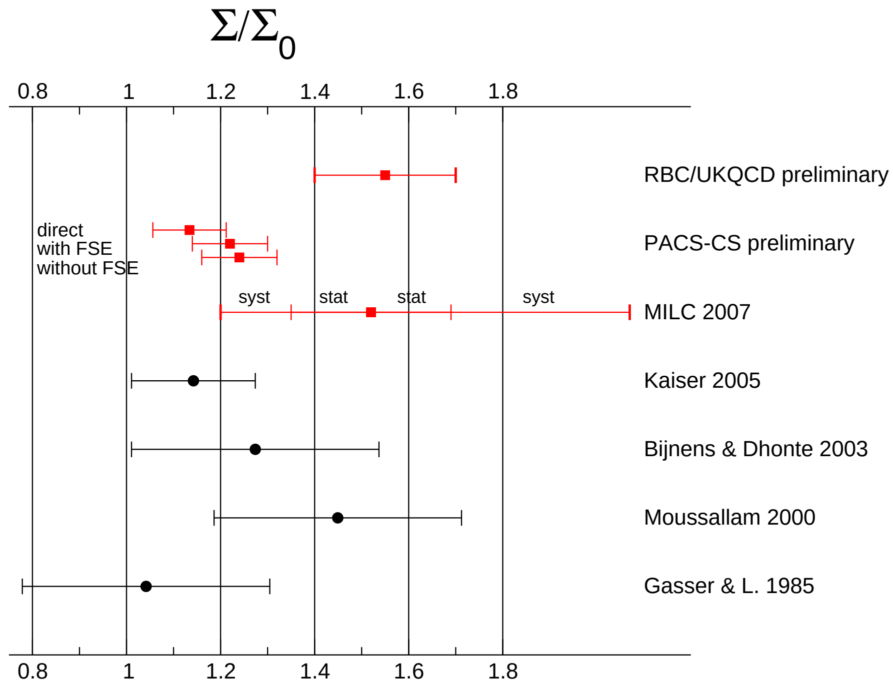
$$F > F_0\,,\ \Sigma > \Sigma_0$$

for a recent discussion, see S. Descotes-Genon at Lattice 2007

- *For* $N_c \longrightarrow \infty$*:* F, Σ *and* B *become independent of* m_s

$\Rightarrow$ $(F/F_0-1), (\Sigma/\Sigma_0-1), (B/B_0-1)$ *violate the OZI rule*

Condensate

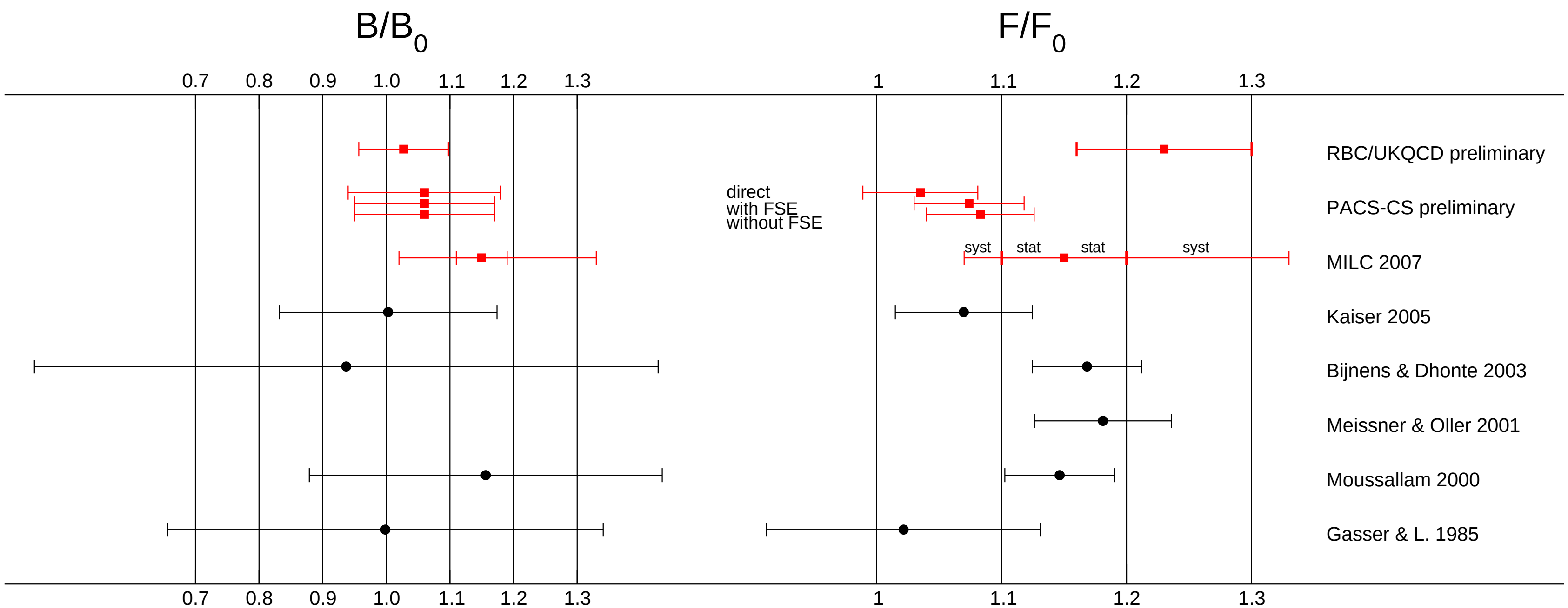


- *Central values of RBC/UKQCD and PACS-CS for* Σ/Σ_0 *lead to qualitatively different conclusions concerning OZI-violations*

