Recent developments in light flavour hadron physics

H. Leutwyler

University of Bern

MIT, April 29, 2008

Motivation

- QCD with massless quarks is the ideal of a theory: no dimensionless free parameters
- Gives rise to a rich structure at low energies
- High energy side looks like what we are used to: relevant degrees of freedom visible in the Lagrangian, can treat interaction as a pertubation
- Low energies are out of reach of perturbation theory

$\Rightarrow$ Progress in understanding is slow

- $\exists$ many models that resemble QCD: instantons, monopoles, bags, superconductivity, gluonic strings, linear σ model, hidden gauge, NJL, AdS/CFT . . .
- Discuss the model independent approach based on chiral perturbation theory + dispersion theory

Le menu

$\sim$ *Le potage* $\sim$

Energy gap of QCD

- Main characteristic of QCD at low energies:
  Energy gap is very small, $M_\pi \simeq$ 140 MeV
- In 1960, Nambu found out why that is so:
  Has to do with a hidden approximate symmetry
- Symmetry becomes exact for $m_u, m_d \to 0$

$\Rightarrow$ Energy gap disappears: pions become massless

- In reality $m_u, m_d \neq 0$, but very small

$\Rightarrow$ Symmetry is not perfect, but nearly so

∼ *L' hors d'oeuvre* ∼

Lattice results for light dynamical quarks

- Progress achieved on the lattice allows to simulate quarks with sufficiently small masses
- Can establish contact with physics, using χPT
- First meaningful determinations of effective coupling constants

∼ *L' entrée* ∼

Puzzling results on $K_L \to \pi\mu\nu$

- NA48 Data are in conflict with a venerable low energy theorem: Callan-Treiman-relation
- Physics beyond the Standard Model or incorrect measurement ?

∼ *La spécialité du patron* ∼

Interaction among the pions

- Significant progress in understanding the interaction among the pions by means of dispersion theory
- Roy equations for the $\pi\pi$ scattering amplitude are particularly well suited
- Two subtraction constants $\leftrightarrow$ S-wave scattering lengths
- Use low energy theorems for the scattering lengths

$\Rightarrow$ Scattering amplitude is fixed to an incredible precision

$\Rightarrow$ Currently, theory is ahead of experiment . . .

$\sim$ *Le dessert* $\sim$

Lowest resonance of QCD

- Masses and widths of the lowest resonances can now be calculated in a reliable manner

$\Rightarrow$ The lightest resonance of QCD has the quantum numbers of the vacuum and sits at

$$M_\sigma - \frac{i}{2}\Gamma_\sigma \simeq 441 - i\,272\,\text{MeV}$$

∼ Le potage ∼

Energy gap of QCD

Massless quarks

- The size of the energy gap is controlled by m_u, m_d
  The remaining flavours do not play an essential role
- Theoretical paradise: only two flavours, no mass

QCD with $N_f = 2$, $m_u = m_d = 0$

- Interactions of u and d are identical
  If the masses are the same, there is no difference at all
- More symmetry: for massless fermions,
  right and left do not communicate

$\Rightarrow$ Lagrangian is invariant under independent rotations of the right– and left–handed quark fields
$SU(2)_R \times SU(2)_L$ chiral symmetry

$\Rightarrow$ 6 conserved "charges"
Q_1^V, Q_2^V, Q_3^V (vector currents)
Q_1^A, Q_2^A, Q_3^A (axial currents)

Asymmetry of the ground state

- Charges commute with the Hamiltonian:

$$[Q_i^{\mathsf{V}}, H_0] = 0 \qquad [Q_i^{\mathsf{A}}, H_0] = 0$$

- The state of lowest energy is asymmetric with respect to chiral rotations: $Q_i^{\mathsf{A}}\,|0\rangle \neq 0$ Nambu 1960

$\Rightarrow$ Chiral symmetry is hidden, “spontaneously broken”

- Very strong experimental evidence $\surd$
  Very strong evidence from lattice calculations $\surd$
  Analytic understanding of the ground state still poor

Goldstone bosons

- Consequence of $Q_i^{\mathsf{A}}\,|0\rangle \neq 0$:

$$H_0\,Q_i^{\mathsf{A}}\,|0\rangle = Q_i^{\mathsf{A}}\,H_0\,|0\rangle = 0$$

$\Rightarrow$ Spectrum of QCD with two massless flavours must contain 3 states $Q_1^{\mathsf{A}}\,|0\rangle,\; Q_2^{\mathsf{A}}\,|0\rangle,\; Q_3^{\mathsf{A}}\,|0\rangle$ with $E = 0$

"Goldstone bosons"

- spin 0, negative parity
- Indeed, the 3 lightest hadrons do have these quantum numbers: π^+, π^0, π^-

  But massless they are not

Back to earth

- world $\neq$ paradise

  Light quark masses break chiral symmetry, allow the left to talk to the right

- Chiral symmetry broken in two ways:

  spontaneously $\quad Q_i^{\mathsf{A}}\,|0\rangle \neq 0$

  explicitly $\quad m_u\,,\,m_d \neq 0$

- Only the diagonal vector currents are strictly conserved in QCD: $N_u, N_d, N_s, N_c, N_b, N_t \;\rightarrow$ baryon number, electric charge, strangeness, charm, ...

- It so happens that m_u and m_d are small

$\Rightarrow$ H_{QCD} has an approximate $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{SU}(2)_{\mathrm{R}}$ symmetry

Quark masses as perturbations

- Masses of u, d enter the Hamiltonian via

$$H_{\rm QCD} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d\}$$

  H_0 describes u, d as massless, s, c, b, t as massive

  H_0 is invariant under $\mathrm{SU(2)_L \times SU(2)_R}$

- H_0 treats the pions as massless particles

  H_1 gives them a mass

Expansion in powers of m_u, m_d	$\Longleftrightarrow$	Perturbation series in powers of H_1

Gell-Mann-Oakes-Renner formula

- First order perturbation theory in H_1 yields:

$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \text{explicit}}}{+} m_d) \times |\langle 0|\, \underset{\substack{\Uparrow \\ \text{spontaneous}}}{\bar{u}u}\, |0\rangle| \times \frac{1}{F_\pi^2}$$

Gell-Mann, Oakes & Renner 1968

Coefficient: decay constant F_π

$$\langle 0|\, \bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\, p^\mu \sqrt{2}\, F_\pi$$

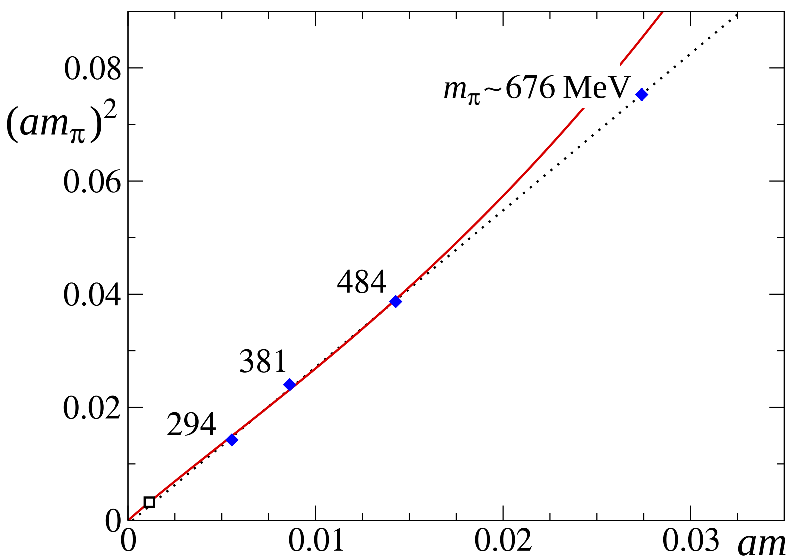
Value of F_π is known from $\pi^+ \to \mu^+\nu$

$\sim$ L' hors d'oevre $\sim$

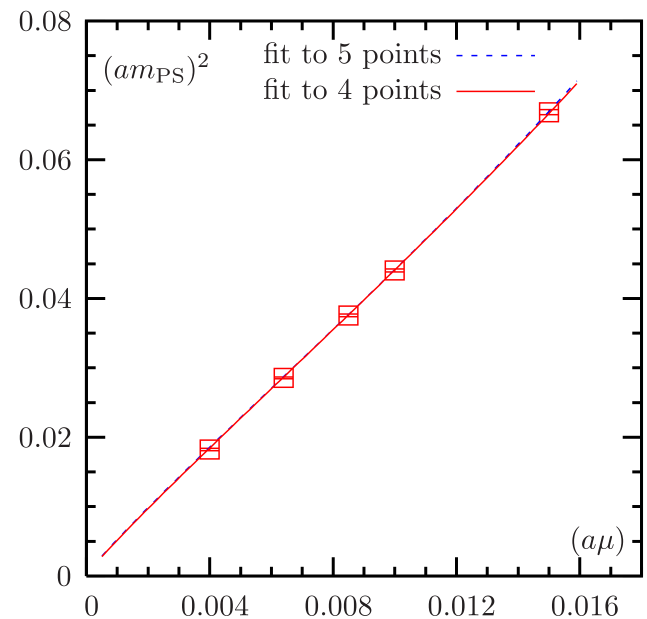
Lattice results for light dynamical quarks

Lattice results for M_π

- GMOR formula can now be checked on the lattice: determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- No quenching, quark masses are sufficiently light

$\Rightarrow$ legitimate to use χPT for the extrapolation to the physical values of m_u, m_d

Lattice

- Quality of data is impressive
- Proportionality of M_π^2 to the quark mass appears to hold out to values of m_u, m_d that are an order of magnitude larger than in nature
- Main limitation: systematic uncertainties
  in particular: $N_f = 2 \rightarrow N_f = 3$

Chiral perturbation theory

- Consequences of hidden, approximate symmetry can be worked out by means of an effective field theory

Weinberg 1979

- Hidden symmetry controls energy gap of QCD

$\Rightarrow$ Can calculate how gap grows as the symmetry breaking parameters m_u, m_d are turned on

