Recent developments in the physics of the light quarks

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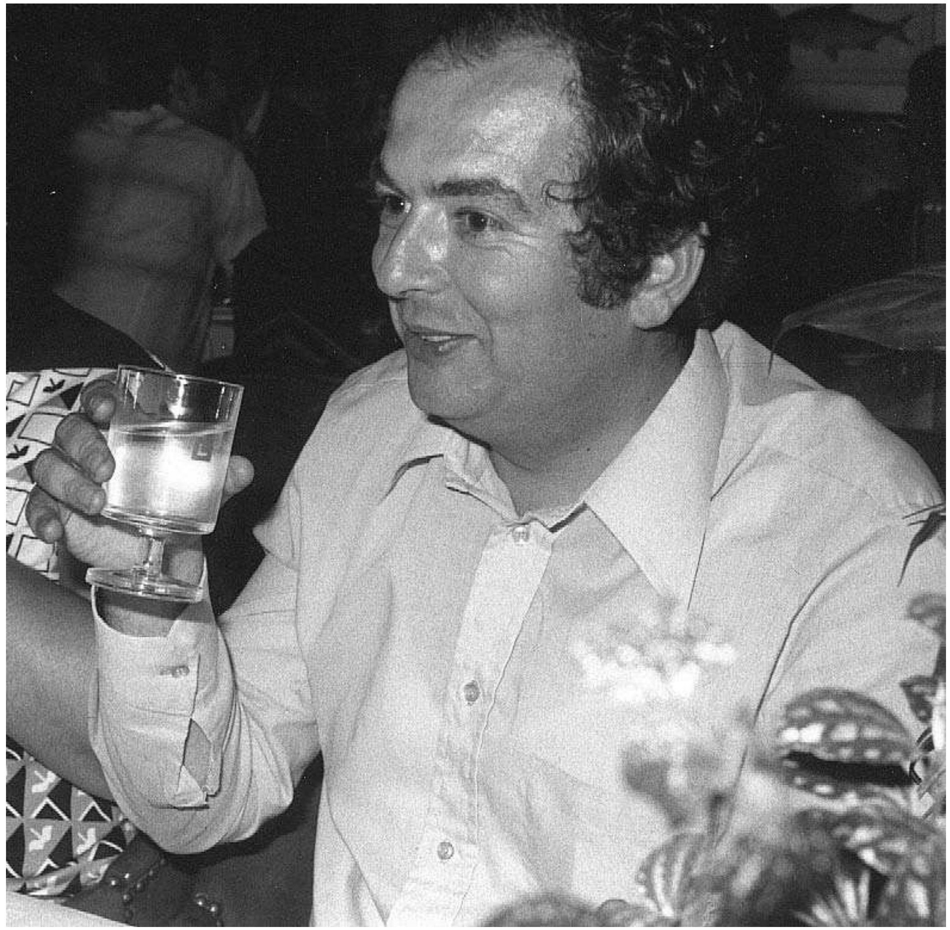
Confinement8

Mainz, September 3, 2008

In memorian Jan Stern



29. 6. 1942 – 2. 7. 2008

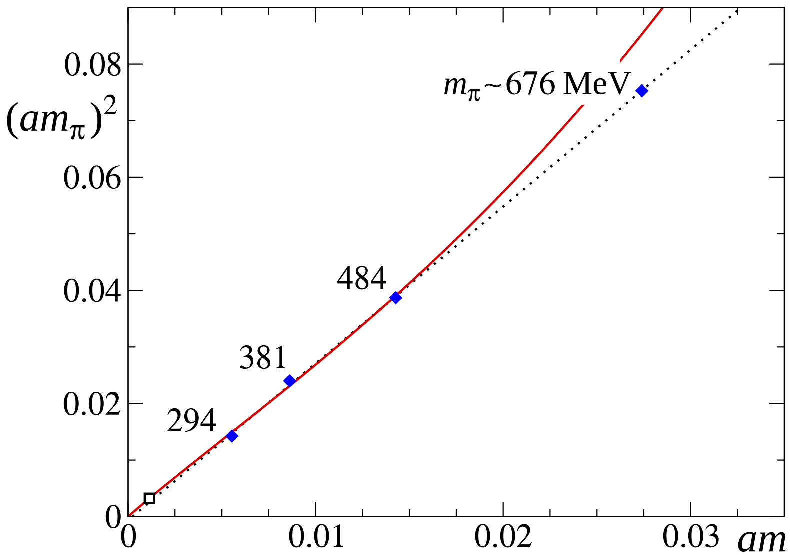
Energy gap of QCD

- *Main characteristic of QCD at low energies: energy gap is very small,* $M_\pi \simeq$ *140 MeV*
- *Nambu found out why this is so: the strong interaction has a hidden, approximate symmetry*
  Nambu 1960
- *Gap is determined by the masses of the two lightest quarks*
  Gell-Mann, Oakes & Renner 1968

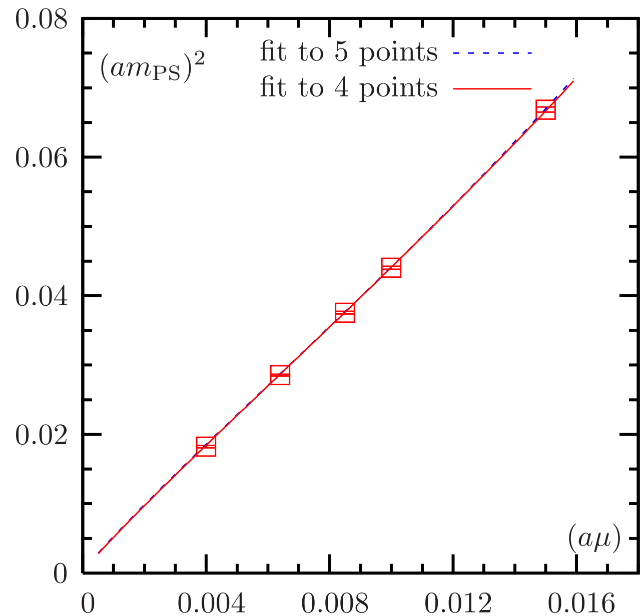
$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \textit{explicit}}}{+} m_d) \times |\langle 0|\, \underset{\substack{\Uparrow \\ \textit{spontaneous}}}{\bar{u}u}\, |0\rangle| \times \frac{1}{F_\pi^2}$$