$\Rightarrow$ *Discrepancy indicates large systematic errors*

- *The lattice results confirm the parametric inequalities, but do not yet allow to draw conclusions about the size of the OZI-violations*

Results for B, F



- *Results for B are coherent, indicate small OZI-violations in B*

$\Rightarrow$ *F is the crucial factor in $\Sigma = F^2 B$*

Conclusions for SU(3)×SU(3)

- *The available lattice data allow for very juicy OZI-violations, but are also consistent with* $B/B_0 \simeq F/F_0 \simeq \Sigma/\Sigma_0 \simeq 1$
- *If the central value of RBC/UKQCD for* F/F_0 *were confirmed within small uncertainties, we would be faced with a qualitative puzzle:*
  - F_π *is the pion wave function at the origin*
  - F_K *is larger because one of the two valence quarks is heavier* $\longrightarrow$ *moves more slowly* $\longrightarrow$ *wave function more narrow* $\longrightarrow$ *higher at the origin:* $F_K/F_\pi \simeq 1.19$
  - $F/F_0 = 1.23$ *indicates that the wave function is more sensitive to the mass of the sea quarks than to the mass of the valence quarks . . . very strange* $\longrightarrow$ *extraordinarily interesting*
- *Note: most of the numbers quoted are preliminary, errors purely statistical, continuum limit, finite size effects, . . .*

Resonances: exact formula for mass and width

- Most of the pole positions quoted by the PDG are obtained by
  (a) parametrizing the data for real values of s
  (b) continuing this parametrization analytically in s

  $\Rightarrow$ Result is sensitive to the parametrization used

- We found a model independent method:
  1. Poles on second sheet are zeros on first sheet
  2. The Roy equations are valid for complex values of s, in a limited region of the first sheet

  $\Rightarrow$ Exact representation of the S-matrix elements in terms of observable quantities, valid for complex values of s

  $\Rightarrow$ Exact formula for the pole position

  3. Can evaluate this formula to good precision
     and determine the pole position numerically

Caprini, Colangelo and L., Phys. Rev. Lett. 96 (2006) 132001

Formula for resonances with $I = \ell = 0$

$$\boxed{S_0^0(s) = 0} \quad \text{for } s = (M - \tfrac{i}{2}\Gamma)^2$$

$$S_0^0(s) = 1 + 2 i \rho\, t_0^0(s)\,, \qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2) b + \sum_{I,\ell} \int_{4M_\pi^2}^{\infty} ds'\, K_{0\ell}^{0I}(s,s')\, \mathrm{Im}\, t_\ell^I(s')$$

- *The subtraction constants are determined by* a_0^0, a_0^2*:*

  $$a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$$

- *The kernels are elementary functions, e.g.*

  $$K_{00}^{00}(s,s') = \underbrace{\frac{1}{\pi(s'-s)}}_{r.h.cut} + \underbrace{\frac{2\, ln\{(s+s'-4M_\pi^2)/s'\}}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}}_{l.h.cut}$$

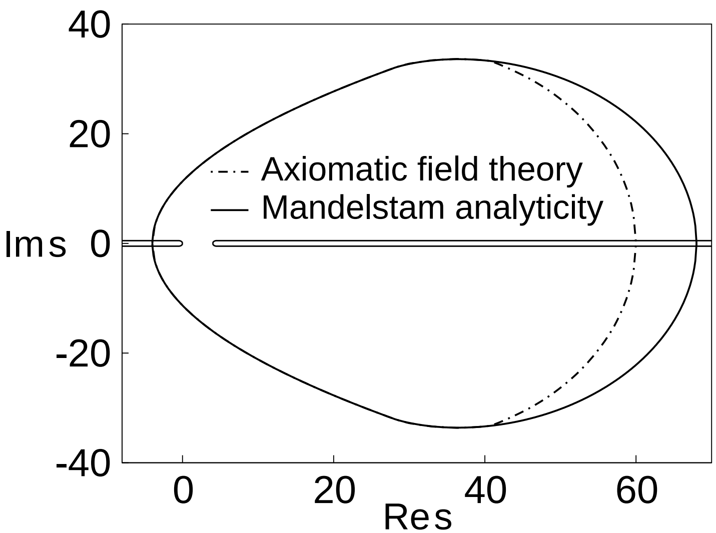
- *Left hand cut is essential for convergence:*

  $K_{00}^{00}(s,s') \sim 1/{s'}^3$ *for large* s'

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$
- Equations are valid for complex s in a limited region of the first sheet