- Hidden symmetry also determines the interaction of the Goldstone bosons at low energies, among themselves, as well as with other hadrons

Expansion of M_π^2 in powers of the quark mass

- GMOR formula represents leading term of χPT
- At NLO, the expansion contains a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

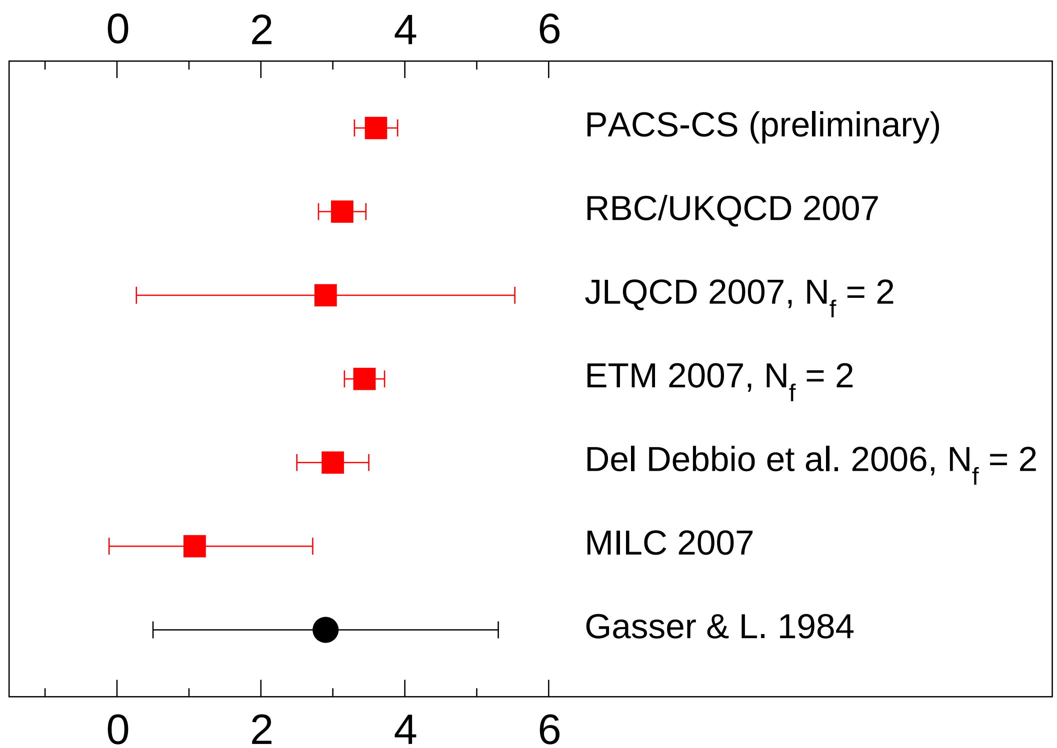
$$M^2 \equiv B(m_u + m_d)$$

Gasser & L. 1983

- Coefficient is determined by pion decay constant
  Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3)×SU(3):
  $0.2\,\text{GeV} \lesssim \Lambda_3 \lesssim 2\,\text{GeV}$

Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



Horizontal axis shows the value of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

Range for Λ_3 obtained in 1984 corresponds to $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of RBC/UKQCD 2007, for instance, is $\bar{\ell}_3 = 3.13 \pm 0.33$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

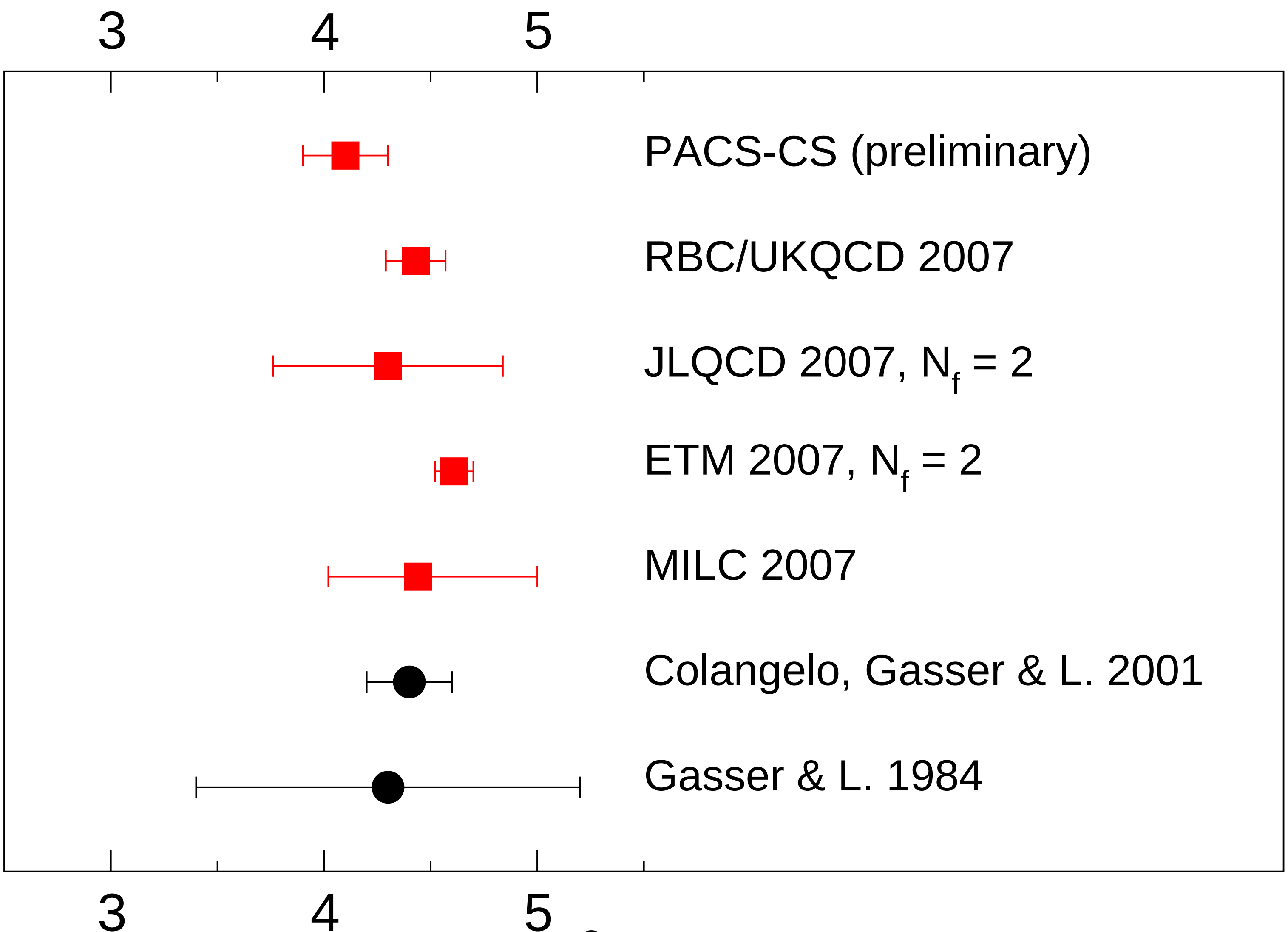
$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

  F is value of pion decay constant in limit $m_u, m_d \rightarrow 0$

- Structure is the same, coefficients and scale of logarithm are different
- Quark mass dependence of F_π can also be measured on the lattice $\Rightarrow$ measurement of Λ_4
- Alternative method: determine the scalar form factor of the pion, radius $\langle r^2 \rangle_s \Leftrightarrow \bar{\ell}_4$

Lattice results for Λ_4



$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

$\sim$ *L' entrée* $\sim$

Puzzling results on $K_L \to \pi\mu\nu$

Callan-Treiman-relation

- Hadronic matrix element of weak current:

  $\langle K^0|\bar{u}\gamma^\mu s|\pi^-\rangle = (p_K+p_\pi)^\mu f_+(t) + (p_K-p_\pi)^\mu f_-(t)$

- Scalar form factor $\sim \langle K^0|\partial_\mu(\bar{u}\gamma^\mu s)|\pi^-\rangle$

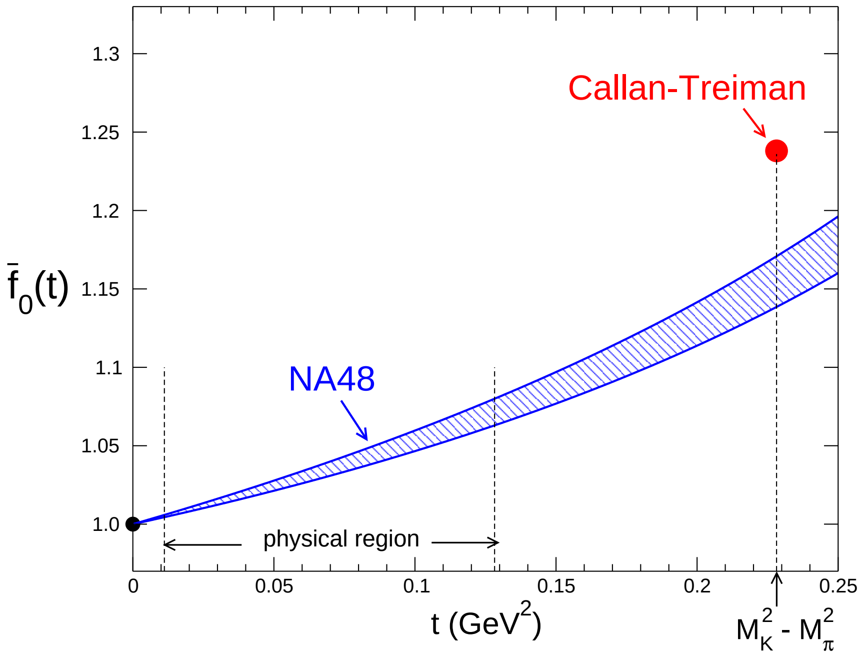
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2}\, f_-(t)$$

- Low energy theorem of Callan and Treiman (1966):

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi}\left\{1 + O(m_u, m_d)\right\} \simeq 1.19$$

