Recent developments in the physics of the light quarks

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University of Bern

Symposium in honour of Harald Fritzsch

München, June 6, 2008

- *in 1972, Murray Gell-Mann lectured at the Schladming Winter School on ”Quarks”*
- *current algebra on the light cone* *work done together with Jan Stern*
- *he invited me to visit Caltech*
- *this is where I met Harald for the first time*
- *I do not have a picture of him from that time*
  make do with a more recent one - he barely changed

Half of a recent picture of Harald

Half of a recent picture of Harald

picture taken when Murray Gell-Mann received the Einstein Medal, Bern 2005

. . . together with the other half

Caltech, spring 1973

- *Harald and Murray had realized that the quarks carry an internal quantum number and had explored the possibility of a nonabelian gauge field coupled to this degree of freedom as the origin of the strong interaction – QCD*
- *I had the chance of taking part in the pursuit of this idea*
- *main problem from my perspective:*
  - *the spectrum of physical states of all quantum field theories encountered in nature thus far was the same as the one of the kinetic part of the Lagrangian*
  - *not the case for the strong interaction ?*
    could a bona fide local field theory confine the degrees of freedom visible in the Lagrangian ?
  - *the gauge field responsible for the weak interaction is of the same mathematical structure, but does not confine flavor . . .*

⇒ *wishful thinking ? asking for a miracle to happen ?*

QCD

- *the arguments in favor of QCD as a viable theory for the strong interaction are outlined in a paper with the title*

  ”Advantages of the color octet gluon picture”

  $\uparrow$ *first appearance of this term*

- *the manuscript was sent to Phys. Lett. on Oct. 1 when I was already back home*

35 years later

- *QCD with massless quarks is the ideal of a theory:*
  no dimensionless free parameters
- *high energy side looks like what we are used to: relevant degrees of freedom are visible in the Lagrangian, can treat the interaction as a perturbation*
- *gives rise to a rich structure at low energies*
- *low energies are out of reach of perturbation theory*

$\Rightarrow$ *progress in understanding is slow*

- $\exists$ *many models that resemble QCD: instantons, monopoles, bags, superconductivity, gluonic strings, linear* σ *model, hidden gauge, NJL, AdS/CFT, but . . .*
- *two model independent methods:*
  - *effective field theory,* χ*PT*
  - *numerical simulation on a lattice*

Energy gap of QCD

- *main characteristic of QCD at low energies:*

  energy gap is very small, $M_\pi \simeq$ *140 MeV*
- *in 1960, Nambu found out why that is so:*
  - *has to do with a hidden approximate symmetry*
  - *symmetry becomes exact for* $m_u, m_d \longrightarrow 0$

  $\Rightarrow$ *energy gap disappears: pions become massless*
  - *in reality* $m_u, m_d \neq 0$, *but very small*

  $\Rightarrow$ *symmetry is not perfect, but nearly so*
  - *the state of lowest energy is not symmetric*

  $\Rightarrow$ *Chiral symmetry is hidden, "spontaneously broken"*
  - *Very strong experimental evidence* $\surd$

    Very strong evidence from lattice calculations $\surd$

    Analytic understanding of the ground state still poor

Quark masses as perturbations

- Masses of u, d enter the Hamiltonian via

$$H_{QCD} = H_0 + H_1$$

$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d\}$$

  H_0 describes u, d as massless, s, c, b, t as massive

  H_0 is invariant under $SU(2)_L \times SU(2)_R$

- H_0 treats the pions as massless particles

  H_1 gives them a mass

Expansion in powers of m_u, m_d	$\Longleftrightarrow$	Perturbation series in powers of H_1

Gell-Mann-Oakes-Renner formula

- *First order perturbation theory in H_1 yields:*

$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \text{explicit}}}{+} m_d) \times |\langle 0|\, \underset{\substack{\Uparrow \\ \text{spontaneous}}}{\bar{u}u}\, |0\rangle| \times \frac{1}{F_\pi^2}$$

Gell-Mann, Oakes & Renner 1968

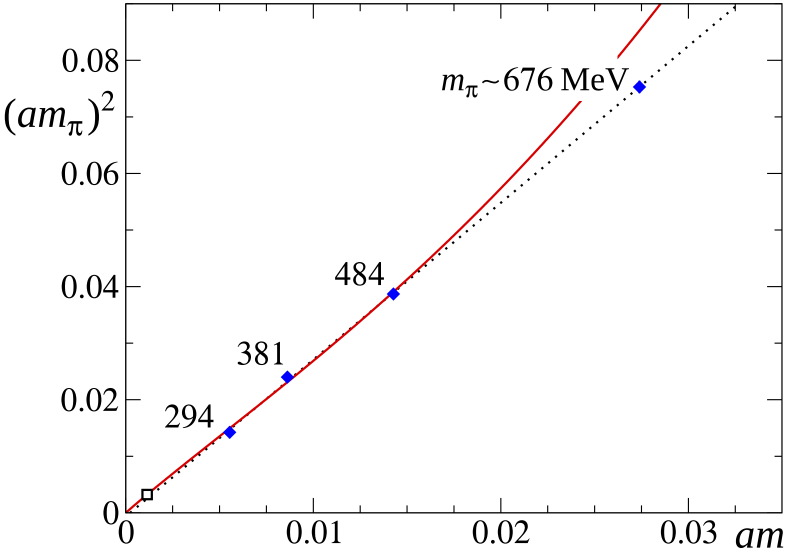
Coefficient: decay constant F_π

$$\langle 0|\, \bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\, p^\mu \sqrt{2}\, F_\pi$$

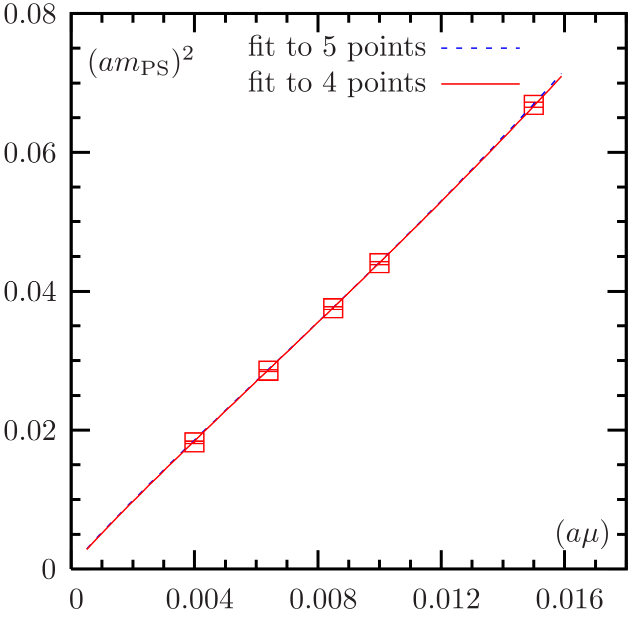
Value of F_π is known from $\pi^+ \to \mu^+\nu$

Checking the formula on the lattice

- can determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- no quenching, quark masses are sufficiently light

⇒ legitimate to use χPT for the extrapolation to the physical values of m_u, m_d

Lattice

- *quality of data is impressive*
- *proportionality of* M_π^2 *to the quark mass appears to hold out to values of* m_u, m_d *that are an order of magnitude larger than in nature*
- *main limitation: systematic uncertainties*
  in particular: $N_f = 2 \longrightarrow N_f = 3$

Expansion of M_π^2 in powers of the quark masses

- Gell-Mann-Oakes-Renner formula represents leading term of the chiral perturbation series
- disregard isospin breaking, set $m_u = m_d = m$
- expand in powers of m, keeping m_s fixed
- at NLO, the expansion contains a logarithm

Langacker & Pagels 1973

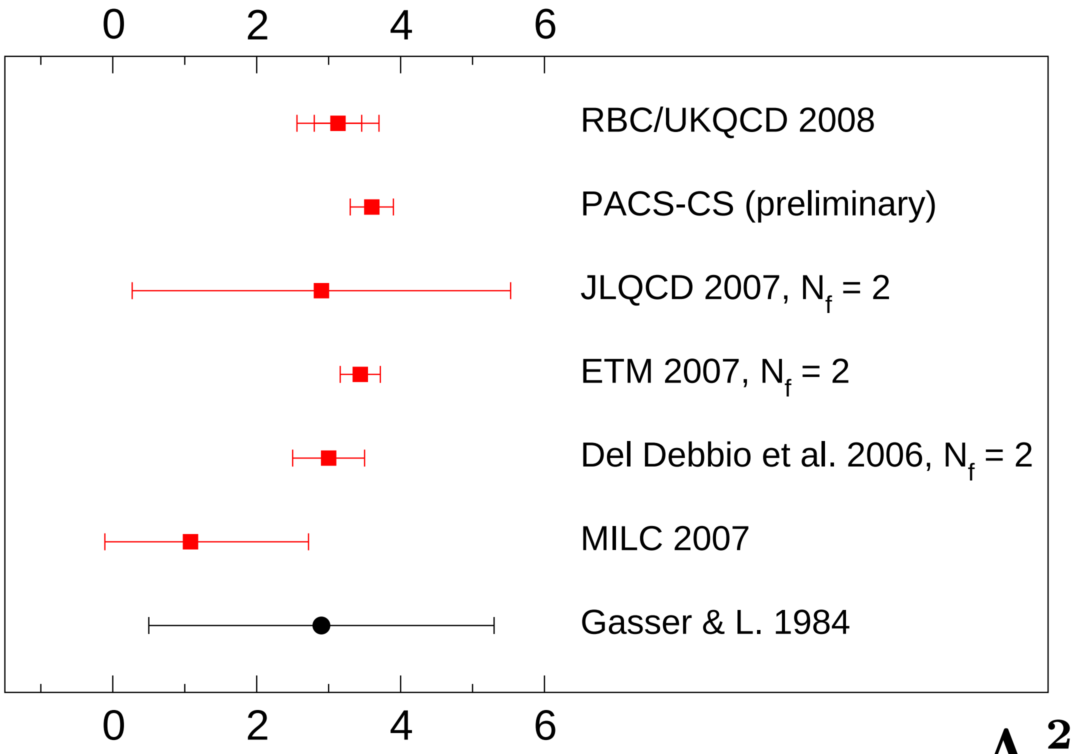
$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

$$M^2 \equiv 2Bm$$

- coefficient is determined by the pion decay constant
  symmetry does not determine the scale Λ_3
- crude result, based on SU(3)×SU(3):
  0.2 GeV $\lesssim \Lambda_3 \lesssim 2$ GeV

Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



horizontal axis shows the value of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

range for Λ_3 *obtained in 1984 corresponds to* $\bar{\ell}_3 = 2.9 \pm 2.4$

result of RBC/UKQCD 2008: $\bar{\ell}_3 = 3.13 \underset{stat}{\pm 0.33} \underset{syst}{\pm 0.24}$

Expansion of F_π in powers of the quark mass

- also contains a logarithm at NLO:

$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

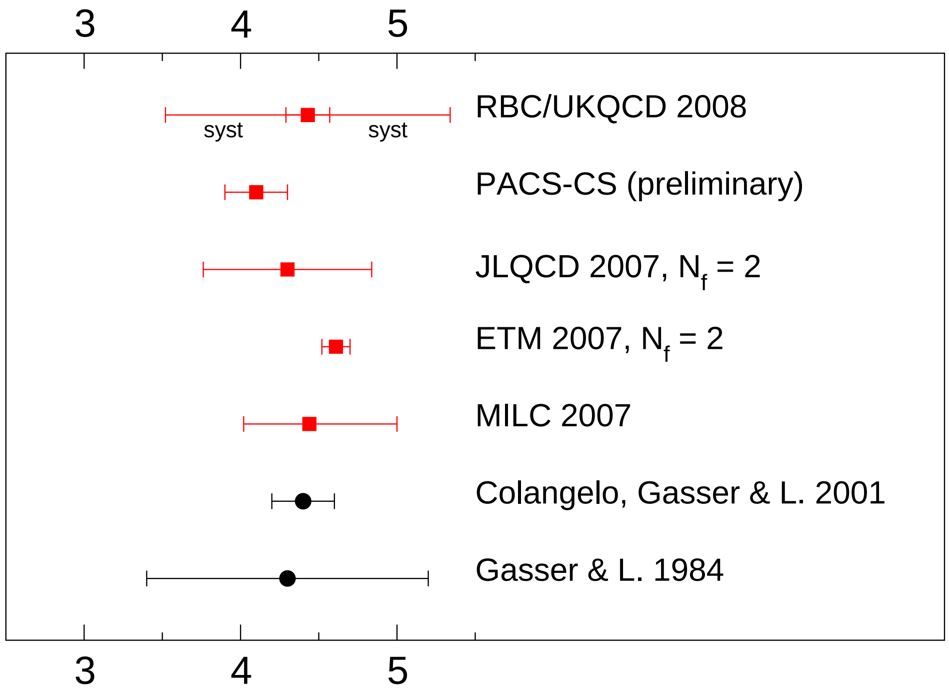
F is value of pion decay constant in limit $m_u, m_d \longrightarrow 0$

- structure is the same, coefficients and scale of logarithm are different
- quark mass dependence of F_π can also be measured on the lattice

$\Rightarrow$ measurement of Λ_4

- alternative method: determine the scalar form factor of the pion, radius $\langle r^2 \rangle_s \Leftrightarrow \bar{\ell}_4$

Lattice results for Λ_4



$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

- *lattice results beautifully confirm the prediction for the sensitivity of* F_π *to* m_u, m_d*:*

$$\frac{F_\pi}{F} = 1.072 \pm 0.007$$

Colangelo and Dürr 2004

Progress in understanding the interaction among the pions

- *dispersion theory*
- $\pi\pi$ *scattering is special: crossed channels are identical*

$\Rightarrow$ *Re* $T(s,t)$ *can be represented as a twice subtracted dispersion integral over Im* $T(s,t)$ *in physical region* S.M. Roy 1971

- *the 2 subtraction constants can be identified with the* S*-wave scattering lengths:*

$$a_0^0 \;,\; a_0^2 \begin{array}{l} \leftarrow \text{isospin} \\ \leftarrow \text{angular momentum} \end{array}$$

- *representation leads to dispersion relations for the individual partial waves:* Roy equations

Roy equations

- *pioneering work on the physics of the Roy equations was done around the time when QCD was discovered*

Pennington & Protopopescu 1973, Basdevant, Froggatt & Petersen 1974

- *dispersion integrals converge rapidly (2 subtractions)*

⇒ *crude phenomenological information on Im* $T(s,t)$ *for energies above 800 MeV suffices*

⇒ *given* a_0^0, a_0^2*, the scattering amplitude can be calculated very accurately*

Ananthanarayan, Colangelo, Gasser & L. 2001

Descotes, Fuchs, Girlanda & Stern 2002

⇒ a_0^0, a_0^2 *are the essential parameters at low energy*

- *main problem in early work:* a_0^0, a_0^2 *poorly known*
  experimental information near threshold is meagre

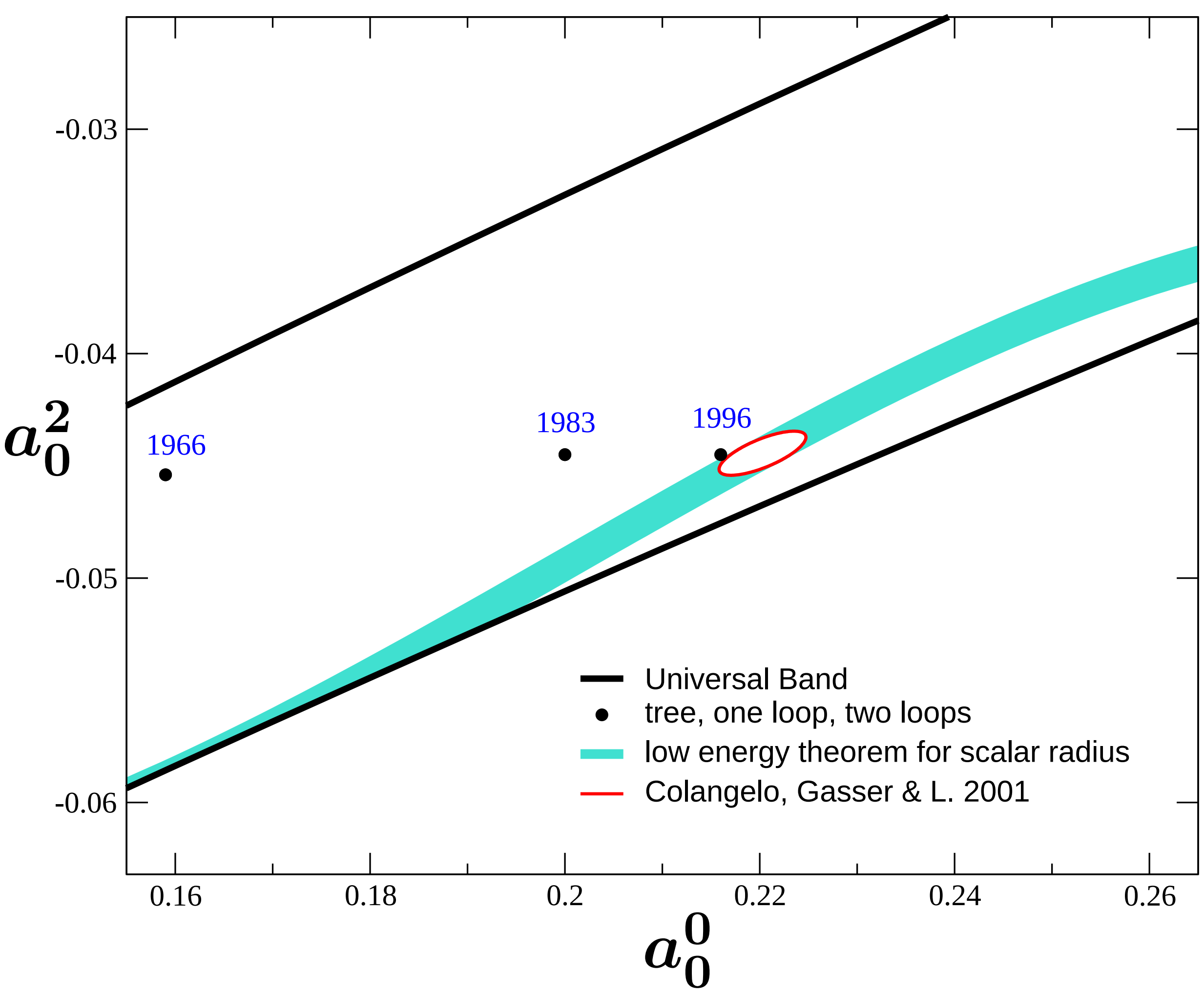
Low energy theorems

- *χPT provides the missing piece: theoretical prediction for* $\boldsymbol{a_0^0, a_0^2}$