Caprini, Colangelo and L. 2006



s in units of M_π^2

- Proof is based on first principles, general quantum field theory

A. Martin, Scattering Theory: Unitarity, Analyticity and Crossing, Lecture Notes in Physics, vol. 3, 1969.

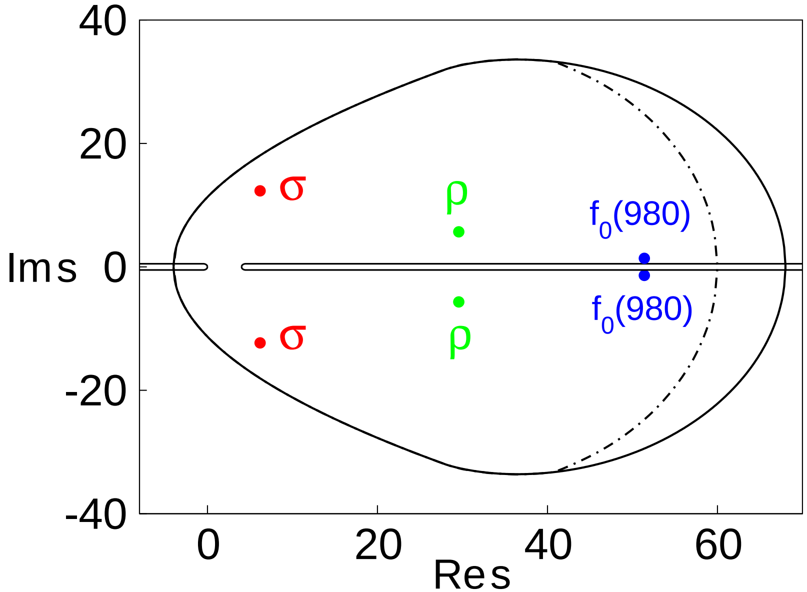
G. Mahoux, S. M. Roy and G. Wanders, Nucl. Phys. B70 (1974) 297.

$\Rightarrow$ Exact representation for the S-matrix elements in this region do not need to parametrize the amplitude

Numerical result for resonances with vacuum quantum numbers

- $S_0^0(s)$ has two pairs of zeros in the region where the formula holds
- For the central solution of the Roy equations, the zeros occur at

$$s = (6.2 \pm i\,12.3)\,M_\pi^2 \quad \sigma$$
$$s = (51.4 \pm i\,1.4)\,M_\pi^2 \quad f_0(980)$$



s in units of M_π^2

Error analysis

- *Formula is exact, evaluation is approximate*
- *Key point: can follow error propagation explicitly*
- *Focus on σ, split the formula into 3 pieces:*

  1. *Contribution from* $\mathrm{Im}\, t_0^0$ *below* $K\bar{K}$ *threshold*
  2. *Subtraction terms*
  3. *Higher energies and other partial waves*

  - *Subtraction constants* (a_0^0, a_0^2) *are known very accurately*
  - *Higher energies barely contribute (2 subtractions)*
  - *Crossed channel contributions are dominated by the* ρ*, excellent experimental information from* e^+e^-, τ
  - *Illustration: replace our representation for the higher energies and other partial waves by the one of KPY III :* σ *pole moves by* $-0.6 - i\,1.2$ *MeV*

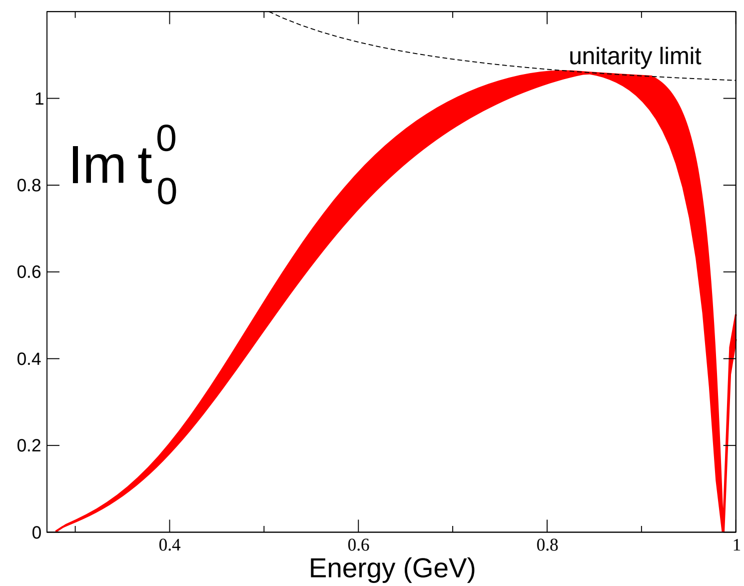
$\Rightarrow$ *Uncertainty in result for* σ *pole is dominated by 1.*

1. Contribution from $\mathrm{Im}\, t_0^0$ below $K\bar{K}$ threshold
2. Subtraction terms
3. Higher energies and other partial waves