$f_0(0) = f_+(0) \simeq 0.96$ relevant for determination of V_{us}

Comparison with experiment



NA48 Phys. Lett. B 647 (2007) 341

141 authors, 2.3×10^6 events

Plot shows normalized scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- Callan-Treiman relation in this normalization:

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)} \left\{1 + O(m_u, m_d)\right\}$$

- Experimental value: $\frac{F_K}{F_\pi f_+(0)} = 1.2446 \pm 0.0041$

Bernard and Passemar 2008

Uncertainty in prediction & extrapolation

- Callan-Treiman-relation is exact only for $m_u, m_d \to 0$

  Corrections of NLO were worked out long ago, are tiny

  Gasser & L. 1985

  Form factor now known to NNLO Post & Schilcher 2002, Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005

  Including the uncertainties from $m_u, m_d \neq 0$:

  $$\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$$

  Bernard, Oertel, Passemar & Stern, preliminary

$\Rightarrow$ Cannot blame the discrepancy on the prediction

- CT-point not in physical region, extrapolation needed

  Curvature can be calculated with dispersion theory

  Jamin, Oller & Pich 2004, Bernard, Oertel, Passemar & Stern 2006

$\Rightarrow$ Cannot blame the discrepancy on the extrapolation

Implications

- NA48 data on $K_L \to \pi\mu\nu$ disagree with SM

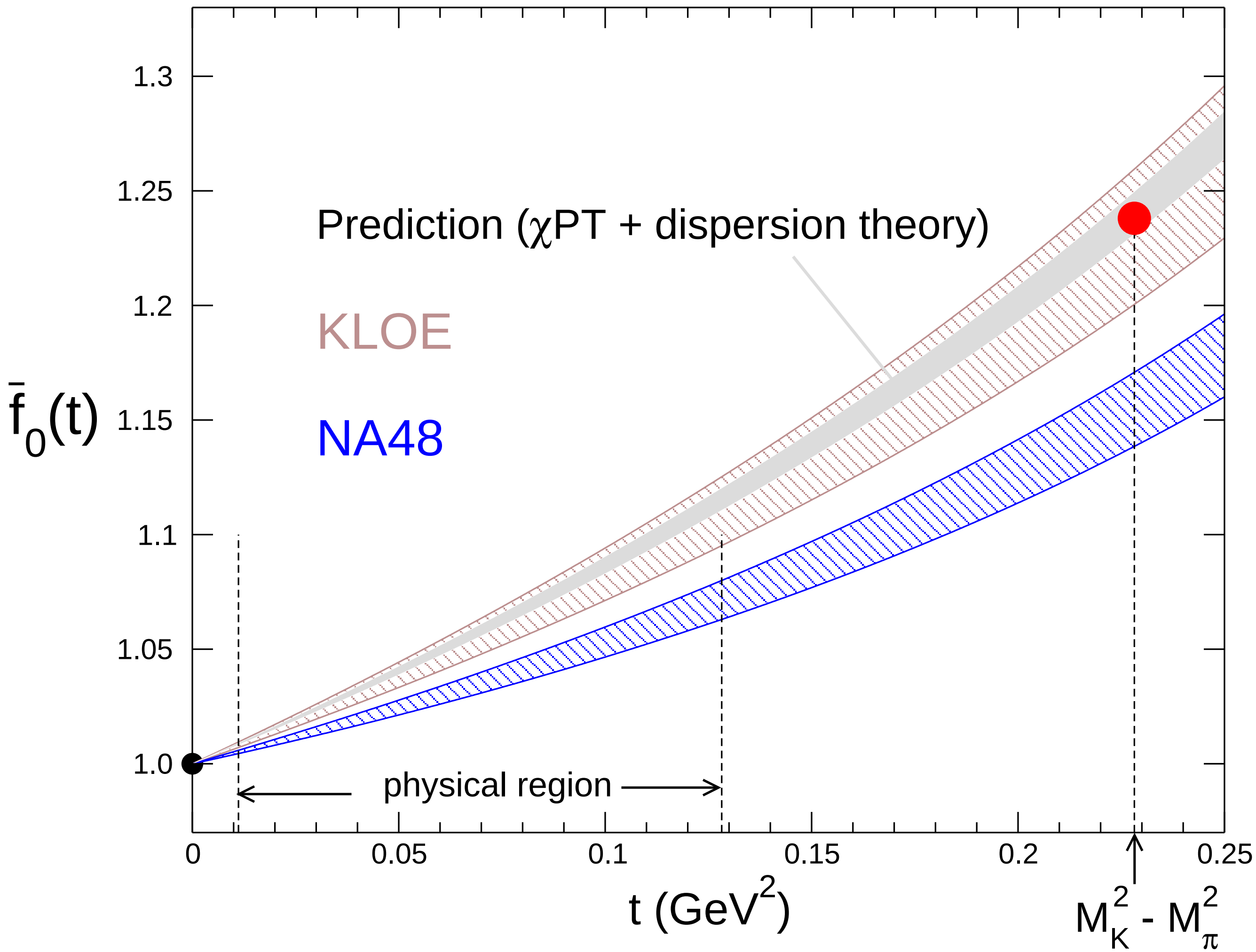
  If confirmed, the implications are dramatic:

$\Rightarrow$ Right–handed currents ? Bernard, Oertel, Passemar & Stern 2006

- In the meantime, new data from KLOE

  Results are consistent with Callan-Treiman-relation

Comparison of KLOE and NA48



I thank Emilie Passemar for some of the material shown in this figure

Slope of the scalar form factor

- Definition of the slope $$\bar{f}_0(t) = 1 + \frac{\lambda_0\, t}{M_{\pi^+}^2} + O(t^2)$$
- Callan-Treiman-relation implies sharp prediction:

$$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$ Jamin, Oller & Pich 2004

- Update with current experimental information

$$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

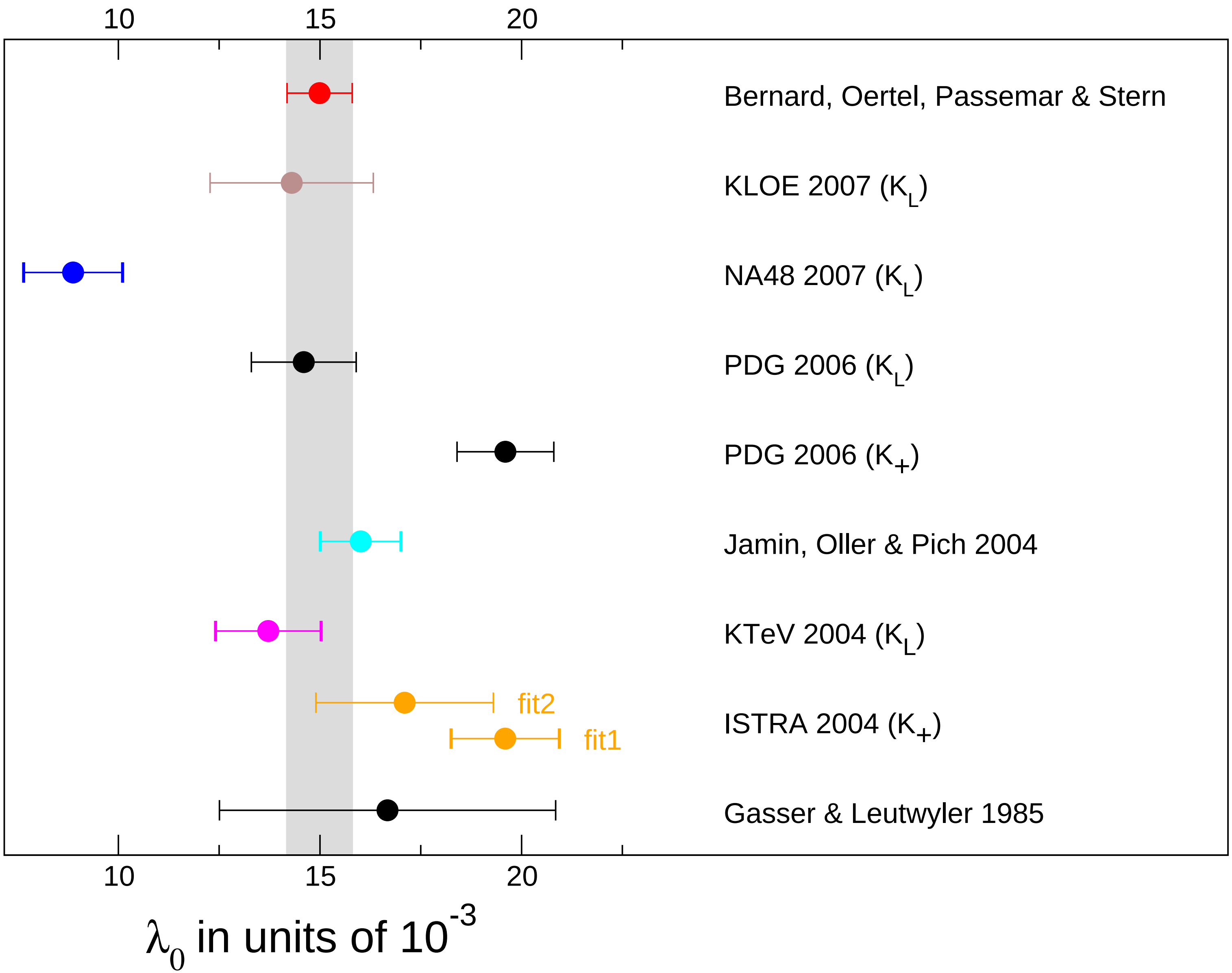
Bernard, Oertel, Passemar & Stern, preliminary

- To be compared with the result of NA48:

$$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$ Fit with dispersive representation of BOPS

$$\lambda_0 = (11.7 \pm 0.7_{\text{stat}} \pm 1.0_{\text{syst}}) \times 10^{-3}$$ Linear fit

Comparison of results for the slope



Conclusions for $K \to \mu\nu\pi$

- Experimental discrepancies need to be resolved
- Important to analyze existing data on charged K-decays (isospin breaking)
- Dispersion theory fixes the shape of the form factors

  The most recent analyses properly account for the curvature
  Publishing linear fits is useless

- KTeV is being reanalyzed, ISTRA ?