  Weinberg 1966, Gasser & L. 1983, Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

- *most accurate results for* $\boldsymbol{a_0^0, a_0^2}$ *are obtained by matching the chiral and dispersive representations in the unphysical region below threshold, near the Adler zero* *Colangelo, Gasser & L. 2001*
- *in combination with the low energy theorems for* $\boldsymbol{a_0^0, a_0^2}$*, the dispersion relations for the partial waves fix the* $\boldsymbol{\pi\pi}$ *scattering amplitude to an incredible degree of accuracy*

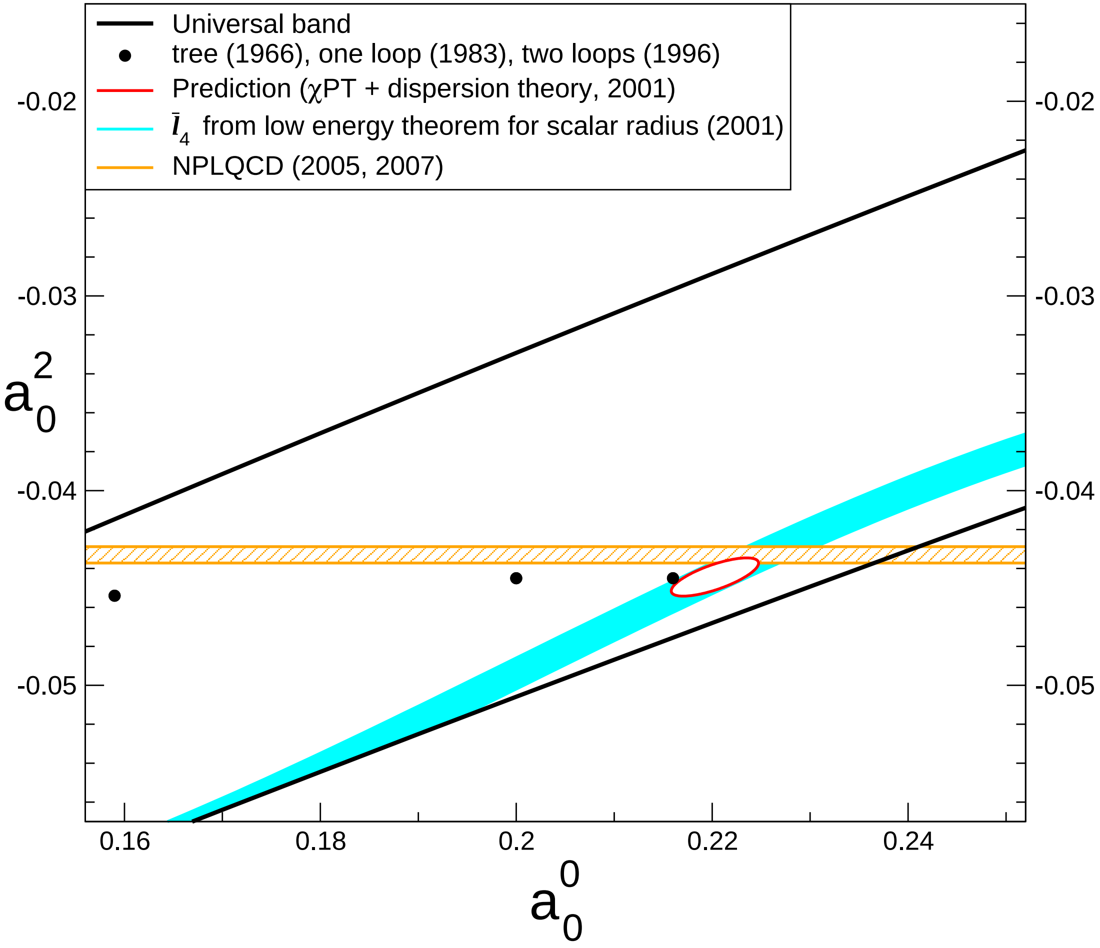
Predictions for the S-wave $\pi\pi$ scattering lengths



sizable corrections in a_0^0, while a_0^2 nearly stays put

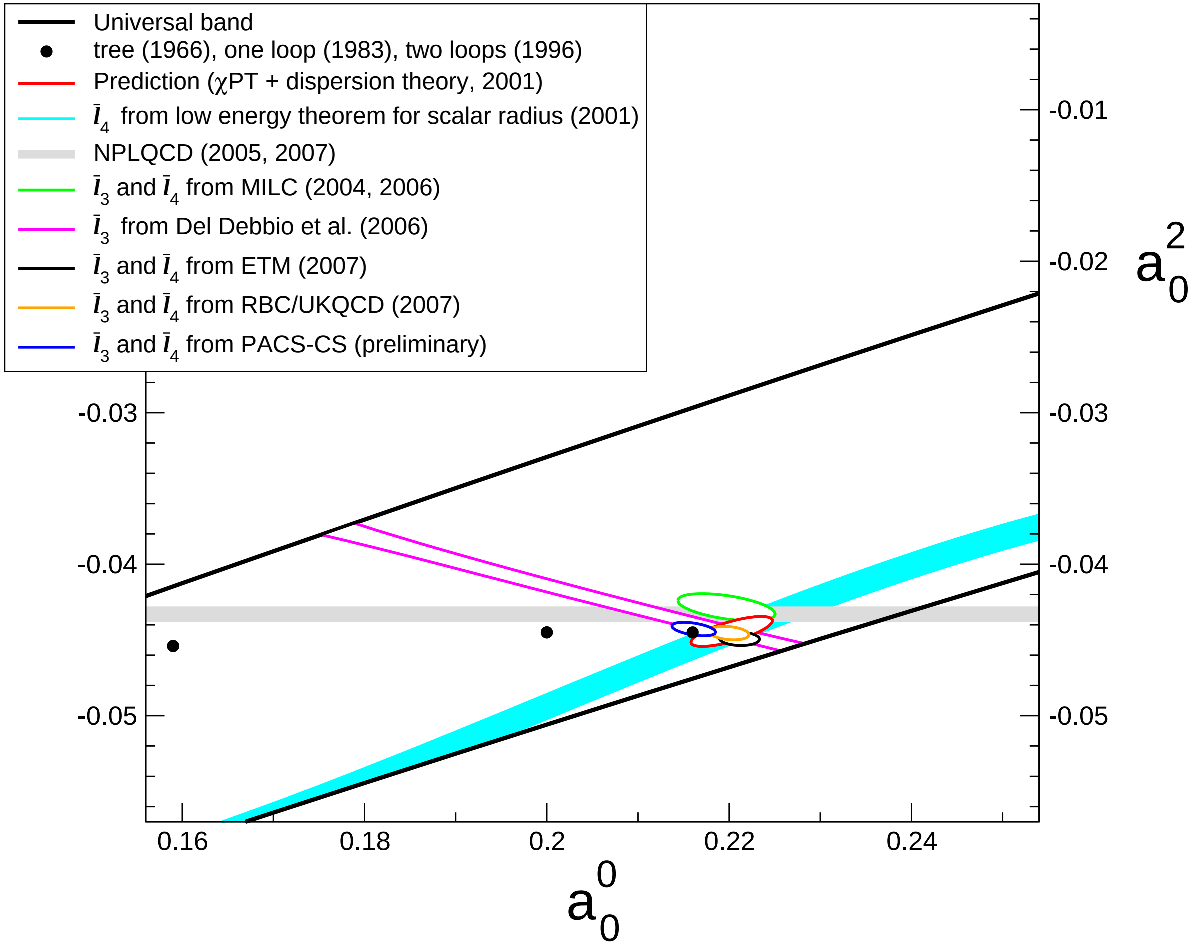
Lattice result for a_0^2

- *lattice allows direct measurement of* a_0^2 *via volume dependence of energy levels*



Consequence of lattice results for ℓ_3, ℓ_4

- uncertainty in prediction for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4
- can make use of the lattice results for these



Experiments on light flavours at low energy

- production experiments $\pi N \longrightarrow \pi\pi N$, $\psi \longrightarrow \pi\pi\omega \ldots$

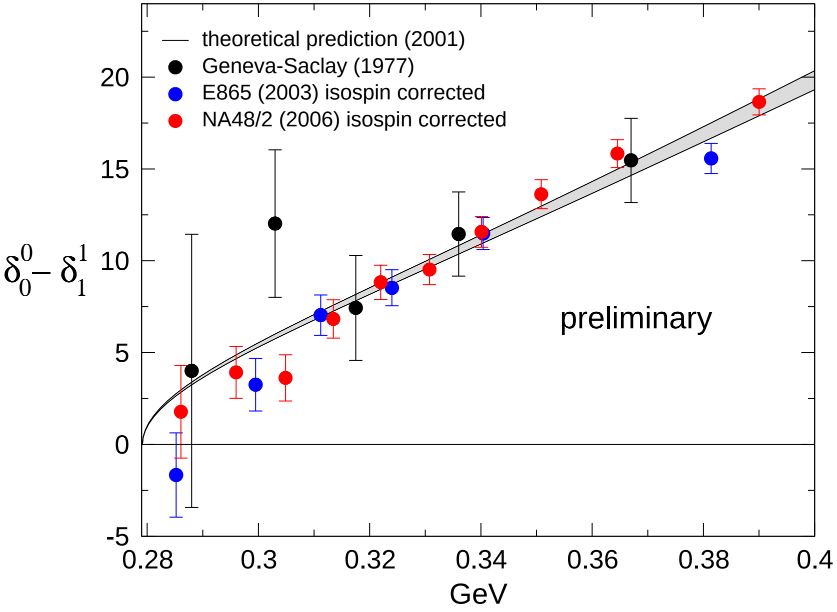
  problem: pions are not produced in vacuo