Unitarity and dip leave little room between 800 MeV and $2M_K$

Only the region below 800 MeV really matters

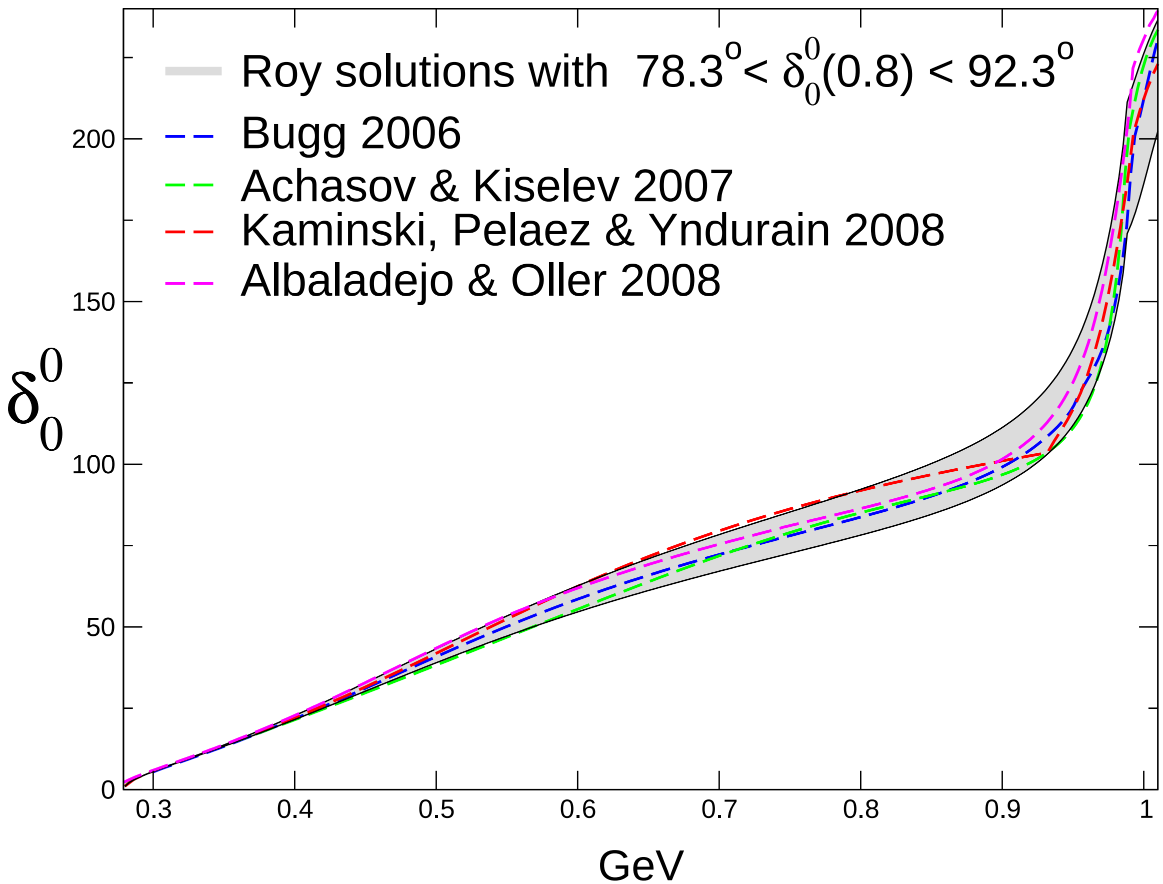
There, S_0^0 *is nearly elastic*



$$\mathrm{Im}\, t_0^0 \simeq \sin^2\delta_0^0/\rho$$

$\Rightarrow$ *Need to know the phase* δ_0^0 *below 800 MeV*

Behaviour of the phase below 800 MeV



Bugg uses BES data for $J/\psi \longrightarrow \omega\pi\pi$

Achasov & Kiselev use KLOE data on $\phi \longrightarrow \gamma\pi\pi$

Kamiński et al.: method and results were discussed in the talk of José Peláez

Albaladejo & Oller: N/D fit to several data sets

Result for pole position

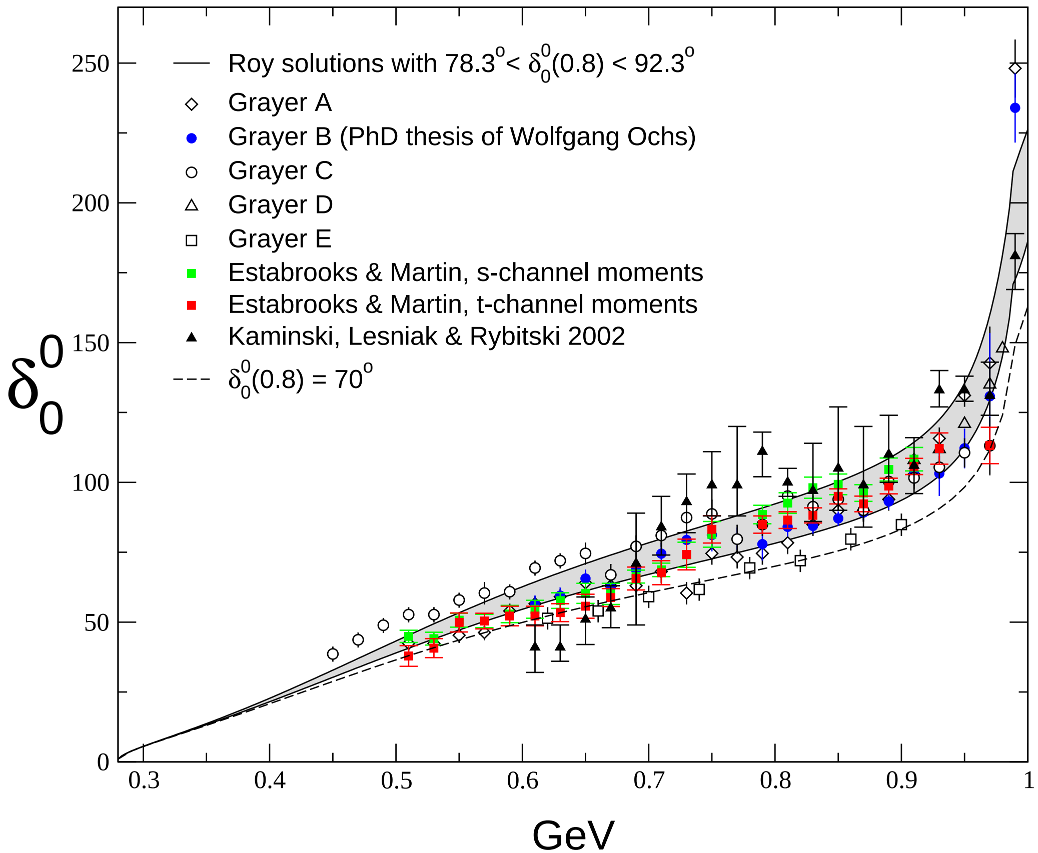
- *Vary the Roy equation input within the experimental uncertainties, use the predictions for the subtraction constants and add errors up*

$\Rightarrow$ *On the lower half of sheet II, the pole closest to the origin sits at*

$$M_\sigma - \tfrac{i}{2}\Gamma_\sigma = 441\,{}^{+16}_{-8} \; - i\, 272\,{}^{+9}_{-13} \text{ MeV}$$

- *Do not need to rely on the solution of the Roy equations:*
  - *Replace the central Roy solution below* $2M_K$ *by the phase representation of Bugg 2006* $\Rightarrow$ *pole moves to* $444 - i\,267$ *MeV*
  - *Ditto with Achasov & Kiselev 2007:* $438 - i\,274$ *MeV*
  - *Ditto with Kamiński, Peláez & Ynduráin 2008:* $458 - i\,253$ *MeV*