  ISTRA: $\mathbf{0.54 \times 10^6}$ events
  KTeV: $\mathbf{1.9 \times 10^6}$ events
  NA48: $\mathbf{2.3 \times 10^6}$ events

- Excellent overview of current experimental situation: Antonelli, talk at Lepton-Photon 07 http://chep.knu.ac.kr/lp07

$\sim$ La spécialité du patron $\sim$

Interaction among the pions

Theory of $\pi\pi$ interaction

- $\pi\pi$ scattering is special: crossed channels are identical

$\Rightarrow$ Re $T(s,t)$ can be represented as a twice subtracted dispersion integral over Im $T(s,t)$ in physical region

S.M. Roy 1971

- The 2 subtraction constants can be identified with the S-wave scattering lengths:

$$a_0^0\ ,\ a_0^2$$

← isospin
← angular momentum

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*

Roy equations

- Pioneering work on the physics of the Roy equations was done around the time when QCD was discovered

  Pennington & Protopopescu 1973, Basdevant, Froggatt & Petersen 1974

- Dispersion integrals converge rapidly (2 subtractions)

$\Rightarrow$ Crude phenomenological information on Im $T(s,t)$ for energies above 800 MeV suffices

$\Rightarrow$ Given a_0^0, a_0^2, the scattering amplitude can be calculated very accurately

Ananthanarayan, Colangelo, Gasser & L. 2001
Descotes, Fuchs, Girlanda & Stern 2002

$\Rightarrow$ a_0^0, a_0^2 are the essential parameters at low energy

- Main problem in early work: a_0^0, a_0^2 poorly known
  Experimental information near threshold is meagre

Low energy theorems

- Chiral perturbation theory provides the missing piece: theoretical prediction for a_0^0, a_0^2

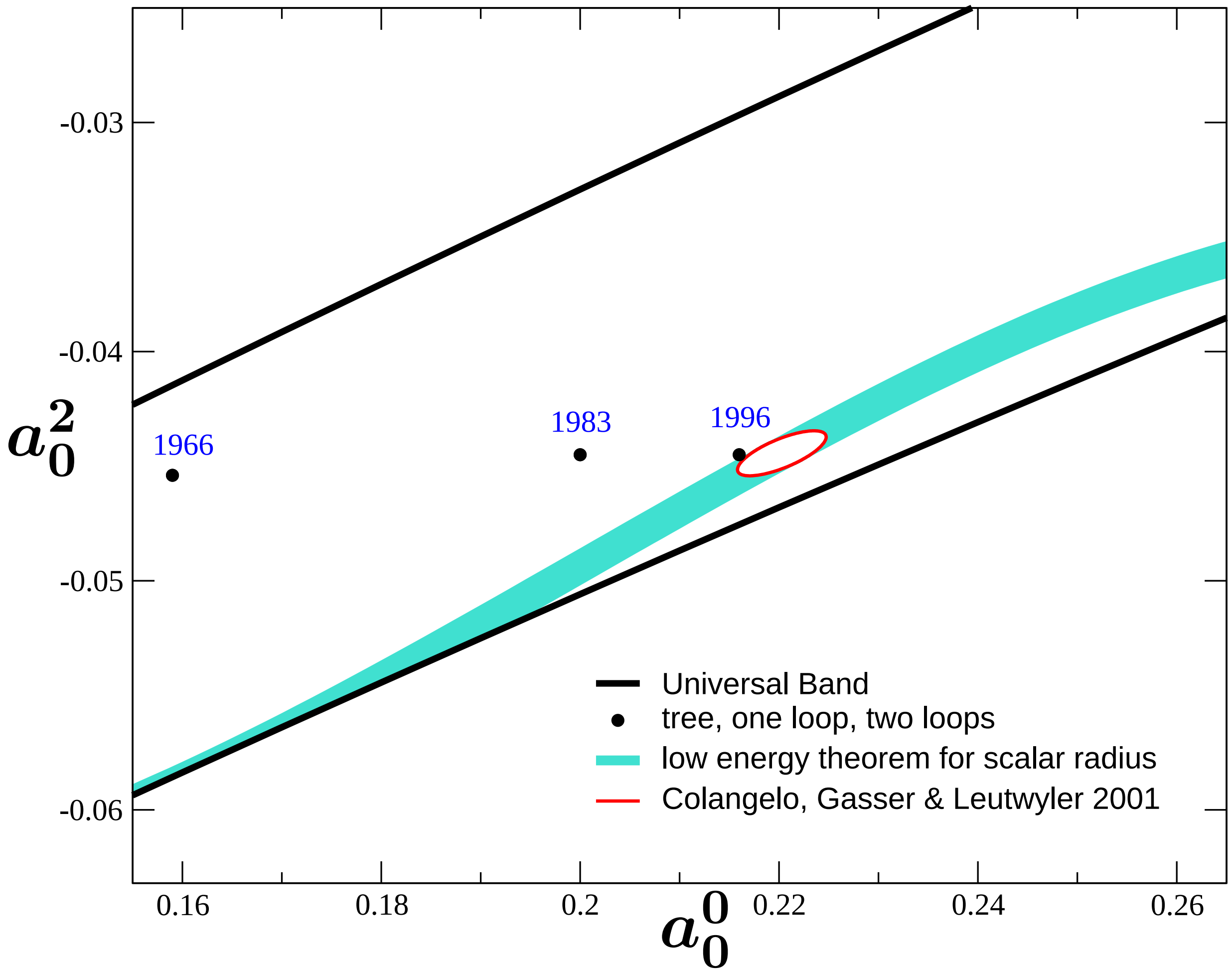
  Weinberg 1966, Gasser & L. 1983, Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

- Most accurate results for a_0^0, a_0^2 are obtained by matching the chiral and dispersive representations in the unphysical region below threshold

  Colangelo, Gasser & L. 2001

- In combination with the low energy theorems for a_0^0, a_0^2, the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to an incredible degree of accuracy

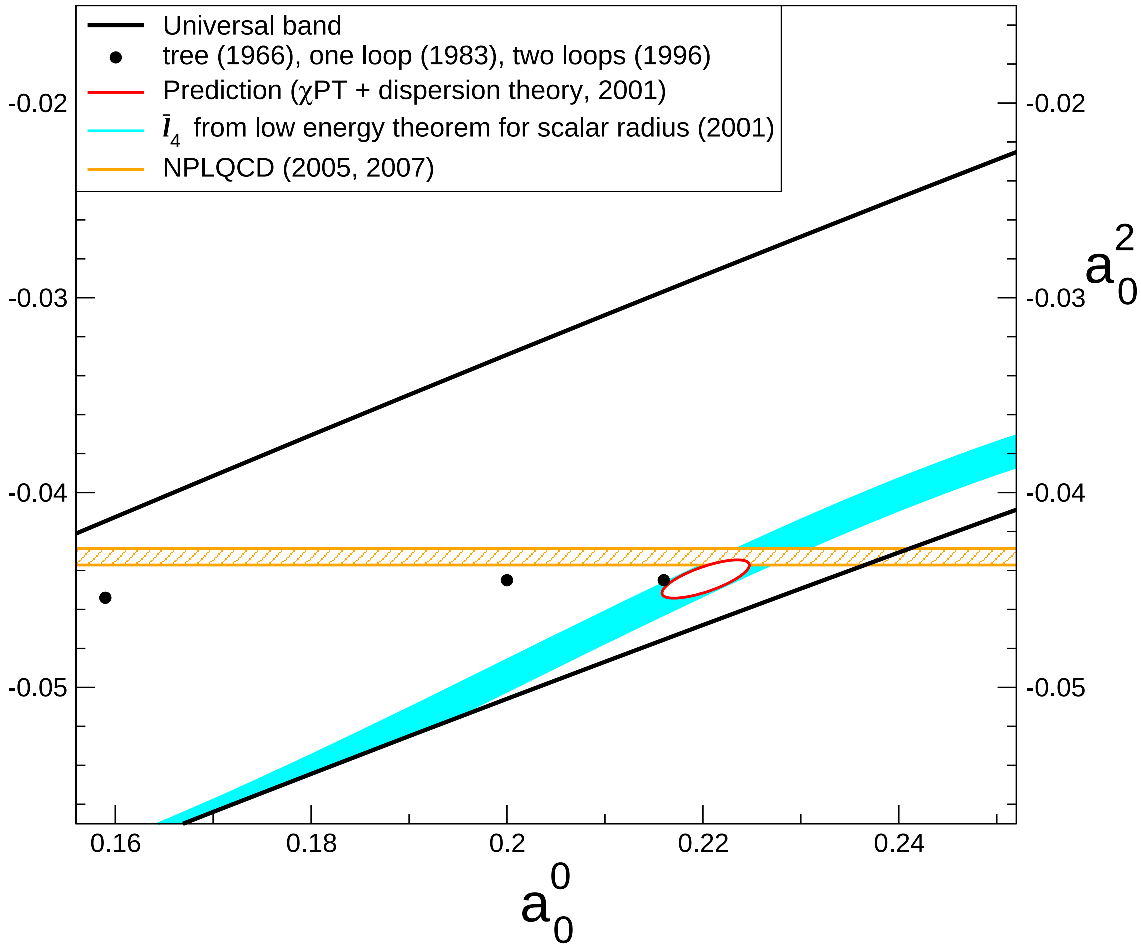
Predictions for the S-wave $\pi\pi$ scattering lengths