  ⇒ extraction of $\pi\pi$ scattering amplitude not simple

  accuracy rather limited
- $K^\pm \longrightarrow \pi^+\pi^- e^\pm \nu$ data: CERN-Saclay, E865, NA48/2
- $K^\pm \longrightarrow \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $\pi^+\pi^-$ atoms, DIRAC

K_{e4} decay

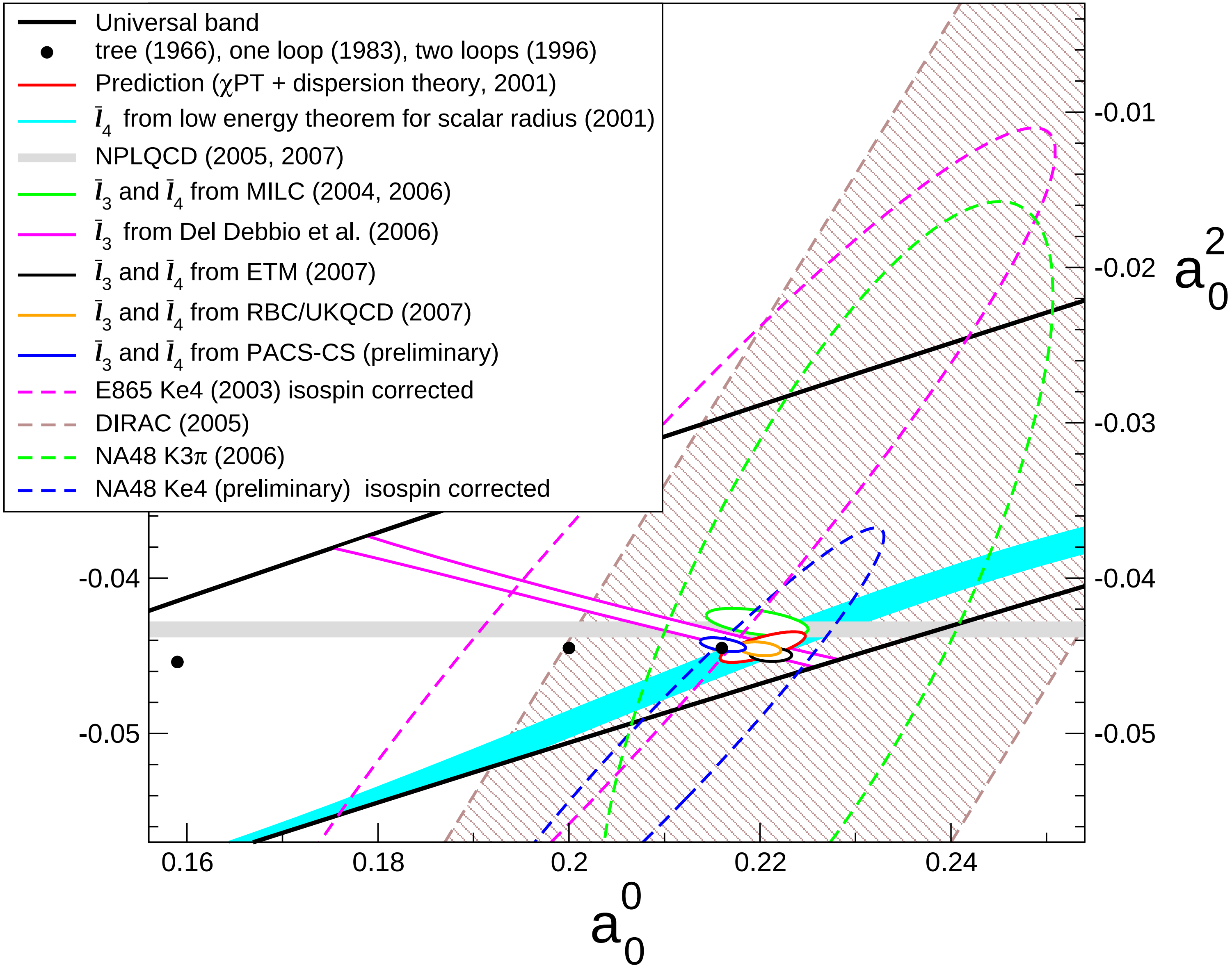
- $K \longrightarrow \pi\pi e \nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- there was a discrepancy here, because a pronounced isospin breaking effect from $K \longrightarrow \pi^0\pi^0 e \nu \longrightarrow \pi^+\pi^- e \nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007

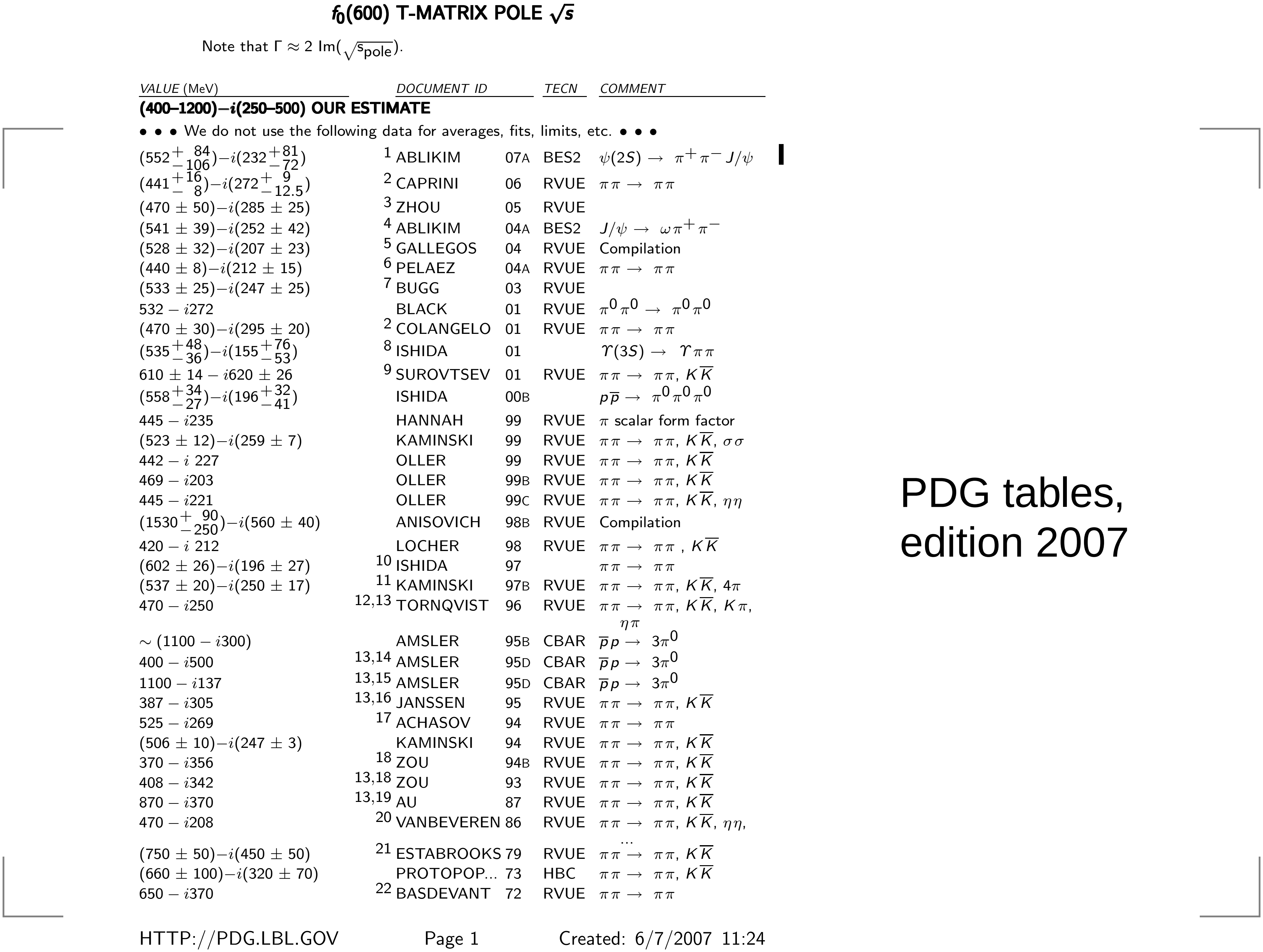
a_0^0, a_0^2 : prediction, lattice & experiment



Resonances: exact formula for mass and width

- where is the lowest resonance of QCD ?
  - concerns the nonperturbative domain of QCD
  - quark and gluon degrees of freedom useless there

  ⇒ understanding very poor, pattern of energy levels ?
  - lowest resonance: σ ? ρ ?
- resonance ↔ pole on second sheet
  - poles are universal
  - pole position is unambiguous, even if width is large
  - where is the pole closest to the origin ?