  - *Ditto with Albaladejo & Oller 2008:* $451 - i\,257$ *MeV*

Comparison with the figure Peter Minkowski showed yesterday



The Roy equations do admit solutions with δ_0^0(0.8 GeV) = 70°. The corresponding pole position is 5 % lower in mass, 10 % higher in width. None of the published data sets is consistent with such a behaviour of the phase.

Conclusion for lowest resonance of QCD

- *Dispersion theory allows to extend the domain where the first few terms of the chiral perturbation series provide a decent approximation*
- *Model independent method for analytic continuation*
- *Crossing symmetry plays an essential role in this method:*
  fixes the contributions from the left hand cut
  ensures fast convergence, low energy dominance

$\Rightarrow$ *The lowest resonance of QCD carries vacuum quantum numbers and occurs at*

$$M_\sigma = 441\,{}^{+16}_{-8}\ \text{MeV} \qquad \Gamma_\sigma = 544\,{}^{+18}_{-25}\ \text{MeV}$$

Puzzling results on $K_L \longrightarrow \pi\mu\nu$

- Hadronic matrix element of weak current:

$$\langle K^0|\bar{u}\gamma^\mu s|\pi^-\rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

- Scalar form factor $\sim \langle K^0|\partial_\mu(\bar{u}\gamma^\mu s)|\pi^-\rangle$

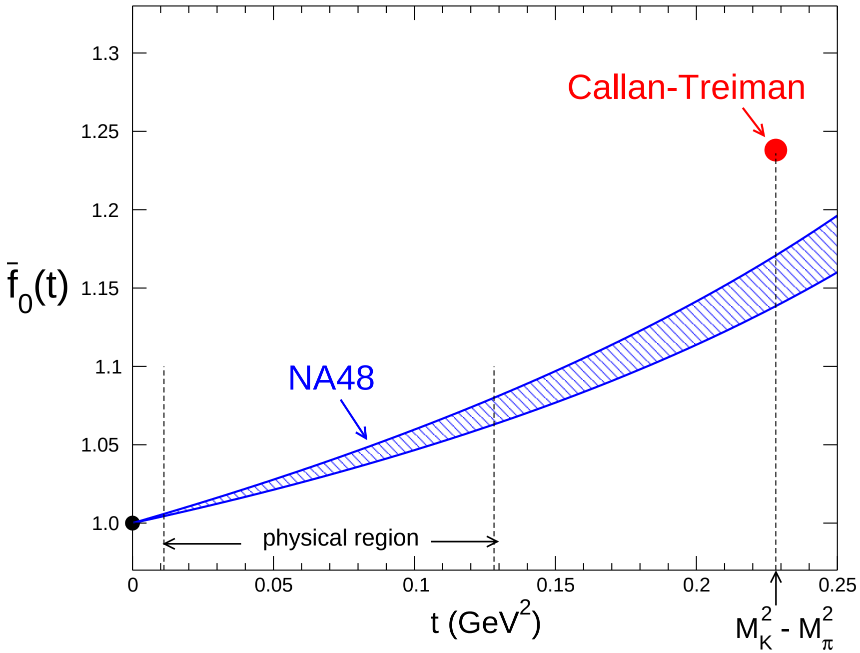
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- Low energy theorem of Callan and Treiman (1966):