Sizable corrections in a_0^0, while a_0^2 nearly stays put

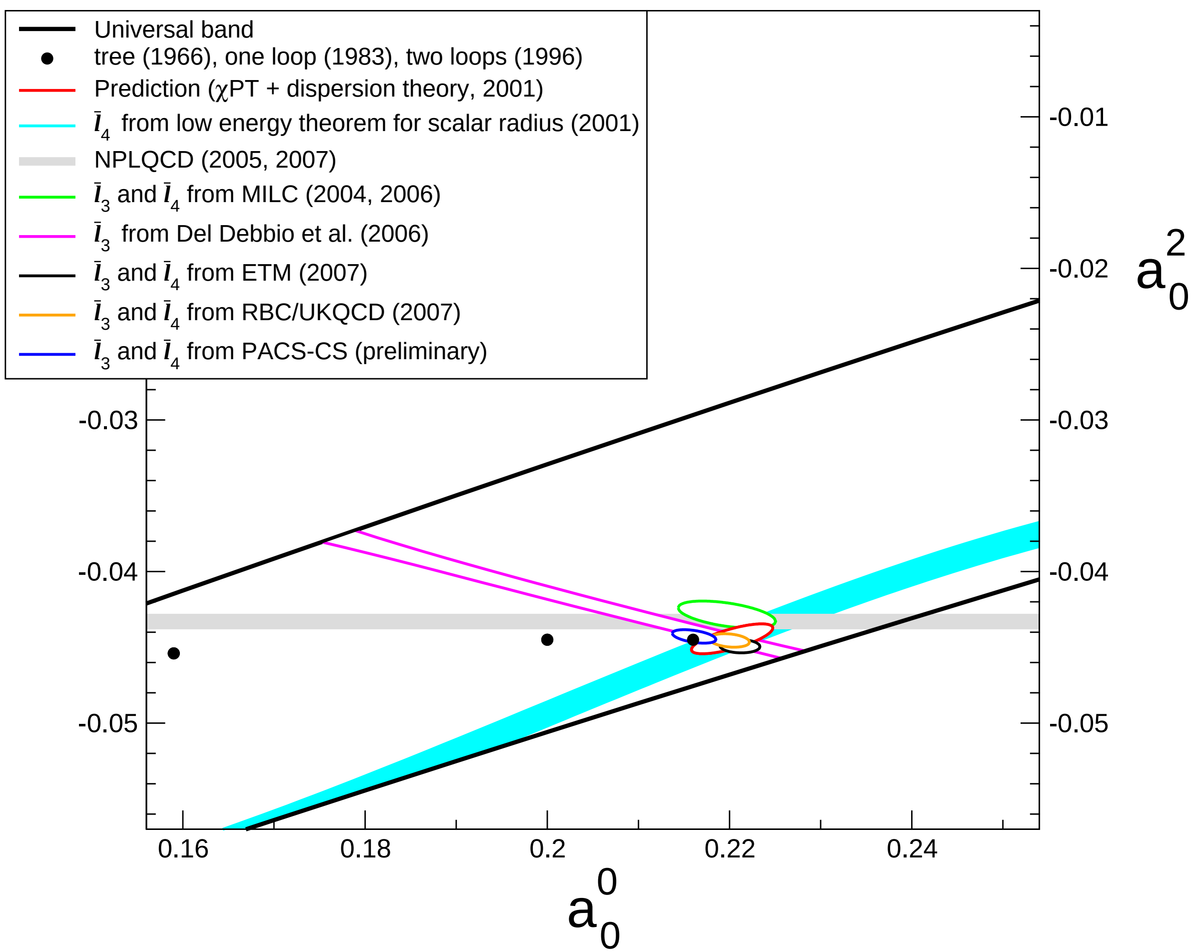
Lattice result for a_0^2

- Lattice allows direct measurement of a_0^2 via volume dependence of energy levels



Consequence of lattice results for ℓ_3, ℓ_4

- Uncertainty in prediction for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4
Can make use of the lattice results for these



Experiments on light flavours at low energy

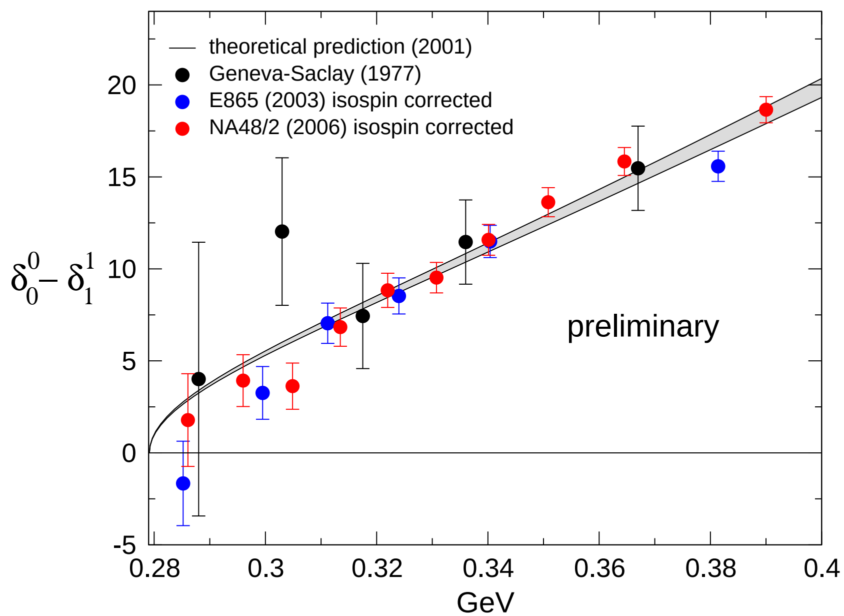
- Production experiments $\pi N \to \pi\pi N$, $\psi \to \pi\pi\omega \dots$
  Problem: pions are not produced in vacuo

$\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple
  Accuracy rather limited

- $\pi^+\pi^-$ atoms, DIRAC
- $K^\pm \to \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $K^\pm \to \pi^+\pi^- e^\pm \nu$ precision data from E865, NA48/2

K_{e4} decay

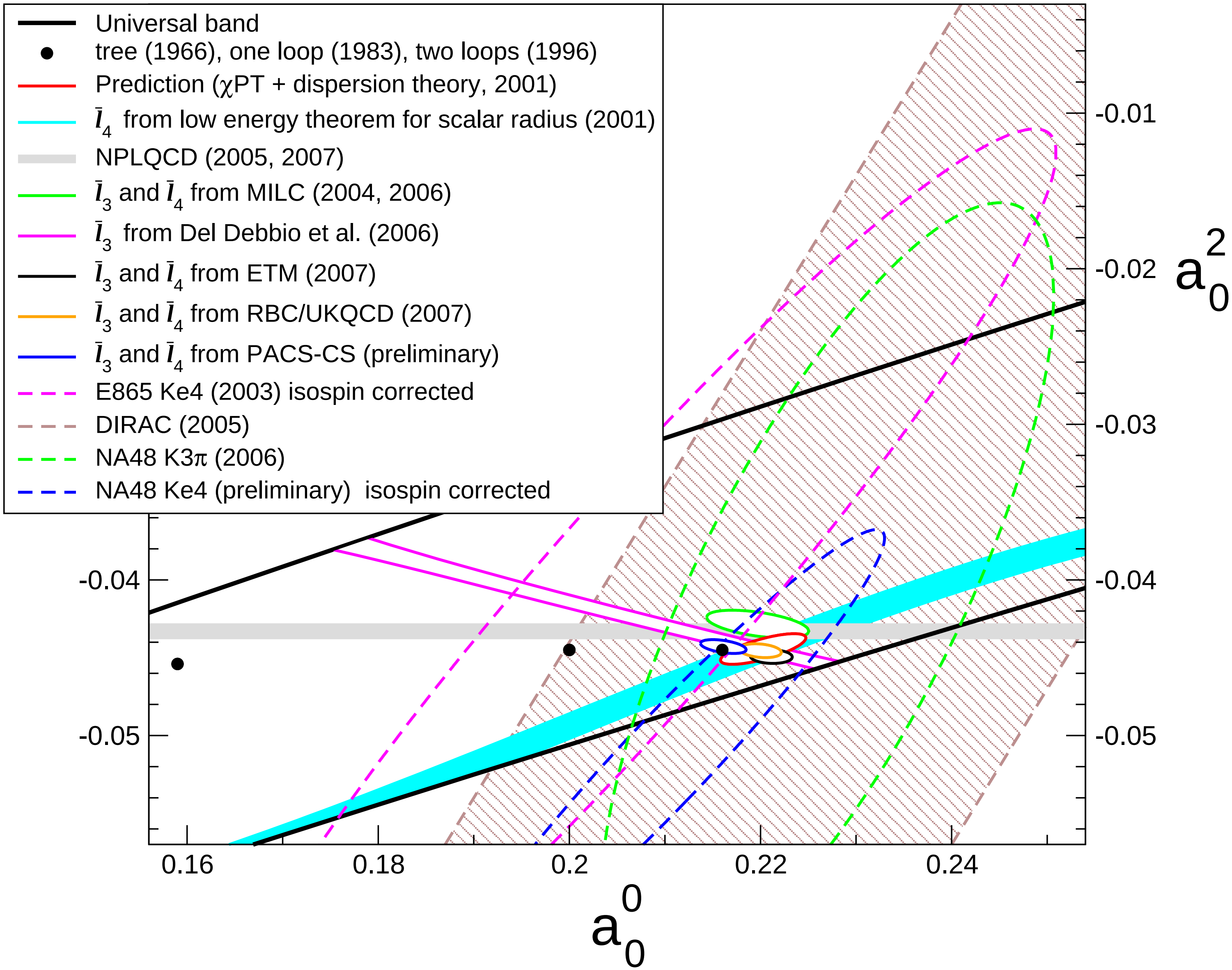
- $K \to \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007

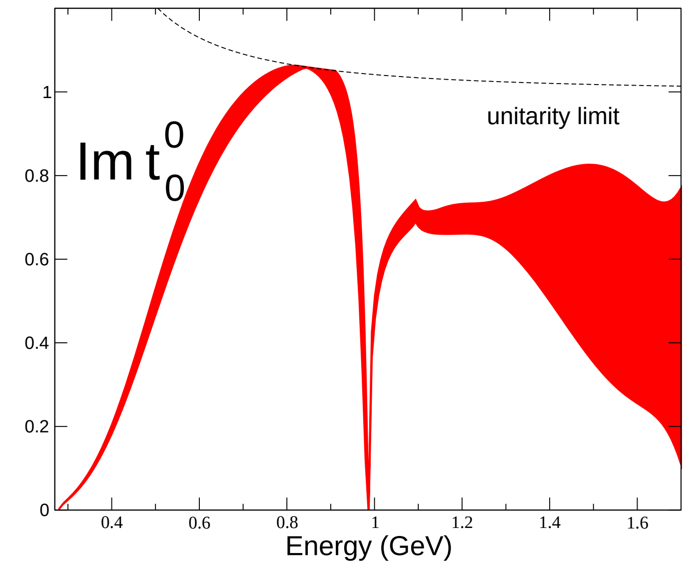
a_0^0, a_0^2 : prediction, lattice & experiment



∼ Le dessert ∼

Lowest resonance of QCD

The red dragon



There is the broad object seen in $\pi\pi$ *scattering, often called "background", which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance*[2] *which we identify as the lightest glueball with quantum numbers* $J^{PC} = 0^{++}$ ...