Checking the GMOR formula on the lattice

- *Can determine* M_π *as a function of* $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- *No quenching, quark masses are sufficiently light*

$\Rightarrow$ *Legitimate to use* χ*PT for the extrapolation to the physical values of* m_u, m_d

Expansion of M_π^2 in powers of the quark masses

- Gell-Mann-Oakes-Renner formula represents leading term of the chiral perturbation series
- Disregard isospin breaking, set $m_u = m_d = m$
- Expand in powers of m, keeping m_s fixed
- At NLO, the expansion contains a logarithm

Langacker & Pagels 1973, Gasser and Zepeda 1980, Gasser 1981

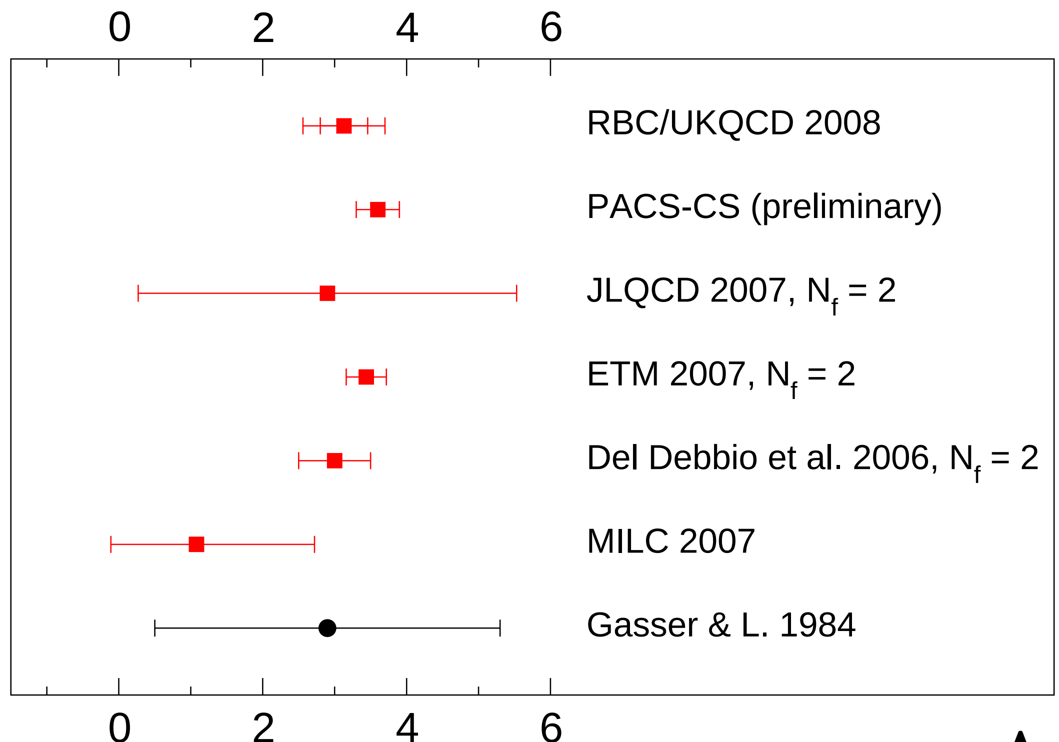
$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

$$M^2 \equiv 2Bm$$

- Coefficient is determined by the pion decay constant
  Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3)×SU(3):
  0.2 GeV $\lesssim \Lambda_3 \lesssim 2$ GeV

Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



Horizontal axis shows the value of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

$\longrightarrow$ *talk by Silvia Necco*

Range for Λ_3 *obtained in 1984 corresponds to* $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of RBC/UKQCD 2008: $\bar{\ell}_3 = 3.13 \underset{stat}{\pm 0.33} \underset{syst}{\pm 0.24}$

Expansion of F_π in powers of the quark mass

- *Also contains a logarithm at NLO:*

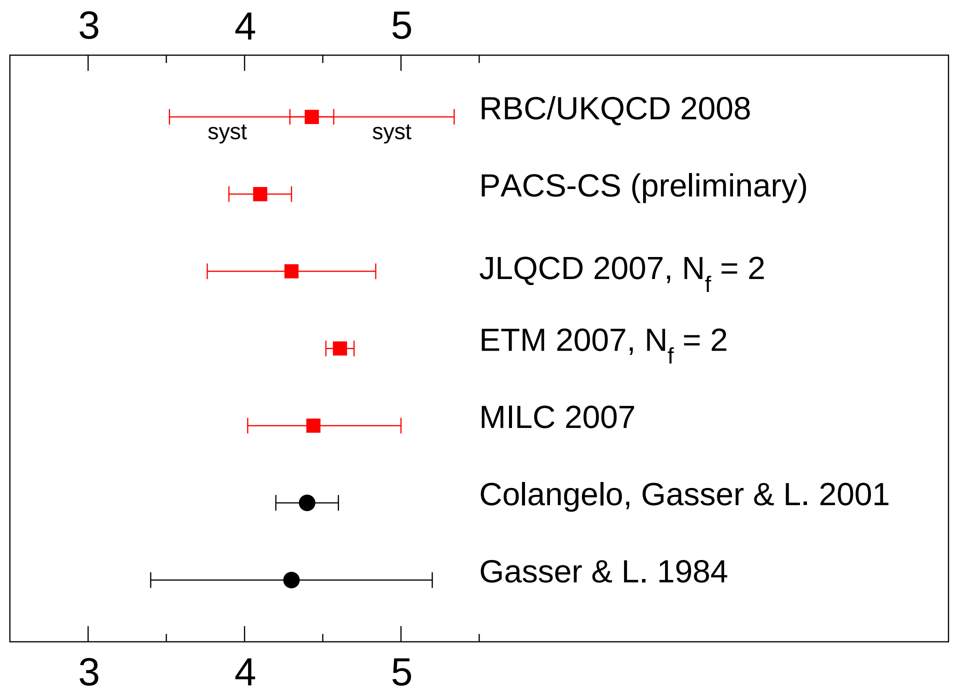
$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

F *is value of pion decay constant in limit* $m_u, m_d \to 0$

- *Structure is the same, coefficients and scale of logarithm are different*
- *Quark mass dependence of* F_π *can also be measured on the lattice* $\Rightarrow$ *measurement of* Λ_4
- *Alternative method: determine the scalar form factor of the pion, radius* $\langle r^2\rangle_s \Leftrightarrow \bar{\ell}_4$