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi}\left\{1 + O(m_u, m_d)\right\} \simeq 1.19$$

$f_0(0) = f_+(0) \simeq 0.96$ relevant for determination of V_{us}

Comparison with experiment



NA48, Phys. Lett. B647 (2007) 341

141 authors, 2.3×10^6 *events*

plot shows normalized scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- *Callan-Treiman relation in this normalization:*

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)}$$

- *Experimental value:* $\dfrac{F_K}{F_\pi f_+(0)} = 1.2446 \pm 0.0041$

Bernard and Passemar 2008

Implications

- NA48 data on $K_L \longrightarrow \pi \mu \nu$ disagree with SM
- If confirmed, the implications are dramatic:

$\Rightarrow$ right-handed currents ? Bernard, Oertel, Passemar & Stern 2006

- There are not many places where the SM disagrees with observation, need to investigate these carefully
- At low energies, high precision is required

Corrections, extrapolation

- *Callan-Treiman-relation is exact only for* $m_u, m_d \longrightarrow 0$

  Corrections of NLO were worked out long ago, are tiny Gasser & L. 1985

  Form factor now known to NNLO Post & Schilcher 2002,

  Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005

  Including the uncertainties from $m_u, m_d \neq 0$*:*

  $\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$ Bernard & Passemar 2008

$\Rightarrow$ *Cannot blame the discrepancy on the prediction*

- *CT–point is not in physical region, extrapolation needed*

  Curvature can be calculated with dispersion theory

  Jamin, Oller & Pich 2004, Bernard, Oertel, Passemar & Stern 2006

$\Rightarrow$ *Cannot blame the discrepancy on the extrapolation*

Slope of the scalar form factor

- *Definition of the slope* $$\bar{f}_0(t) = 1 + \frac{\lambda_0\, t}{M_{\pi^+}^2} + O(t^2)$$
- *Callan-Treiman-relation implies sharp prediction:*

  $$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$ *Jamin, Oller & Pich 2004*

- *Update with current experimental information*

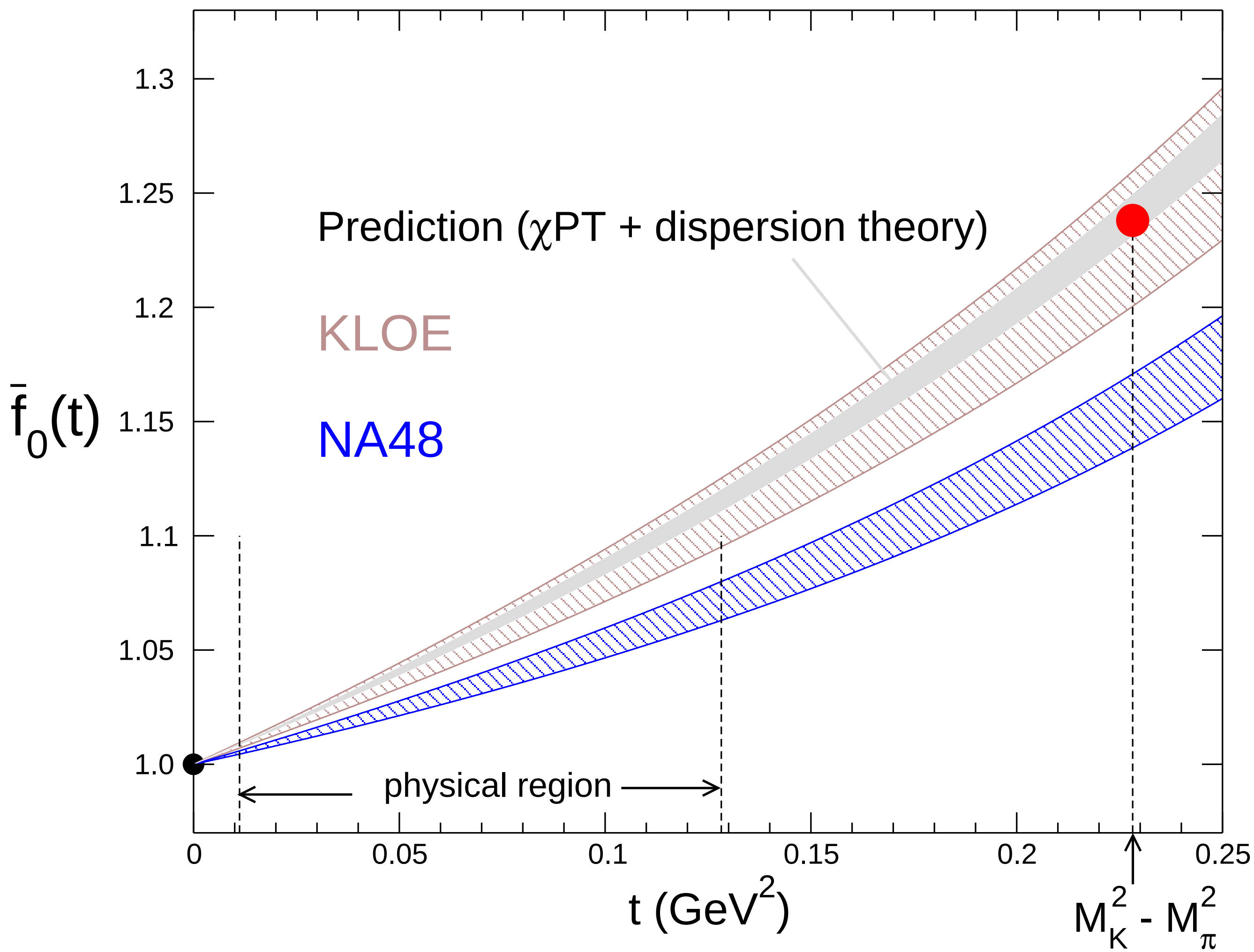
  $$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