[2] *we refer to it as red dragon*

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

Where is the lowest resonance of QCD ?

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001

- Does QCD have a resonance near threshold ?
  - Concerns the nonperturbative domain of QCD
  - Quark and gluon degrees of freedom useless there

  $\Rightarrow$ Understanding very poor, pattern of energy levels ?
  - Lowest resonance: σ ? ρ ?
- Resonance $\leftrightarrow$ pole on second sheet
  - Poles are universal
  - Pole position is unambiguous, even if width is large
  - Where is the pole closest to the origin ?

$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
(400–1200)$-i$(250–500) OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$(552^{+84}_{-106})-i(232^{+81}_{-72})$	[1] ABLIKIM	07A	BES2	$\psi(2S) \to \pi^+\pi^- J/\psi$
$(441^{+16}_{-8})-i(272^{+9}_{-12.5})$	[2] CAPRINI	06	RVUE	$\pi\pi \to \pi\pi$
$(470 \pm 50)-i(285 \pm 25)$	[3] ZHOU	05	RVUE	
$(541 \pm 39)-i(252 \pm 42)$	[4] ABLIKIM	04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528 \pm 32)-i(207 \pm 23)$	[5] GALLEGOS	04	RVUE	Compilation
$(440 \pm 8)-i(212 \pm 15)$	[6] PELAEZ	04A	RVUE	$\pi\pi \to \pi\pi$
$(533 \pm 25)-i(247 \pm 25)$	[7] BUGG	03	RVUE	
$532 - i272$	BLACK	01	RVUE	$\pi^0\pi^0 \to \pi^0\pi^0$
$(470 \pm 30)-i(295 \pm 20)$	[2] COLANGELO	01	RVUE	$\pi\pi \to \pi\pi$
$(535^{+48}_{-36})-i(155^{+76}_{-53})$	[8] ISHIDA	01		$\Upsilon(3S) \to \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	[9] SUROVTSEV	01	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(558^{+34}_{-27})-i(196^{+32}_{-41})$	ISHIDA	00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH	99	RVUE	π scalar form factor
$(523 \pm 12)-i(259 \pm 7)$	KAMINSKI	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\sigma\sigma$
$442 - i\ 227$	OLLER	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$469 - i203$	OLLER	99B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$445 - i221$	OLLER	99C	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$
$(1530^{+90}_{-250})-i(560 \pm 40)$	ANISOVICH	98B	RVUE	Compilation
$420 - i\ 212$	LOCHER	98	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(602 \pm 26)-i(196 \pm 27)$	[10] ISHIDA	97		$\pi\pi \to \pi\pi$
$(537 \pm 20)-i(250 \pm 17)$	[11] KAMINSKI	97B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, 4π
$470 - i250$	[12,13] TORNQVIST	96	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
$\sim (1100 - i300)$	AMSLER	95B	CBAR	$\overline{p}p \to 3\pi^0$
$400 - i500$	[13,14] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$1100 - i137$	[13,15] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$387 - i305$	[13,16] JANSSEN	95	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$525 - i269$	[17] ACHASOV	94	RVUE	$\pi\pi \to \pi\pi$
$(506 \pm 10)-i(247 \pm 3)$	KAMINSKI	94	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$370 - i356$	[18] ZOU	94B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$408 - i342$	[13,18] ZOU	93	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$870 - i370$	[13,19] AU	87	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$470 - i208$	[20] VANBEVEREN	86	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, ...
$(750 \pm 50)-i(450 \pm 50)$	[21] ESTABROOKS	79	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(660 \pm 100)-i(320 \pm 70)$	PROTOPOP...	73	HBC	$\pi\pi \to \pi\pi$, $K\overline{K}$
$650 - i370$	[22] BASDEVANT	72	RVUE	$\pi\pi \to \pi\pi$

HTTP://PDG.LBL.GOV Page 1 Created: 6/7/2007 11:24

PDG tables, edition 2007

Model independent determination of the pole

- Most of the results quoted by the PDG are obtained by
  (a) parametrizing the data for real values of s
  (b) continuing this parametrization analytically in s

  $\Rightarrow$ Result is sensitive to the parametrization used

- We found a model independent method:
  1. Poles on second sheet are zeros on first sheet
  2. The Roy equations are valid for complex values of s, in a limited region of the first sheet

  $\Rightarrow$ Exact representation of the partial waves in terms of observable quantities, valid for complex values of s

  3. Can evaluate this representation to good precision and determine the zeros numerically

Roy equation for the isoscalar S-wave

Two S-waves: I = 0 and I = 2

$$S_0^0(s)=\eta_0^0(s)\exp 2i\delta_0^0(s)$$ S-matrix element

$$S_0^0(s)=1+2\,i\rho\,t_0^0(s) \qquad \rho=\sqrt{1-4M_\pi^2/s}$$

↑ partial wave amplitude

$$t_0^0(s)=a+(s-4M_\pi^2)\,b+ \\ +\sum_{I=0}^{2}\sum_{\ell=0}^{\infty}\int_{4M_\pi^2}^{\infty}ds'K_{0\ell}^{0I}(s,s')\,\mathrm{Im}\,t_\ell^I(s')$$ Roy equation

- The subtraction constants are determined by a_0^0, a_0^2:

$$a=a_0^0\,,\qquad b=(2a_0^0-5a_0^2)/(12M_\pi^2)$$

Roy equation for the isoscalar S-wave

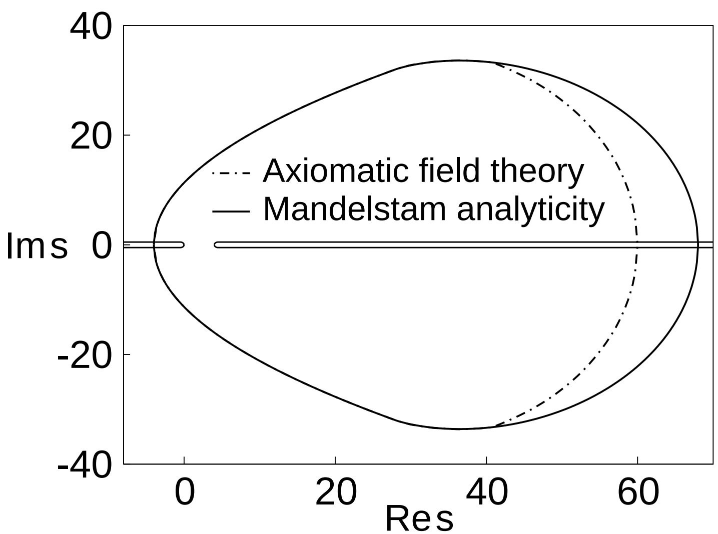
$$t_0^0(s) = a + (s - 4M_\pi^2)\, b + \\ + \sum_{I=0}^{2} \sum_{\ell=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds'\, K_{0\ell}^{0I}(s,s')\, \mathrm{Im}\, t_\ell^I(s')$$

- The kernels are elementary functions, e.g.
$$K_{00}^{00}(s,s') = \underbrace{\frac{1}{\pi(s'-s)}}_{r.h.cut} + \underbrace{\frac{2\, ln\{(s+s'-4M_\pi^2)/s'\}}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}}_{l.h.cut}$$
- Left hand cut is essential for convergence:
$K_{00}^{00}(s,s') \sim 1/s'^3$ for large s'

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$
- Equations are valid for complex s in a limited region of the first sheet

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001



s in units of M_π^2

- Proof is based on first principles, general quantum field theory

A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing*, Lecture Notes in Physics, vol. 3, 1969.

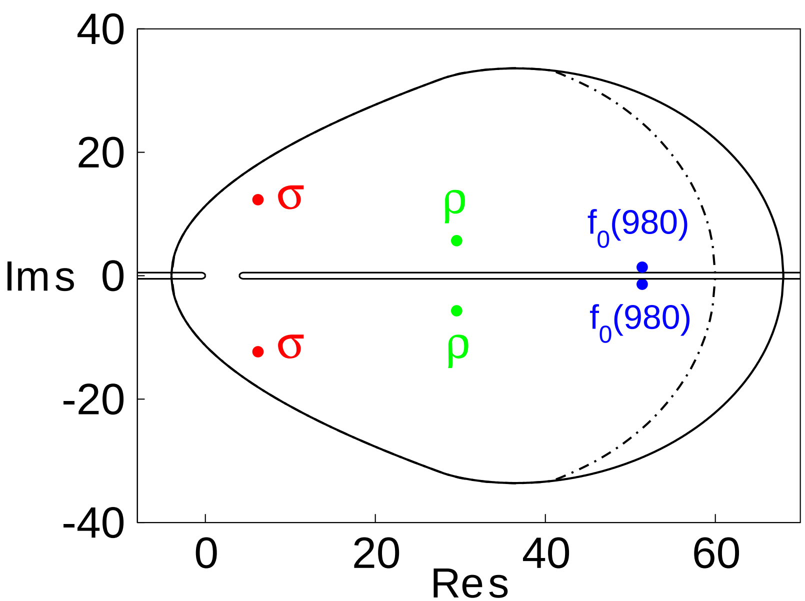
G. Mahoux, S. M. Roy and G. Wanders, Nucl. Phys. B70 (1974) 297.