Lattice results for Λ_4



$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

⟶ talk by Silvia Necco

- *Lattice results beautifully confirm the prediction for the sensitivity of F_π to m_u, m_d:*

$$\frac{F_\pi}{F} = 1.072 \pm 0.007$$

Colangelo and Dürr 2004

$\pi\pi$ interaction

- Sharp predictions for S-wave $\pi\pi$ scattering lengths
  Uncertainty in prediction is dominated by uncertainty in ℓ_3, ℓ_4

$\Rightarrow$ Can make use of the lattice results for these

$\longrightarrow$ talk by Gerhard Ecker

- Experimental tests
  - Production experiments
    $\pi N \longrightarrow \pi\pi N$, $\psi \longrightarrow \pi\pi\omega \ldots$
    Problem: pions are not produced in vacuo

  $\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple

  $\Rightarrow$ Accuracy rather limited

  - Precision measurements
    $K^\pm \longrightarrow \pi^+\pi^- e^\pm \nu$ CERN-Saclay, E865, NA48/2
    $K^\pm \longrightarrow \pi^0\pi^0\pi^\pm$ cusp near threshold NA48/2
    $\pi^+\pi^-$ atoms DIRAC

$\longrightarrow$ talk by Brigitte Bloch-Devaux

Conclusions for SU(2)×SU(2)

- *Expansion in powers of* m_u, m_d *yields a very accurate low energy representation of QCD*
- *Lattice results confirm the GMOR relation*

$\Rightarrow$ M_π *is dominated by the contribution from the quark condensate*

$\Rightarrow$ *Energy gap of QCD is understood very well*

- *Lattice approach allows an accurate measurement of the effective coupling constant* ℓ_3 *already now*
- *Even for* ℓ_4*, the lattice starts becoming competitive with dispersion theory*
- *Precision experiments test the theory to high accuracy*

Expansion in powers of m_s

- *Theoretical reasoning:*
  - *The eightfold way is an approximate symmetry*
  - *The only coherent way to understand this within QCD:* $m_s - m_d$, $m_d - m_u$ *can be treated as perturbations*
  - *Since* $m_u, m_d \ll m_s$

$\Rightarrow$ m_s *can be treated as a perturbation*

$\Rightarrow$ *Expect expansion in powers of* m_s *to work, but convergence to be comparatively slow*

- *In principle, this can now also be checked on the lattice*

Paramagnetic inequalities

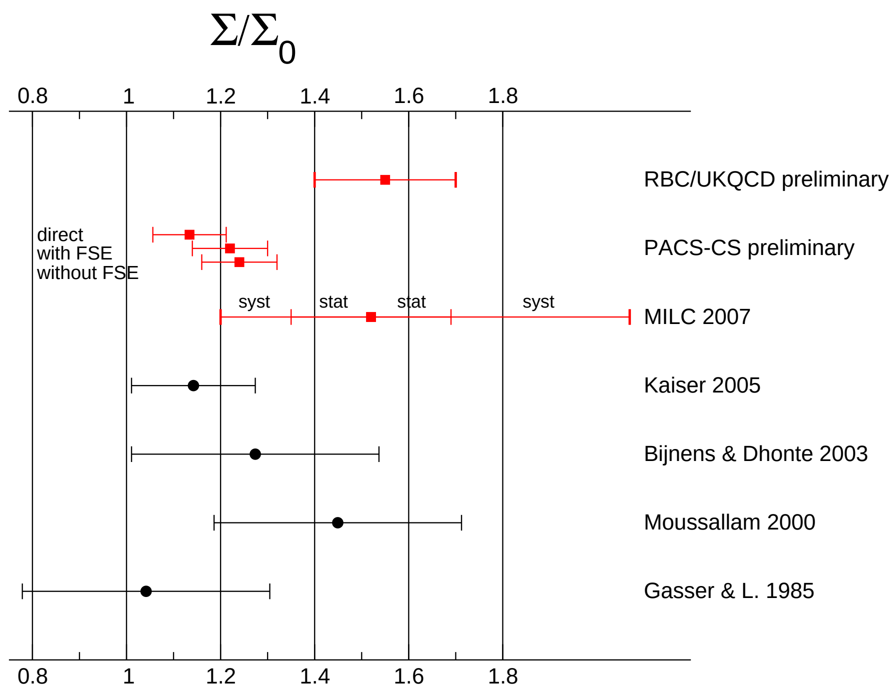
- *Consider the limit* $m_u, m_d \longrightarrow 0$, m_s *physical*
  - F *is value of* F_π *in this limit*
  - Σ *is value of* $|\langle 0|\,\bar{u}u|\,|0\rangle$ *in this limit*
  - B *is value of* $M_\pi^2/(m_u + m_d)$ *in this limit*
- *Exact relation:* $\Sigma = F^2 B$
- F_0, B_0, Σ_0*: values for* $m_u = m_d = m_s = 0$
- *Inequalities set up by Jan Stern and collaborators:*
  both F *and* Σ *should decrease if* m_s *is taken smaller*

$$F > F_0\,,\ \Sigma > \Sigma_0$$

- *For* $N_c \longrightarrow \infty$*:* F, Σ *and* B *become independent of* m_s

$\Rightarrow$ $(F/F_0 - 1), (\Sigma/\Sigma_0 - 1), (B/B_0 - 1)$ *violate the OZI rule*