  Bernard, Oertel, Passemar & Stern, preliminary

- *To be compared with the result of NA48:*

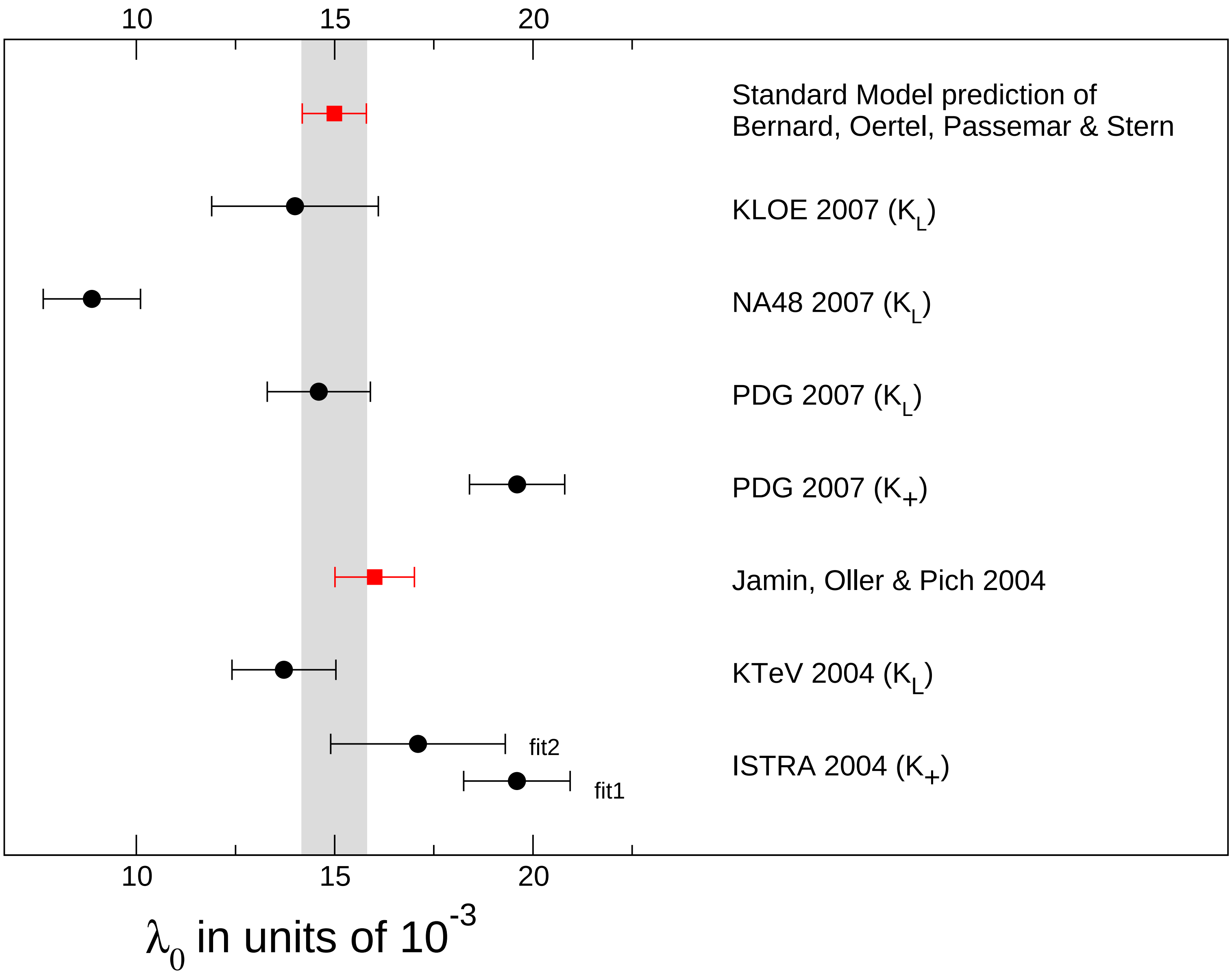
  $$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$ *Fit with dispersive representation of BOPS*