$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(400–1200)−*i*(250–500) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(552^{+84}_{-106})-i(232^{+81}_{-72})$	[1] ABLIKIM 07A	BES2	$\psi(2S) \to \pi^+\pi^- J/\psi$
$(441^{+16}_{-8})-i(272^{+9}_{-12.5})$	[2] CAPRINI 06	RVUE	$\pi\pi \to \pi\pi$
$(470 \pm 50)-i(285 \pm 25)$	[3] ZHOU 05	RVUE	
$(541 \pm 39)-i(252 \pm 42)$	[4] ABLIKIM 04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528 \pm 32)-i(207 \pm 23)$	[5] GALLEGOS 04	RVUE	Compilation
$(440 \pm 8)-i(212 \pm 15)$	[6] PELAEZ 04A	RVUE	$\pi\pi \to \pi\pi$
$(533 \pm 25)-i(247 \pm 25)$	[7] BUGG 03	RVUE	
$532 - i272$	BLACK 01	RVUE	$\pi^0\pi^0 \to \pi^0\pi^0$
$(470 \pm 30)-i(295 \pm 20)$	[2] COLANGELO 01	RVUE	$\pi\pi \to \pi\pi$
$(535^{+48}_{-36})-i(155^{+76}_{-53})$	[8] ISHIDA 01		$\Upsilon(3S) \to \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	[9] SUROVTSEV 01	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(558^{+34}_{-27})-i(196^{+32}_{-41})$	ISHIDA 00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH 99	RVUE	π scalar form factor
$(523 \pm 12)-i(259 \pm 7)$	KAMINSKI 99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\sigma\sigma$
$442 - i\ 227$	OLLER 99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$469 - i203$	OLLER 99B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$445 - i221$	OLLER 99C	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$
$(1530^{+90}_{-250})-i(560 \pm 40)$	ANISOVICH 98B	RVUE	Compilation
$420 - i\ 212$	LOCHER 98	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(602 \pm 26)-i(196 \pm 27)$	[10] ISHIDA 97		$\pi\pi \to \pi\pi$
$(537 \pm 20)-i(250 \pm 17)$	[11] KAMINSKI 97B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, 4π
$470 - i250$	[12,13] TORNQVIST 96	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
$\sim (1100 - i300)$	AMSLER 95B	CBAR	$\overline{p}p \to 3\pi^0$
$400 - i500$	[13,14] AMSLER 95D	CBAR	$\overline{p}p \to 3\pi^0$
$1100 - i137$	[13,15] AMSLER 95D	CBAR	$\overline{p}p \to 3\pi^0$
$387 - i305$	[13,16] JANSSEN 95	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$525 - i269$	[17] ACHASOV 94	RVUE	$\pi\pi \to \pi\pi$
$(506 \pm 10)-i(247 \pm 3)$	KAMINSKI 94	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$370 - i356$	[18] ZOU 94B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$408 - i342$	[13,18] ZOU 93	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$870 - i370$	[13,19] AU 87	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$470 - i208$	[20] VANBEVEREN 86	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, ...
$(750 \pm 50)-i(450 \pm 50)$	[21] ESTABROOKS 79	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(660 \pm 100)-i(320 \pm 70)$	PROTOPOP... 73	HBC	$\pi\pi \to \pi\pi$, $K\overline{K}$
$650 - i370$	[22] BASDEVANT 72	RVUE	$\pi\pi \to \pi\pi$

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PDG tables,
edition 2007

Model independent determination of the pole

- most of the results quoted by the PDG are obtained by
  (a) parametrizing the data for real values of s
  (b) continuing this parametrization analytically in s

  ⇒ result is sensitive to the parametrization used

- we found a model independent method:
  1. poles on second sheet are zeros on first sheet
  2. the Roy equations are valid for complex values of s, in a limited region of the first sheet

  ⇒ exact representation of the S-matrix elements in terms of observable quantities, valid for complex values of s

  ⇒ exact formula for the pole position

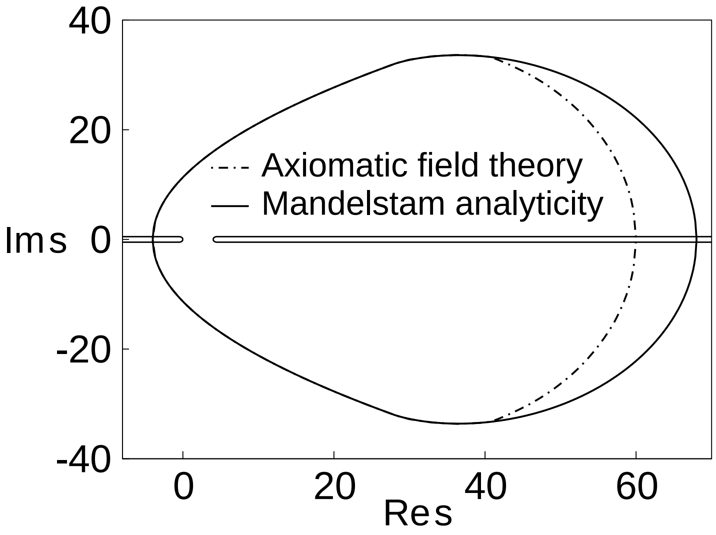
  3. can evaluate this formula to good precision and determine the pole position numerically

Caprini, Colangelo and L., Phys. Rev. Lett. 96 (2006) 132001

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$
- equations are valid for complex s in a limited region of the first sheet

Caprini, Colangelo and L. 2006



s in units of M_π^2

- proof is based on first principles, general quantum field theory

A. Martin, Scattering Theory: Unitarity, Analyticity and Crossing, Lecture Notes in Physics, vol. 3, 1969.

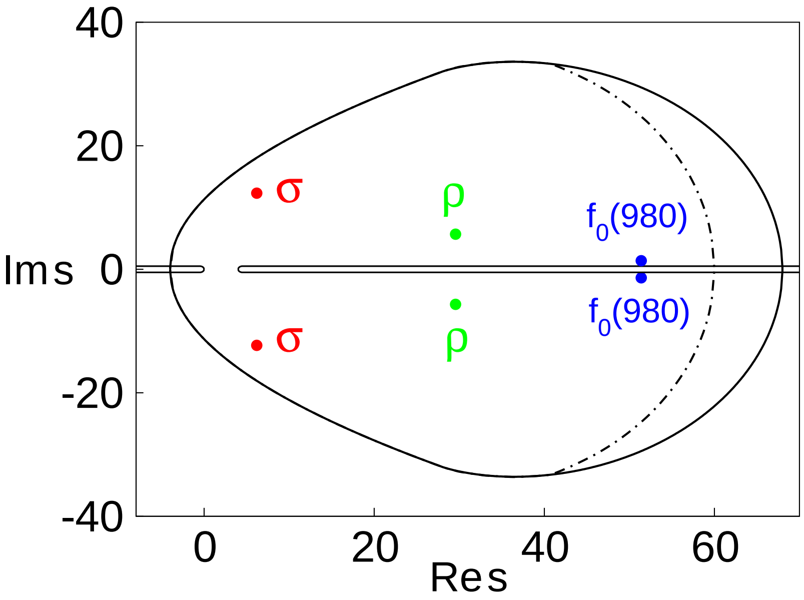
G. Mahoux, S. M. Roy and G. Wanders, Nucl. Phys. B70 (1974) 297.

⇒ exact representation for the S-matrix elements in this region do not need to parametrize the amplitude

Numerical result

- *the S-matrix element with* $I=\ell=0$ *has two pairs of zeros in the region where the formula holds*
- *for the central solution of the Roy equations, the zeros occur at*

$$s=(6.2\pm i\,12.3)\,M_\pi^2 \quad \sigma$$
$$s=(51.4\pm i\,1.4)\,M_\pi^2 \quad f_0(980)$$



s in units of M_π^2

Error estimate

- formula is exact, evaluation is approximate
  can explicitly follow error propagation

⇒ on the lower half of sheet II, the pole closest to the origin sits at

$$M_\sigma - \frac{i}{2}\Gamma_\sigma = 441\,{}^{+16}_{-8} \; - i\, 272\,{}^{+9}_{-13} \text{ MeV}$$

Conclusion I: expansion in powers of m_u, m_d

- *first few terms of the expansion in powers of m_u, m_d yield a very accurate approximation for the key low energy observables,* $M_\pi, F_\pi, a_0^0, a_0^2, \langle r^2 \rangle_v, \langle r^2 \rangle_s, \ldots$
- *lattice makes slow, but steady progress*
- *precision experiments carried out and under way*

⇒ *energy gap of QCD is understood very well:*

M_π *is dominated by the contribution from the quark condensate*

⇒ *despite the beautiful new experimental results in low energy pion physics, theory is still ahead of experiment*

Conclusion II: dispersion theory

- *dispersion theory allows to extend the domain where the first few terms of the chiral perturbation series provide a decent approximation*
- *limitations of this method*
  - *calculations cannot be done on the back of an envelope*
  - *analysis only covers low energies*
    extension to higher energies is under way
  - *only a few applications have been worked out:*
    - $\pi\pi$ *scattering*
    - *pion form factors*
    - *hadronic vacuum polarization in muon* $g-2$
  - *much is yet to be done:* $J/\psi \longrightarrow \omega\pi\pi$, $D \longrightarrow 3\pi$,
    $\gamma\gamma \longrightarrow \pi\pi$, πK, πN, $\ldots$

Conclusion III: exact formula for mass and width of resonances

- *model independent method for analytic continuation*
- *crossing symmetry plays an essential role:*
  fixes contributions from left hand cut
  ensures fast convergence, low energy dominance

⇒ *the lowest resonance of QCD carries vacuum quantum numbers and occurs at*

$$M_\sigma = 441\,{}^{+16}_{-8}\ \text{MeV} \qquad \Gamma_\sigma = 544\,{}^{+18}_{-25}\ \text{MeV}$$