$\Rightarrow$ Exact representation for $S_0^0(s)$ in this region
Do not need to parametrize the amplitude

Evaluation of the pole position

- Have an exact formula for the pole position in terms of physical quantities: $S_0^0(s) = 0$
- For the central solution of the Roy equations, $S_0^0(s)$ has two pairs of zeros in the region where the formula holds:

$$s = (6.2 \pm i\,12.3)\,M_\pi^2 \quad \sigma$$
$$s = (51.4 \pm i\,1.4)\,M_\pi^2 \quad f_0(980)$$



s in units of M_π^2

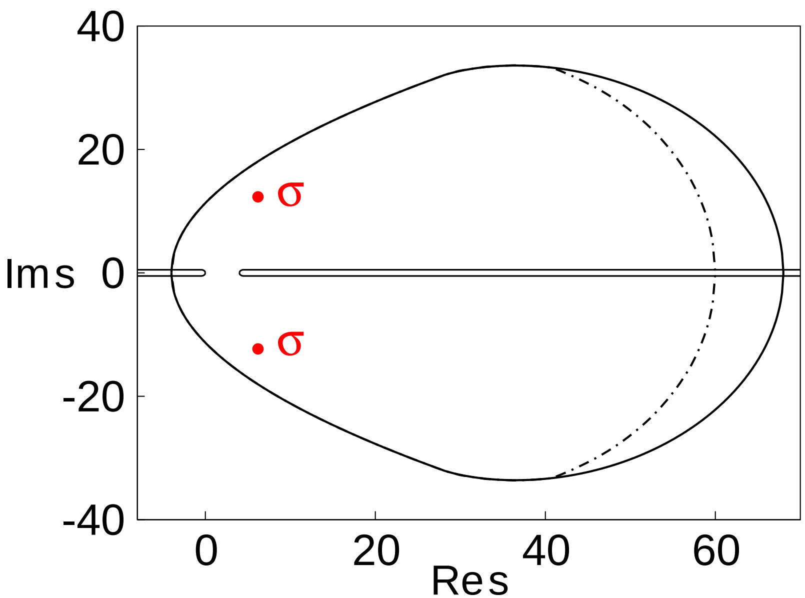
The eyes of the red dragon

Tail at 1.7 GeV: $s \simeq 150\,M_\pi^2$

Result

- Lowest resonance of QCD has vacuum quantum numbers
- Pole on lower half of sheet II occurs in vicinity of

$$m_\sigma = 441 - i\,272\,\text{MeV} = M_\sigma - \tfrac{i}{2}\Gamma_\sigma$$



s in units of M_π^2

Loci Oculorum Draconis Rutili

T. Barnes, Theory summary, MESON 2006

Error analysis

- Formula is exact, evaluation is approximate
- Key point: can follow error propagation explicitly
- Split the formula into 3 pieces:
  1. Subtraction terms
  2. Contribution from $\text{Im}\, t_0^0(s)$ below $K\bar{K}$ threshold
  3. Higher energies and other partial waves
- Subtractions terms are determined by a_0^0, a_0^2

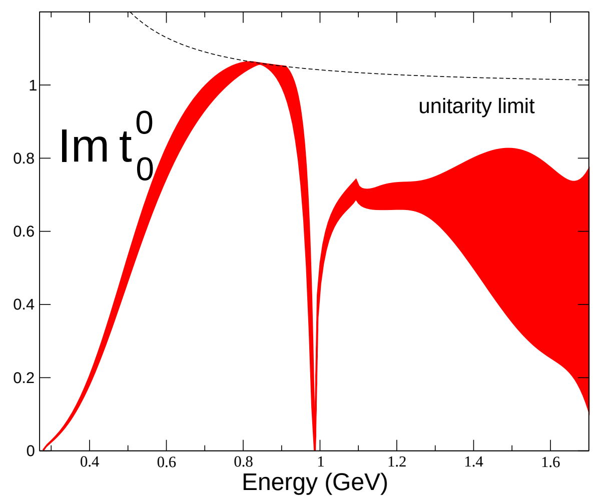
$\Rightarrow$ Errors from there are very small

- Slowly varying background from 3. is dominated by the ρ, also accurately known

$\Rightarrow$ Uncertainty in result for σ pole is dominated by 2.

Behaviour of Im $t_0^0(s)$ below $K\bar{K}$ threshold

↓ $K\bar{K}$ treshold



$\Rightarrow$ Head of dragon determines position of the σ pole

- Unitarity and dip leave little room above 0.8 GeV

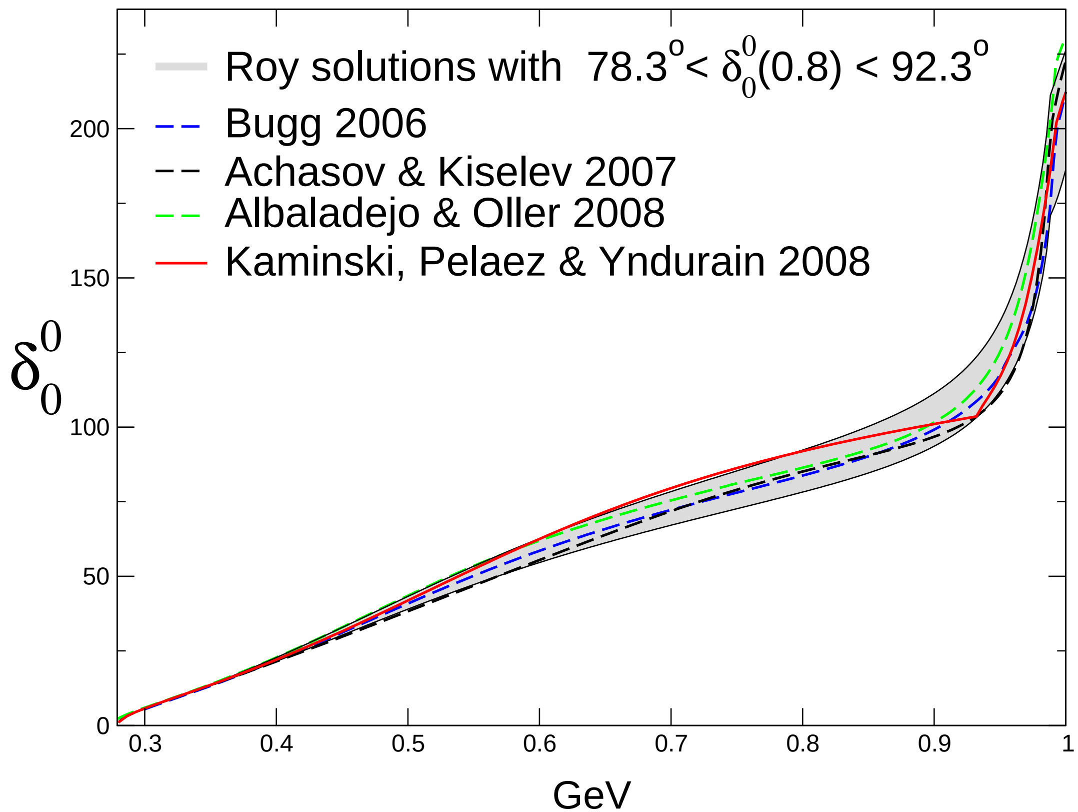
$\Rightarrow$ Only the region below 0.8 GeV really matters

- There, the partial waves are nearly elastic:

$$\text{Im}\, t_0^0(s) \simeq \sin^2 \delta_0^0(s) / \sqrt{1 - 4M_\pi^2/s}$$

$\Rightarrow$ Need to know the phase $\delta_0^0(s)$ below 0.8 GeV

How well do we know $\delta_0^0(s)$ below $K\bar{K}$ threshold ?



Bugg uses BES data for $J/\psi \longrightarrow \omega\pi\pi$

Achasov & Kiselev use KLOE data on $\phi \longrightarrow \gamma\pi\pi$

Albaladejo & Oller: N/D fit to several data sets

Kaminski, Pelaez and Yndurain: $\pi\pi$, K_{e4}, $K \longrightarrow \pi\pi$

Thorough discussion of the phenomenological uncertainties in $\delta_0^0(s)$:
Caprini, arXiv:0804.3504

Illustration

- Replace the central Roy solution below $2M_K$ by the phase representation of Bugg 2006 $\Rightarrow$ pole moves from $441 - i\,272$ to $444 - i\,267$ MeV
- Ditto with Achasov & Kiselev 2007:
$438 - i\,274$ MeV
- Ditto with Albaladejo & Oller 2008:
$451 - i\,257$ MeV
- Ditto with Kamiński, Peláez and Yndúráin 2008:
$458 - i\,253$ MeV