Condensate

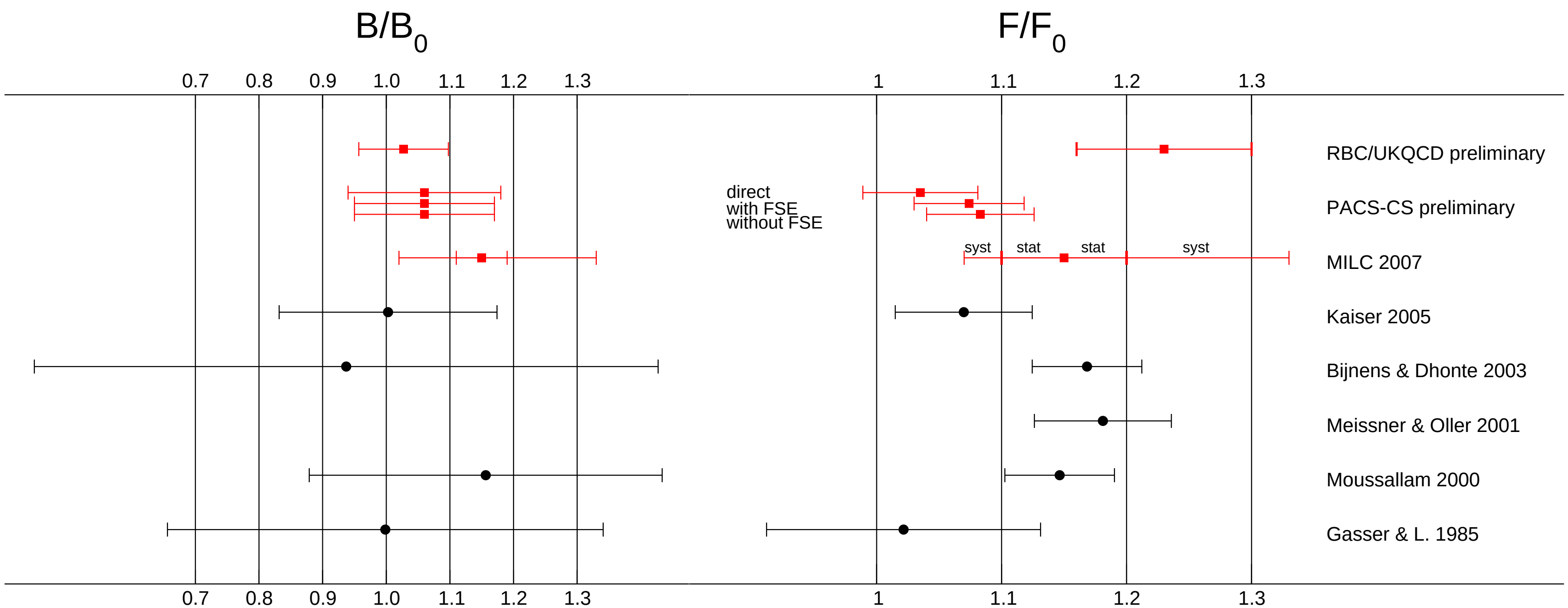


- *Central values of RBC/UKQCD and PACS-CS for* Σ/Σ_0 *lead to qualitatively different conclusions concerning OZI-violations*

$\Rightarrow$ *Discrepancy indicates large systematic errors*

- *The lattice results confirm the parametric inequalities, but do not yet allow to draw conclusions about the size of the OZI-violations*

Results for B, F



- *Results for* B *are coherent, indicate small OZI-violations in* B

$\Rightarrow$ F *is the crucial factor in* $\Sigma = F^2 B$

Conclusion for validity of Zweig rule ?

- *The available lattice data allow for very juicy OZI-violations, but are also consistent with* $B/B_0 \simeq F/F_0 \simeq \Sigma/\Sigma_0 \simeq 1$
- *The central value of RBC/UKQCD for* F/F_0 *is* 1.23. *If this result were confirmed within small uncertainties, we would be faced with a qualitative puzzle:*
  - F_π *is the pion wave function at the origin*
  - F_K *is larger because one of the two valence quarks is heavier* $\longrightarrow$ *moves more slowly* $\longrightarrow$ *wave function more narrow* $\longrightarrow$ *higher at the origin:* $F_K/F_\pi \simeq 1.19$
  - $F/F_0 = 1.23$ *indicates that the wave function is more sensitive to the mass of the sea quarks than to the mass of the valence quarks . . . very strange* $\longrightarrow$ *extraordinarily interesting*

Two examples where χPT appears to fail

First example:

- $K_{\ell 3}$ *decay: value of* $f_+^{K^0\pi^-}(0)$
  - V_{ud} *from nuclear* β *decay + CKM unitarity +* $K_{\ell 3}$ *decay rate*

  $\Rightarrow f_+^{K^0\pi^-}(0) = 0.9594 \pm 0.0049$

  - *Form factor known to NNLO.* *Post & Schilcher, Bijnens & Talavera*

    Result obtained with resonance estimates for the effective coupling constants:

  $\Rightarrow f_+^{K^0\pi^-}(0) = 0.986 \pm 0.007_{1/N_c} \pm 0.002_{M_S,M_P}.$

Cirigliano et al., Kastner & Neufeld

$\longrightarrow$ *talk by Gerhard Ecker*

Two examples where χPT appears to fail

Second example:

- $K \to \pi\pi$ *decay: value of* $\delta_0^0 - \delta_0^2$ *at* $s = M_K^2$
  - $\pi\pi$ *phase shifts accurately known from dispersion theory*
    $\delta_0^0 - \delta_0^2 = 47.5^\circ \pm 1.5^\circ$ *Colangelo et al.*
  - *In the determination from* $K \to \pi\pi$ *via Watson theorem, isospin breaking is enhanced because of the* $\Delta I = \frac{1}{2}$ *rule*
  - *Complete analysis to NLO* *Cirigliano, Ecker, Neufeld & Pich*
  - *Recent update of the numerics yields*
    $\delta_0^0 - \delta_0^2 = 57.5^\circ \pm 3.4^\circ$ *FlaviaNet Kaon Working Group*

Problems with scalar meson dominance ?

- *In both examples, resonance estimates are used*
  In my opinion, this is the weakest point
  $\longrightarrow$ *talks by Gerhard Ecker and Toni Pich*
- *Lowest resonances do dominate the momentum dependence*
  Vector meson dominance $\surd$
- *Quark mass term in* $\mathcal{L}_{QCD}$ *is a scalar operator*
- *Matrix elements dominated by scalar resonances ?*
  Can the dependence on the quark masses be accounted for with scalar meson dominance ?
  To my knowledge, this assumption has not been tested
- *Rapidly rising* $\pi\pi$ *continuum (large chiral logs),* σ *makes a broad bump, narrow peak from* $f_0(980)$*, glueballs, etc.*
- *Failure of scalar meson dominance may be the origin of the problem, but more work is needed to resolve the two puzzles*

more detailed discussion in Erice lectures, arXiv:0808.2825

Resonances: exact formula for mass and width

- *Most of the pole positions quoted by the PDG are obtained by (a) parametrizing the data for real values of s*
  (b) continuing this parametrization analytically in s