Harald had a wonderful time in physics

learning

discovering

discussing

teaching

writing

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learning

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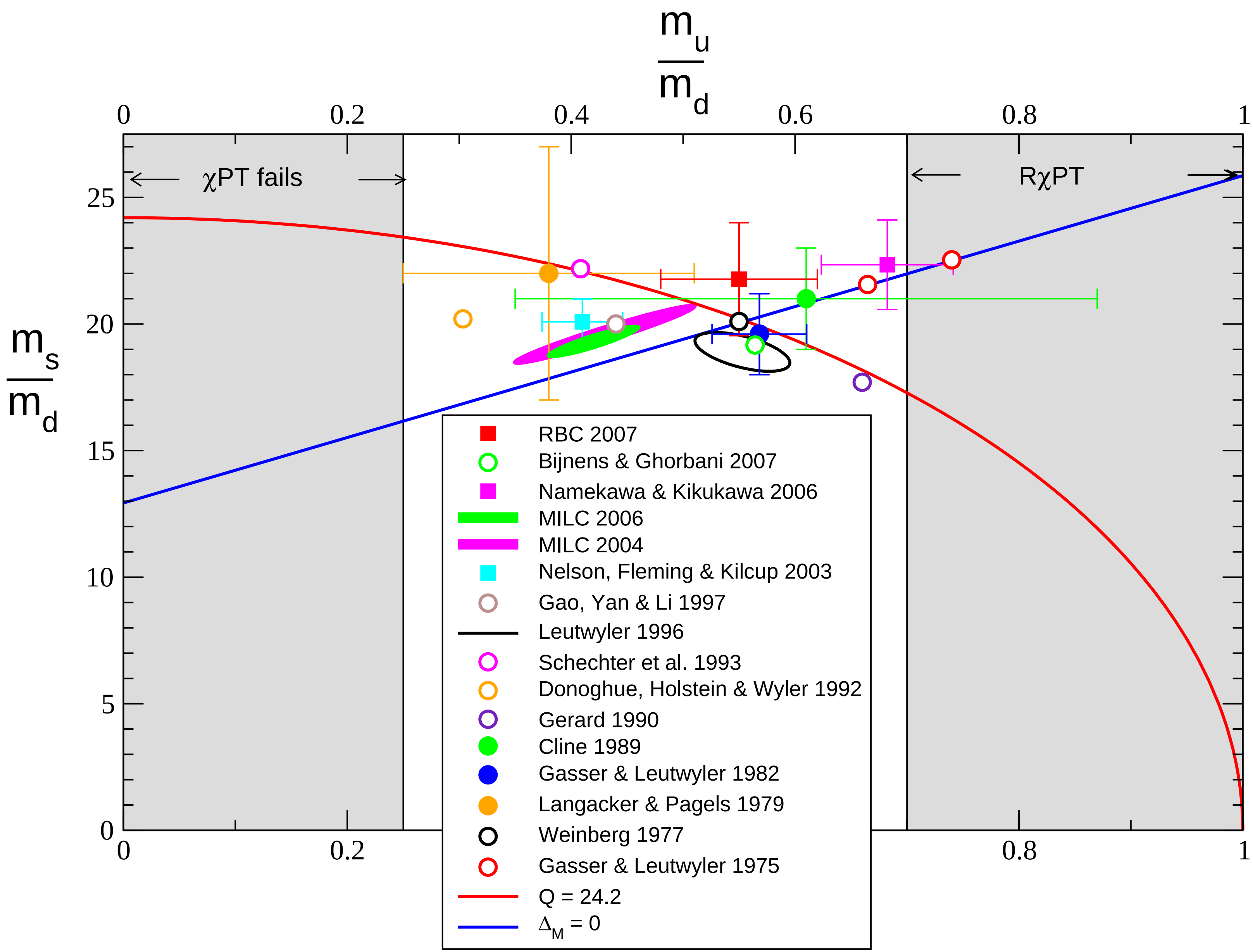
I wish him a happy

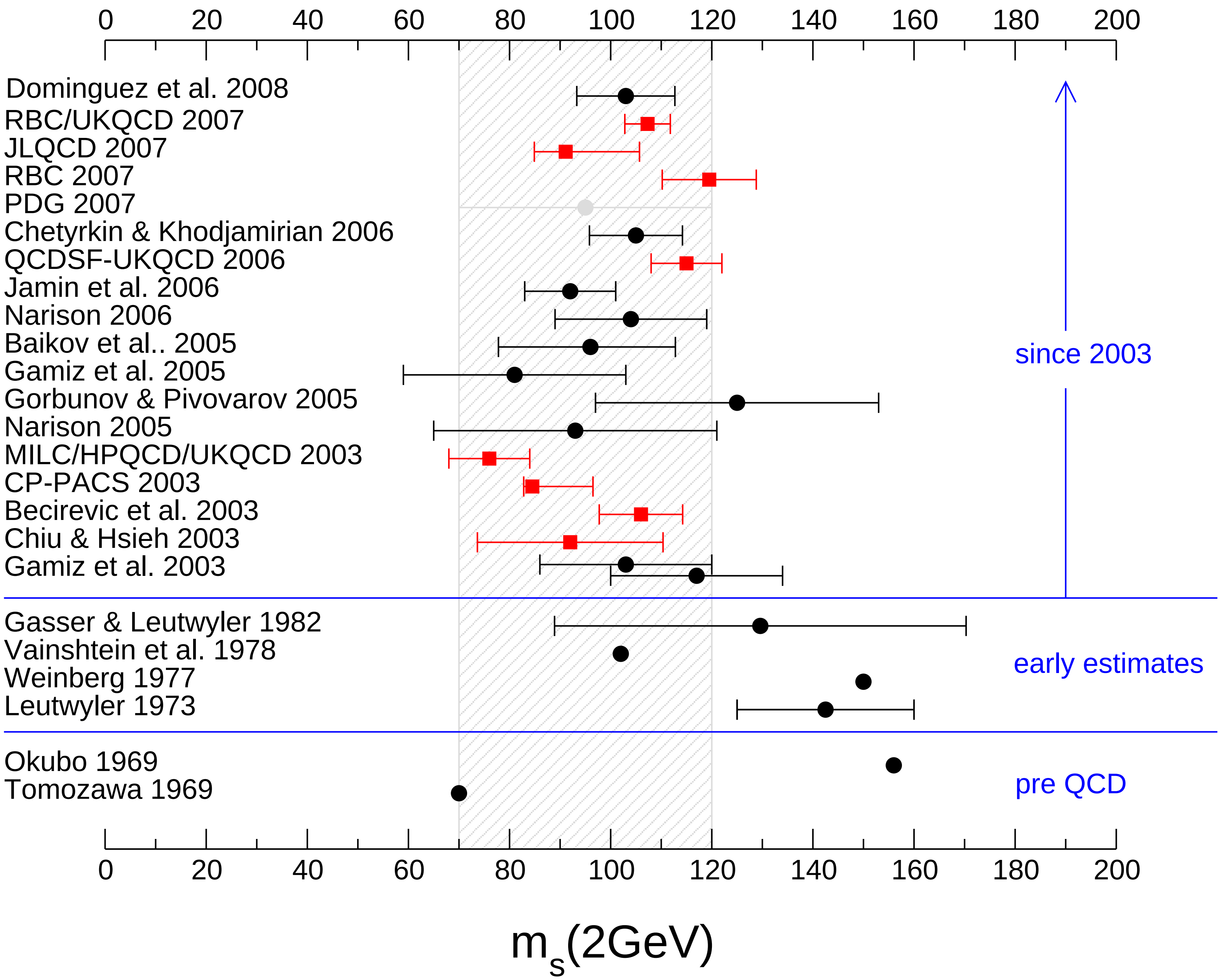
analytic continuation !