New data from KLOE

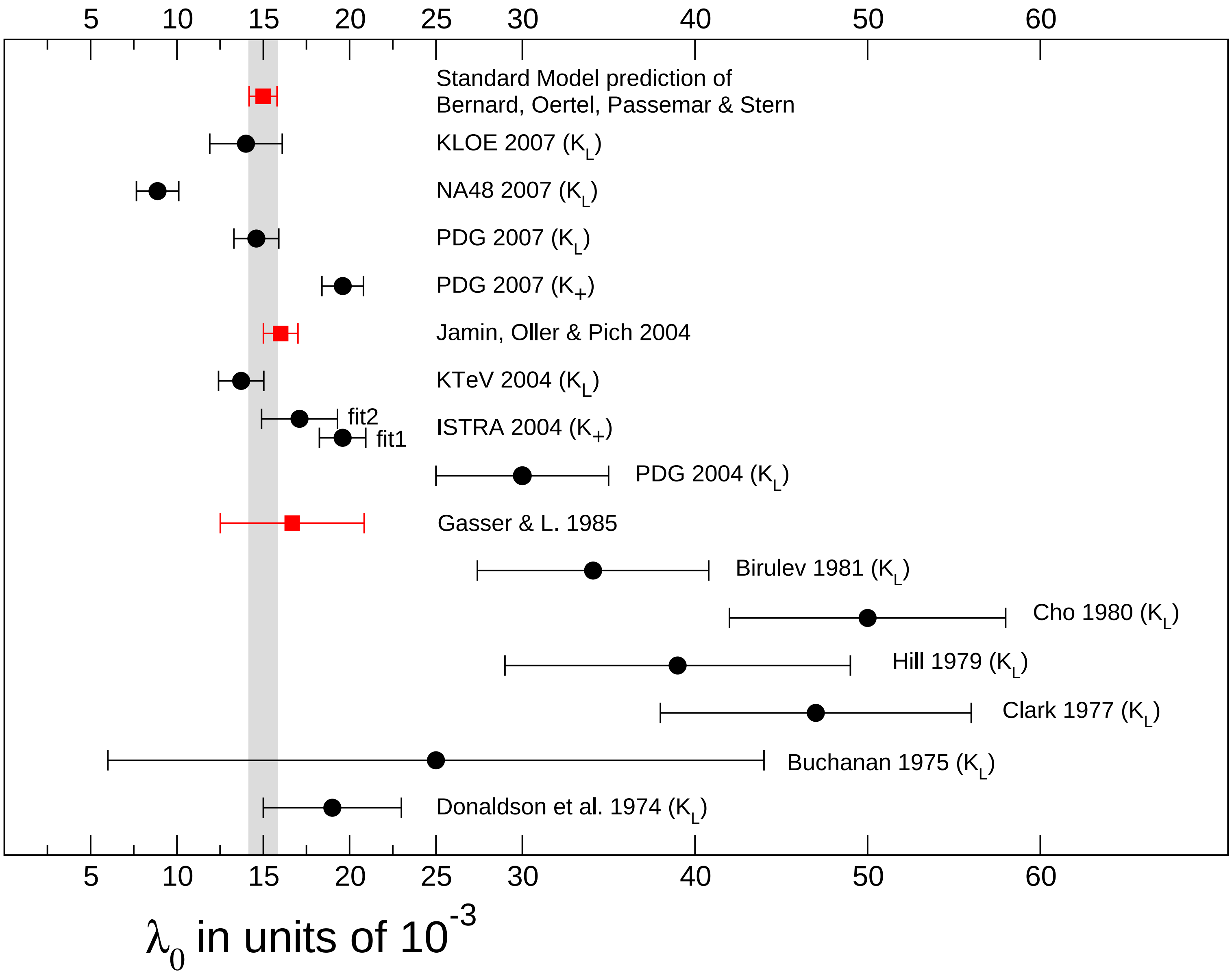


I thank Emilie Passemar for some of the material shown in this figure

Comparison of results for the slope



Older measurements



Conclusions for $K \to \mu\nu\pi$

- *Experiment is difficult, discrepancies need to be resolved*

  Donaldson 1974: 1.6×10^6 *events*

  ISTRA 2004: 0.54×10^6 *events*

  KTeV 2004: 1.5×10^6 *events*

  NA48 2007: 2.3×10^6 *events*

- *Dispersion theory fixes the shape of the form factors*
  publishing linear fits is nonsensical

- *NA48 should improve their data analysis . . . and extend it to charged kaons (isospin breaking)*