  $\Rightarrow$ *Result is sensitive to the parametrization used*

- *We found a model independent method:*
  1. *Poles on second sheet are zeros on first sheet*
  2. *The Roy equations are valid for complex values of s, in a limited region of the first sheet*

  $\Rightarrow$ *Exact representation of the S-matrix elements in terms of observable quantities, valid for complex values of s*

  $\Rightarrow$ *Exact formula for the pole position*

  3. *Can evaluate this formula to good precision and determine the pole position numerically*

Caprini, Colangelo and L., Phys. Rev. Lett. 96 (2006) 132001

Formula for resonances with $I = \ell = 0$

$$\boxed{S_0^0(s) = 0} \quad \textit{for} \;\; s = (M - \tfrac{i}{2}\Gamma)^2$$

$$S_0^0(s) = 1 + 2\,i\rho\, t_0^0(s)\,, \qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2)b + \sum_{I,\ell} \int_{4M_\pi^2}^{\infty} ds'\, K_{0\ell}^{0I}(s,s')\, \mathrm{Im}\, t_\ell^I(s')$$

- *The subtraction constants are determined by* a_0^0, a_0^2*:*

  $$a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$$

- *The kernels are elementary functions, e.g.*

  $$K_{00}^{00}(s,s') = \underbrace{\frac{1}{\pi(s'-s)}}_{r.h.cut} + \underbrace{\frac{2\,ln\{(s+s'-4M_\pi^2)/s'\}}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}}_{l.h.cut}$$

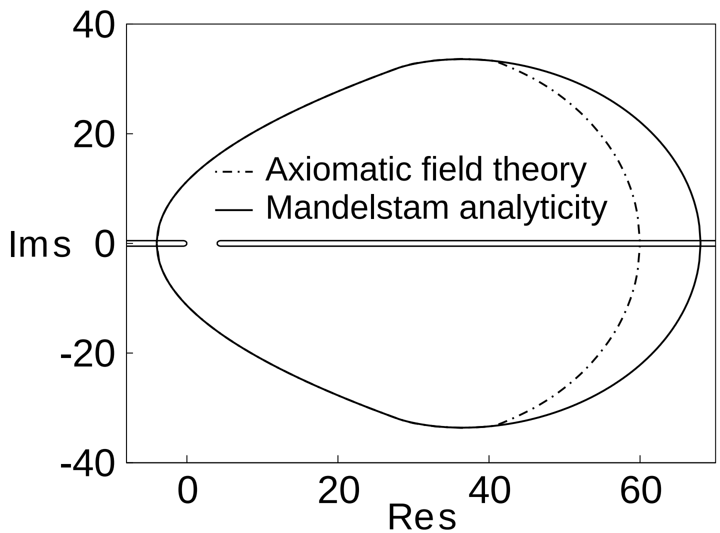
- *Left hand cut is essential for convergence:*

  $K_{00}^{00}(s,s') \sim 1/{s'}^3$ *for large* s'

Domain of validity of the Roy equations

- *Roy derived his equations for real energies in the interval* $-4M_\pi^2 < s < 60M_\pi^2$
- *Equations are valid for complex* s *in a limited region of the first sheet*

Caprini, Colangelo and L. 2006



s in units of M_π^2

- *Proof is based on first principles, general quantum field theory*

A. Martin, Scattering Theory: Unitarity, Analyticity and Crossing, Lecture Notes in Physics, vol. 3, 1969.

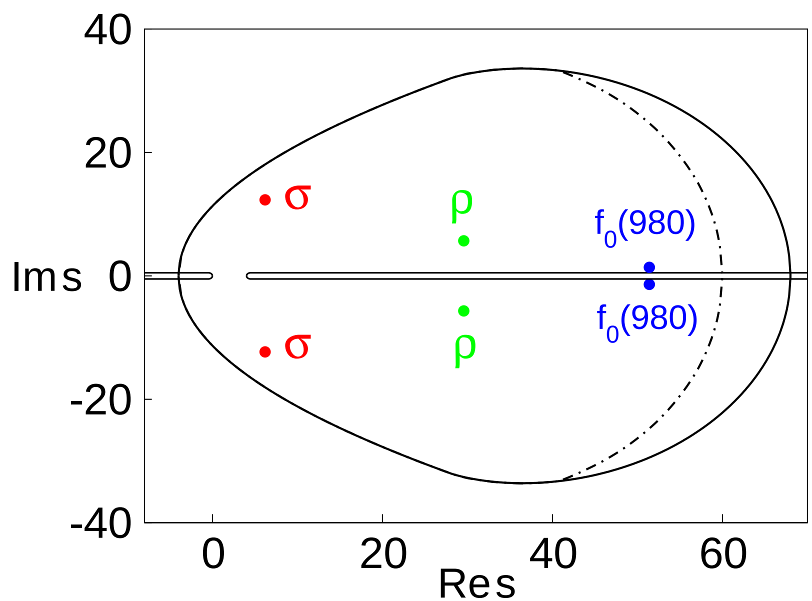
G. Mahoux, S. M. Roy and G. Wanders, Nucl. Phys. B70 (1974) 297.

$\Rightarrow$ *Exact representation for the S-matrix elements in this region do not need to parametrize the amplitude*

Numerical result for resonances with vacuum quantum numbers

- $S_0^0(s)$ has two pairs of zeros in the region where the formula holds
- For the central solution of the Roy equations, the zeros occur at

$$s = (6.2 \pm i\,12.3)\,M_\pi^2 \quad \sigma$$
$$s = (51.4 \pm i\,1.4)\,M_\pi^2 \quad f_0(980)$$



s in units of M_π^2

Error analysis

- Formula is exact, evaluation is approximate
- Key point: can follow error propagation explicitly
- Focus on σ, split the formula into 3 pieces:

> 1. Contribution from $\operatorname{Im} t_0^0$ below $K\bar{K}$ threshold
> 2. Subtraction terms
> 3. Higher energies and other partial waves

  - Subtraction constants (a_0^0, a_0^2) are known very accurately
  - Higher energies barely contribute (2 subtractions)
  - Crossed channel contributions are dominated by the ρ, excellent experimental information from e^+e^-, τ
  - Illustration: replace our representation for the higher energies and other partial waves by the one of KPY III :
    σ pole moves by $-0.6 - i\,1.2$ MeV