SPARES

Quark mass ratios



Value of m_s



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Dominguez et al. 2008
RBC/UKQCD 2007
JLQCD 2007
RBC 2007
PDG 2007
Chetyrkin & Khodjamirian 2006
QCDSF-UKQCD 2006
Jamin et al. 2006
Narison 2006
Baikov et al.. 2005
Gamiz et al. 2005
Gorbunov & Pivovarov 2005
Narison 2005
MILC/HPQCD/UKQCD 2003
CP-PACS 2003
Becirevic et al. 2003
Chiu & Hsieh 2003
Gamiz et al. 2003
since 2003
Gasser & Leutwyler 1982
Vainshtein et al. 1978
Weinberg 1977
Leutwyler 1973
early estimates
Okubo 1969
Tomozawa 1969
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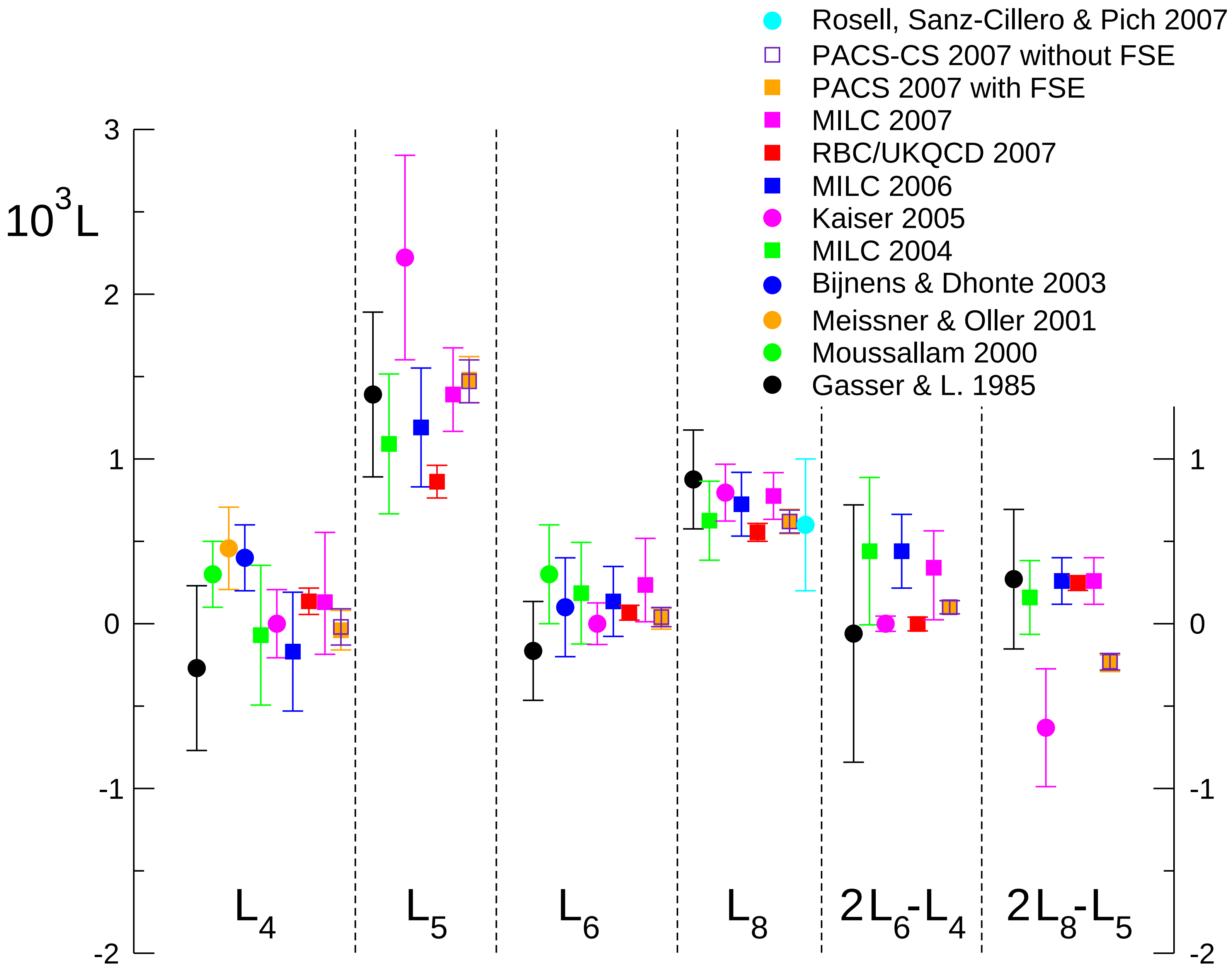
Effective coupling constants of SU(3)×SU(3)



Puzzling results on $K_L \longrightarrow \pi\mu\nu$

- *hadronic matrix element of weak current:*

$$\langle K^0|\bar{u}\gamma^\mu s|\pi^-\rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

- *scalar form factor* $\sim \langle K^0|\partial_\mu(\bar{u}\gamma^\mu s)|\pi^-\rangle$

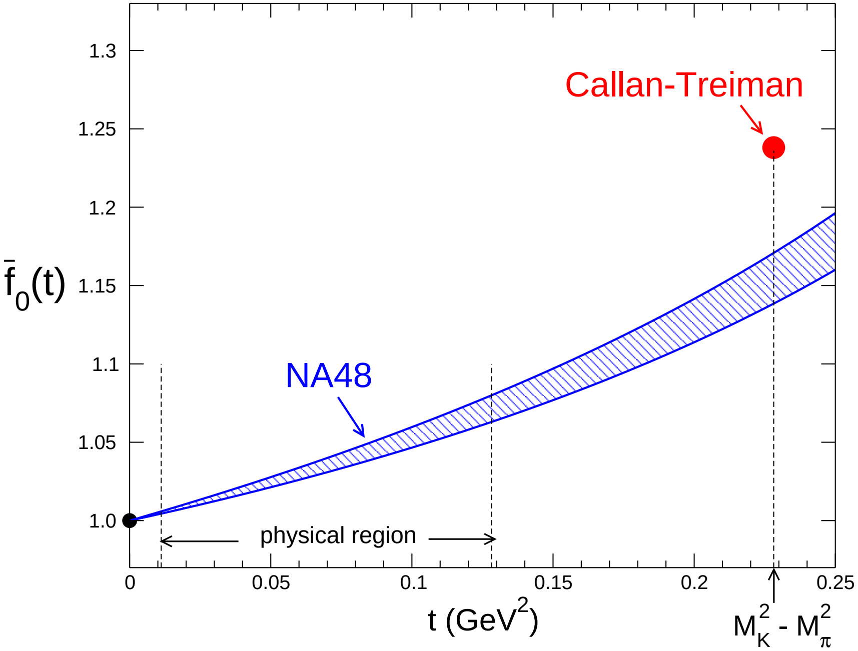
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- *low energy theorem of Callan and Treiman (1966):*

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi}\left\{1 + O(m_u, m_d)\right\} \simeq 1.19$$

$f_0(0) = f_+(0) \simeq 0.96$ *relevant for determination of* V_{us}

Comparison with experiment



NA48, Phys. Lett. B647 (2007) 341

141 authors, 2.3×10^6 *events*

plot shows normalized scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- *Callan-Treiman relation in this normalization:*

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)}$$

- *experimental value:* $\frac{F_K}{F_\pi f_+(0)} = 1.2446 \pm 0.0041$

Bernard and Passemar 2008

Implications

- NA48 data on $K_L \longrightarrow \pi\mu\nu$ disagree with SM

  if confirmed, the implications are dramatic:

⇒ right-handed currents ? Bernard, Oertel, Passemar & Stern 2006

there are not many places where the SM disagrees with observation

need to investigate these carefully

at low energies, high precision is required

Corrections, extrapolation

- Callan-Treiman-relation is exact only for $m_u, m_d \to 0$

  corrections of NLO were worked out long ago, are tiny Gasser & L. 1985

  form factor now known to NNLO Post & Schilcher 2002,

  Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005

  including the uncertainties from $m_u, m_d \neq 0$:

  $\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$ Bernard & Passemar 2008

$\Rightarrow$ cannot blame the discrepancy on the prediction

- CT–point is not in physical region, extrapolation needed

  curvature can be calculated with dispersion theory

  Jamin, Oller & Pich 2004, Bernard, Oertel, Passemar & Stern 2006

$\Rightarrow$ cannot blame the discrepancy on the extrapolation

Slope of the scalar form factor

- *definition of the slope* $$\bar{f}_0(t) = 1 + \frac{\lambda_0\, t}{M^2_{\pi^+}} + O(t^2)$$

- *Callan-Treiman-relation implies sharp prediction:*

  $$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$

  Jamin, Oller & Pich 2004

- *Update with current experimental information*

  $$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

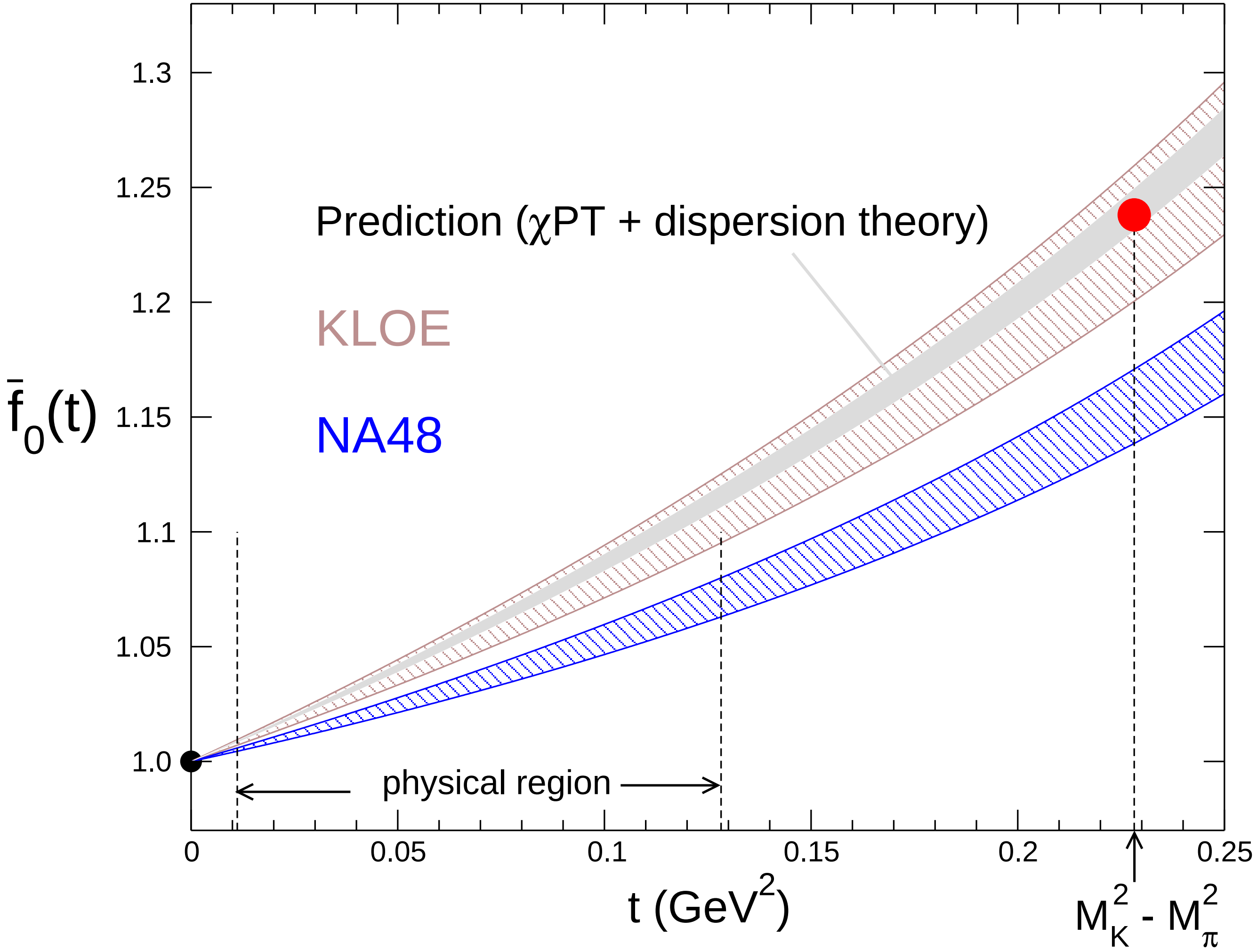
  Bernard, Oertel, Passemar & Stern, preliminary