Final result for the pole position

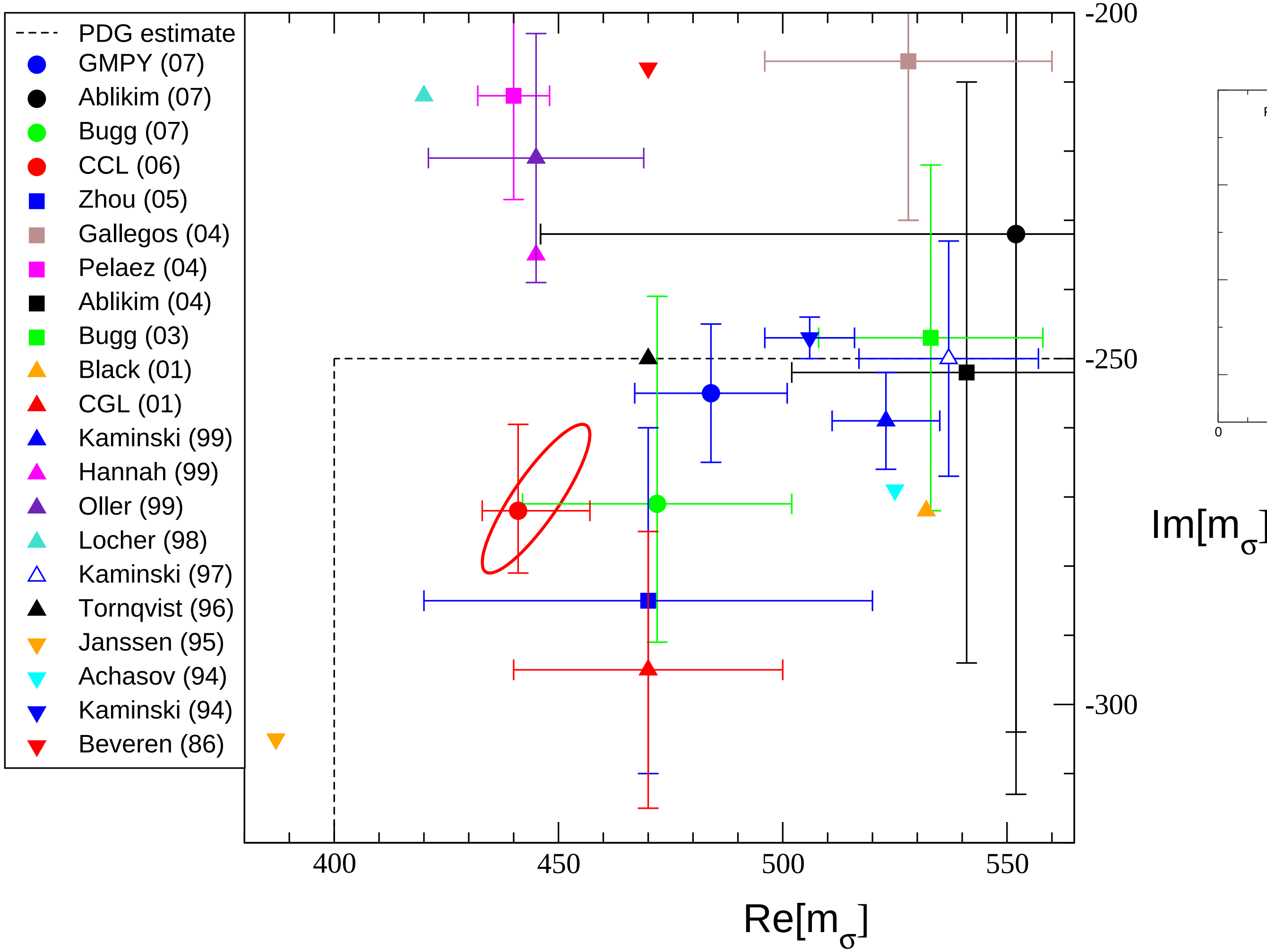
1. Subtraction terms
2. Contribution from Im $t_0^0(s)$ below $K\bar{K}$ threshold
3. Higher energies and other partial waves

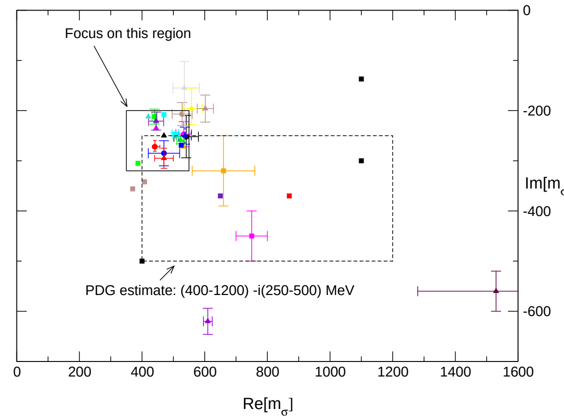
- Vary these contributions wihin their uncertainties, follow error propagation and sum the errors up

$$\Rightarrow \quad m_\sigma = 441\,{}^{+16}_{-8} \; - i\, 272\,{}^{+9}_{-13}\ \text{MeV}$$

Caprini, Colangelo and L. 2006

Comparison with compilation in PDG 2007





Physical interpretation of the σ

- Glueball ? $\bar{q}q$? $\bar{q}q\bar{q}q$?
- Hadrons in terms of quarks and gluons ?
- Fock space is basis of perturbation theory – not clear how to use it quantitatively in nonperturbative domain
- My qualitative picture of the σ:
  - Can understand position of σ pole in terms of F_π

  $\Rightarrow$ Low energy properties of the pions are relevant

  $\Rightarrow$ Physics of the $\sigma \in$ Goldstone boson dynamics

  $\Rightarrow$ Head of the dragon contains only little glue

  $\Rightarrow$ Wave function has large tetra-quark component
- This picture is by no means commonly accepted

  Törnqvist, Ishida, Jaffe, Minkowski, Ochs, Bugg, Pennington, Peláez, Oller, Hannah, Guo, Su, Xiao, Zheng, Zhou, Chen, Hosaka, Zhu, Liu, Maiani, Polosa, Piccinini, Riquer, Isidori, Nicolaci, Pacetti, Menessier, Narison, Fariborz, Jora, Schechter, van Beveren, Kleefeld, Rupp, Scadron, Ynduráin, García-Martín, . . .

$\Rightarrow$ Comprehensive review : Klempt & Zaitsev, arXiv:0708.4016

VISIT THE RED DRAGON

GENTLE ANIMAL
LOOK IN HIS EYES FROM CLOSE
SMELL HIS GOOD BREATH
BRING YOUR PIONS ALONG AND
FEED HIM YOURSELF

The management denies responsibility for incidents involving the dragon's tail

Conclusion

- Low energy pion physics: theory ahead of experiment
  - Precision experiments carried out and under way
  - Lattice makes slow, but steady progress
  - With one exception ($K_{\mu 3}$) all tests confirm the theory
- Limitations of our approach:
  - Calculations cannot be done on back of an envelope
  - Analysis only covers low energies
    Extension to higher energies is under way
  - Only a few applications have been worked out:
    $\pi\pi$ scattering, pion form factors, hadronic vacuum polarization in muon $g-2$
  - Much is yet to be done: $J/\psi \to \omega\pi\pi$, $D \to 3\pi$,
    $\gamma\gamma \to \pi\pi$, πK, πN, $\ldots$

Conclusion

- Model independent method for analytic continuation
  - The lowest resonance of QCD occurs at

    $$M_\sigma = 441\,{}^{+16}_{-8}\ \text{MeV} \qquad \Gamma_\sigma = 544\,{}^{+18}_{-25}\ \text{MeV}$$

    and carries vacuum quantum numbers
  - Crossing symmetry plays an essential role:
    Fixes contributions from left–hand cut
    Ensures fast convergence, low energy dominance
  - Pole occurs at low value of s, closer to left–hand cut than to singularities from $K\bar{K}$, $f_0(980)$
  - Result for Γ_σ relies on theory for a_0^2
    Experiments concerning a_0^2 would be most welcome

SPARES

NNLO

- The next order contains the square of a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{x}{2} \ln \frac{M^2}{\Lambda_3^2} + \frac{17x^2}{8} \left(\ln \frac{M^2}{\Lambda_M^2} \right)^2 + x^2 k_M + O(M^6) \right\}$$

$$F_\pi = F \left\{ 1 - x \ln \frac{M^2}{\Lambda_4^2} - \frac{5x^2}{4} \left(\ln \frac{M^2}{\Lambda_F^2} \right)^2 + x^2 k_F + O(M^6) \right\}$$

$$x \equiv \left(\frac{M}{4\pi F} \right)^2$$

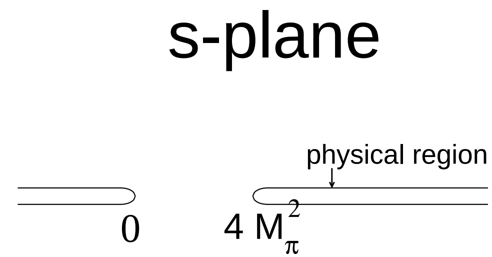
Colangelo 1995, Bijnens et al. 1996, Bürgi 1996

- For physical value of m_u, m_d, the NNLO terms are tiny

$\Rightarrow$ Size of $\Lambda_M, k_M, \Lambda_F, k_F$ barely known

- Must become clearly visible if m_u, m_d are made larger

Pole on second sheet $\leftrightarrow$ zero on first sheet



- $S^0_0(s) = \eta^0_0(s)\exp 2i\delta^0_0(s)$

  $S^0_0(s)$ is analytic in the cut plane

- For $0 < s < 4M_\pi^2$, $S^0_0(s)$ is real

$\Rightarrow$ $S^0_0(s^\star) = S^0_0(s)^\star$

  x in elastic interval: $S^0_0(x \pm i\epsilon) = \exp \pm 2i\delta^0_0(x)$

- Second sheet is reached by continuation across the elastic interval of the right–hand cut

  $$S^0_0(x - i\epsilon)^{II} = S^0_0(x + i\epsilon)^{I} = 1/S^0_0(x - i\epsilon)^{I}$$

  Analyticity $\Rightarrow$ $\boxed{S^0_0(s)^{II} = 1/S^0_0(s)^{I}}$ valid $\forall s$

  Pole in $S^0_0(s)^{II}$ $\Longleftrightarrow$ zero in $S^0_0(s)^{I}$

The κ

- $K\pi$ scattering amplitude obeys an analog of the Roy equations. Pole from κ can be calculated on this basis

$$m_\kappa = (658 \pm 13) - i\,(278.5 \pm 12)\,\text{MeV}$$

Descotes-Genon and Moussallam 2006

- Confirms an earlier calculation, where the l.h. cut was estimated with χPT Zhou and Zheng 2006
- Back-of-the-envelope calculation for $K\pi$ gives

$$m_\kappa = 671 - i\,262\,\text{MeV}$$

$\Rightarrow$ Physics of σ and κ is very similar

Qualitative picture for κ, $f_0(980)$, ...

- Can also understand the κ pole in terms of F_π
- $f_0(980)$ and $a_0(980)$
  - Suspiciously close to $K\bar{K}$ threshold - an accident ?
  - Interaction among two kaons plays important role
- Multiplet pattern ?
  - The $\pi\pi$, πK, $K\bar{K}$ thresholds strongly break SU(3)

  $\Rightarrow$ Expect strong symmetry breaking in the masses and widths of the lowest 0^+ resonances
  - Do these form complete SU(3) multiplets at all ?