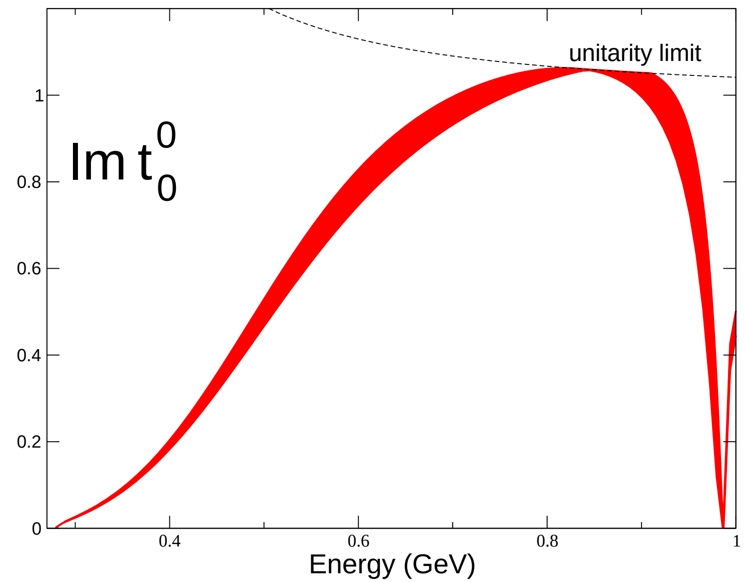
$\Rightarrow$ Uncertainty in result for σ pole is dominated by 1.

1. Contribution from $\text{Im}\, t_0^0$ below $K\bar{K}$ threshold
2. Subtraction terms
3. Higher energies and other partial waves

Unitarity and dip leave little room between 800 MeV and $2M_K$

Only the region below 800 MeV really matters

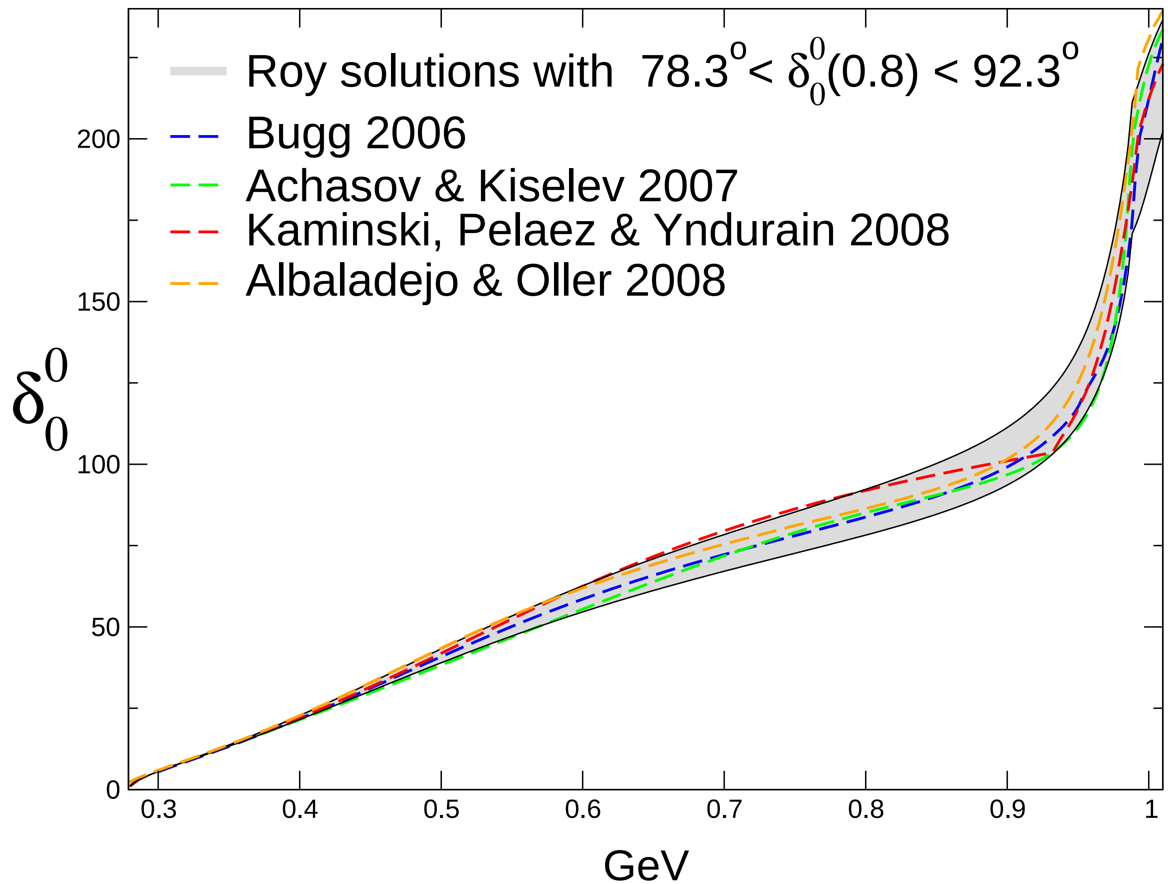
There, S_0^0 *is nearly elastic*



$$\text{Im}\, t_0^0 \simeq \sin^2\delta_0^0/\rho$$

$\Rightarrow$ *Need to know the phase* δ_0^0 *below 800 MeV*

Behaviour of the phase below 800 MeV



Bugg uses BES data for $J/\psi \longrightarrow \omega\pi\pi$

Achasov & Kiselev use KLOE data on $\phi \longrightarrow \gamma\pi\pi$

Kamiński, Peláez & Yndurán $\longrightarrow$ *talk by Stefan Narison*

Albaladejo & Oller: N/D fit to several data sets

Result for pole position

- *Vary the Roy equation input within the experimental uncertainties, use the predictions for the subtraction constants and add errors up*

$\Rightarrow$ *On the lower half of sheet II, the pole closest to the origin sits at*

$$M_\sigma - \tfrac{i}{2}\Gamma_\sigma = 441\,{}^{+16}_{-8} \; - i\; 272\,{}^{+9}_{-13} \text{ MeV}$$

- *Do not need to rely on the solution of the Roy equations:*
  - *Replace the central Roy solution below* $2M_K$ *by the phase representation of Bugg 2006* $\Rightarrow$ *pole moves to* $444 - i\,267$ *MeV*
  - *Ditto with Achasov & Kiselev 2007:* $438 - i\,274$ *MeV*
  - *Ditto with Kamiński, Peláez & Ynduráin 2008:* $458 - i\,253$ *MeV*
  - *Ditto with Albaladejo & Oller 2008:* $451 - i\,257$ *MeV*