- *To be compared with the result of NA48:*

  $$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$

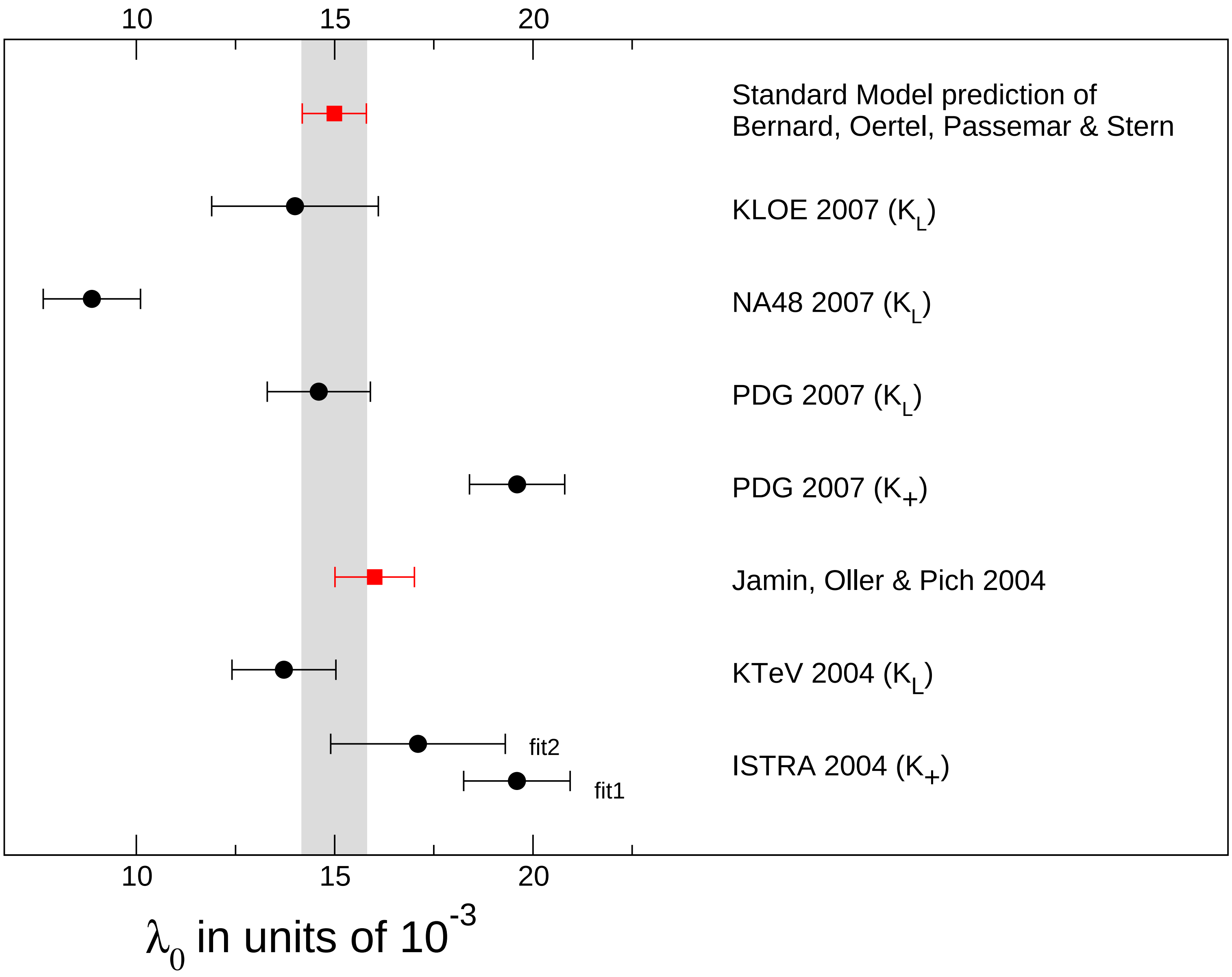
  Fit with dispersive representation of BOPS

New data from KLOE

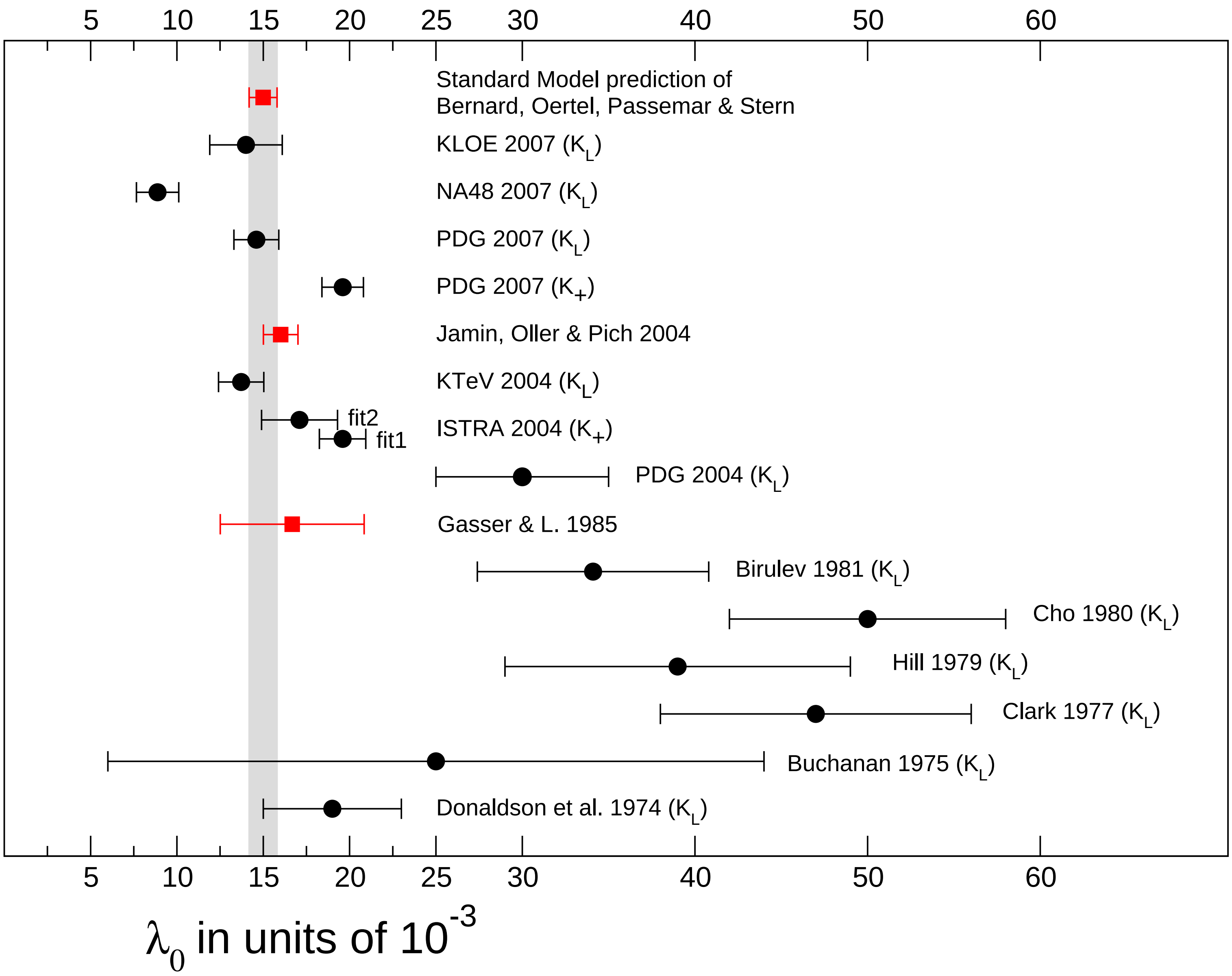


I thank Emilie Passemar for some of the material shown in this figure

Comparison of results for the slope



Older measurements



Conclusions for $K \to \mu\nu\pi$

- *experiment is difficult, discrepancies need to be resolved*

Donaldson 1974:	1.6×10^6 *events*
ISTRA 2004:	0.54×10^6 *events*
KTeV 2004:	1.5×10^6 *events*
NA48 2007:	2.3×10^6 *events*

- *dispersion theory fixes the shape of the form factors*
  publishing linear fits is nonsensical
- *NA48 should improve their data analysis . . . and extend it to charged kaons (isospin breaking)*