Conclusion for lowest resonance of QCD

- *Dispersion theory allows to extend the domain where the first few terms of the chiral perturbation series provide a decent approximation*
- *Model independent method for analytic continuation*
- *Crossing symmetry plays an essential role in this method:*
  fixes the contributions from the left hand cut
  ensures fast convergence, low energy dominance

$\Rightarrow$ *The lowest resonance of QCD carries vacuum quantum numbers and occurs at*

$$M_\sigma = 441\,^{+16}_{-8}\ \text{MeV} \qquad \Gamma_\sigma = 544\,^{+18}_{-25}\ \text{MeV}$$

Puzzling results on $K_L \to \pi\mu\nu$

- *Hadronic matrix element of weak current:*

$$\langle K^0 | \bar{u}\gamma^\mu s | \pi^- \rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

- *Scalar form factor* $\sim \langle K^0 | \partial_\mu (\bar{u}\gamma^\mu s) | \pi^- \rangle$

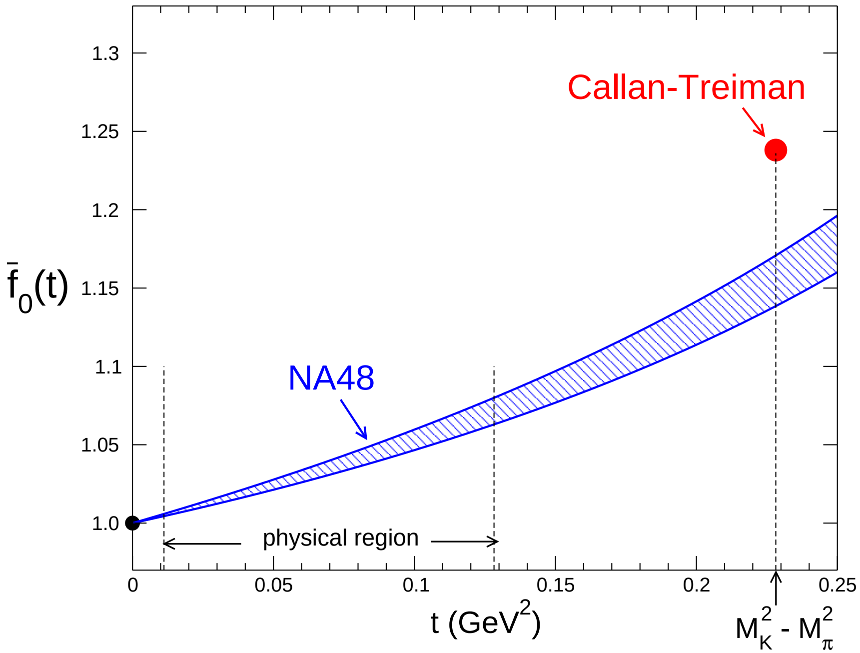
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- *Low energy theorem of Callan and Treiman (1966):*

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi} \left\{ 1 + O(m_u, m_d) \right\} \simeq 1.19$$

$f_0(0) = f_+(0) \simeq 0.96$ *relevant for determination of* V_{us}

Comparison with experiment



NA48, Phys. Lett. B647 (2007) 341

141 authors, 2.3×10^6 *events*

plot shows normalized scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- *Callan-Treiman relation in this normalization:*

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)}$$

- *Experimental value:* $\frac{F_K}{F_\pi f_+(0)} = 1.2446 \pm 0.0041$

Bernard and Passemar 2008

Implications

- NA48 data on $K_L \longrightarrow \pi\mu\nu$ disagree with SM
- If confirmed, the implications are dramatic:

$\Rightarrow$ right-handed currents ? Bernard, Oertel, Passemar & Stern 2006

- There are not many places where the SM disagrees with observation, need to investigate these carefully
- At low energies, high precision is required

Slope of the scalar form factor

- *Definition of the slope* $$\bar{f}_0(t) = 1 + \frac{\lambda_0\, t}{M_{\pi^+}^2} + O(t^2)$$
- *Callan-Treiman-relation implies sharp prediction:*

  $$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$

  Jamin, Oller & Pich 2004

- *Update with current experimental information*

  $$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

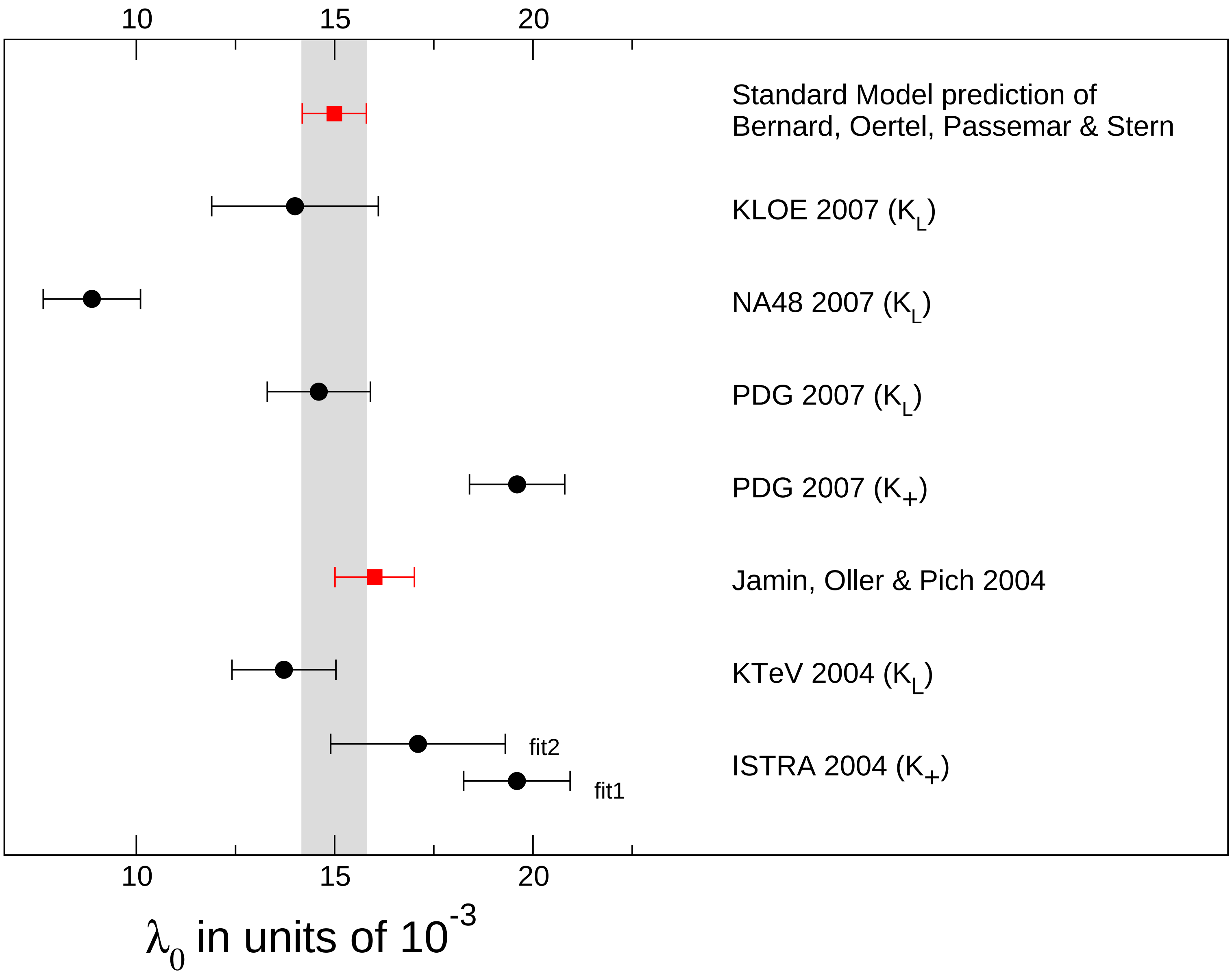
  Bernard, Oertel, Passemar & Stern, preliminary

- *To be compared with the result of NA48:*

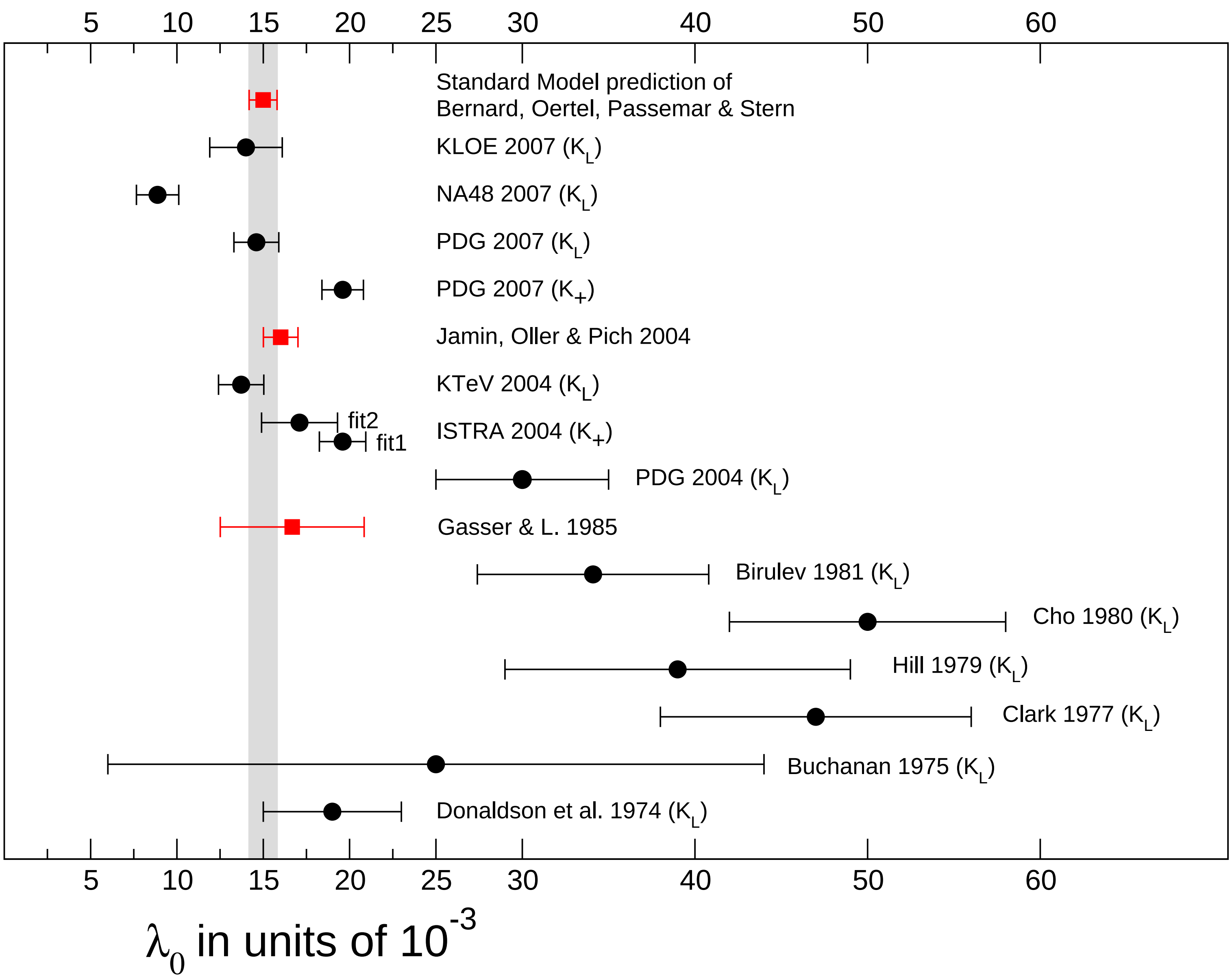
  $$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$

  Fit with dispersive representation of BOPS

Comparison of results for the slope



Older measurements



Conclusions for $K \to \pi\mu\nu$

- *Experiment is difficult, discrepancies need to be resolved*

  Donaldson 1974: 1.6×10^6 *events*

  ISTRA 2004: 0.54×10^6 *events*

  KTeV 2004: 1.5×10^6 *events*

  NA48 2007: 2.3×10^6 *events*

- *Dispersion theory fixes the shape of the form factors*
  publishing linear fits is nonsensical

- *NA48 should improve their data analysis . . . and extend it to charged kaons (isospin breaking)*