Recent developments in light flavor hadron physics

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QCD AND HIGH ENERGY HADRONIC INTERACTIONS

Rencontres de Moriond, La Thuile (Aosta) - Italy, March 19, 2007

Light flavour hadrons

- Main characteristic: small energy gap, $M_\pi \simeq 140$ MeV
- Hidden, approximate symmetry
- Symmetry becomes exact for $m_u, m_d \to 0$

$\Rightarrow$ Energy gap disappears: pions become massless

- In reality $m_u, m_d \neq 0$, but small

$\Rightarrow$ Symmetry is nearly perfect

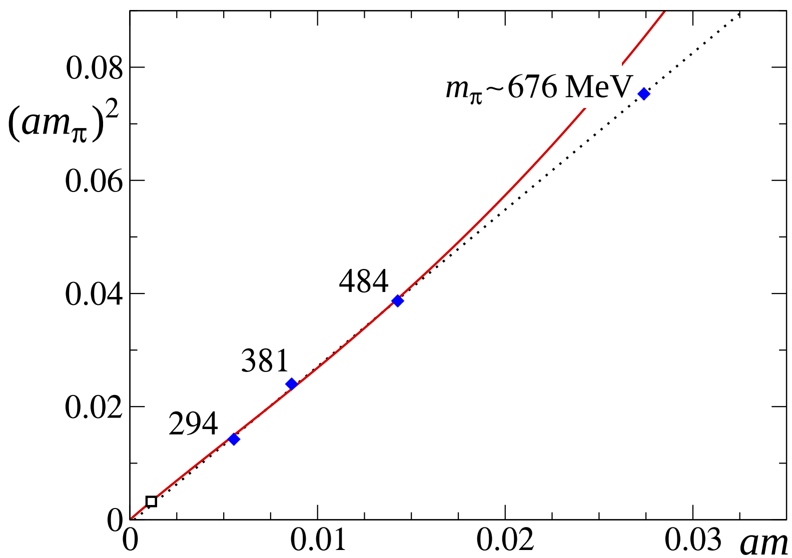
$$M_\pi^2 \simeq (m_u + m_d)B$$

Gell-Mann, Oakes, Renner 1968

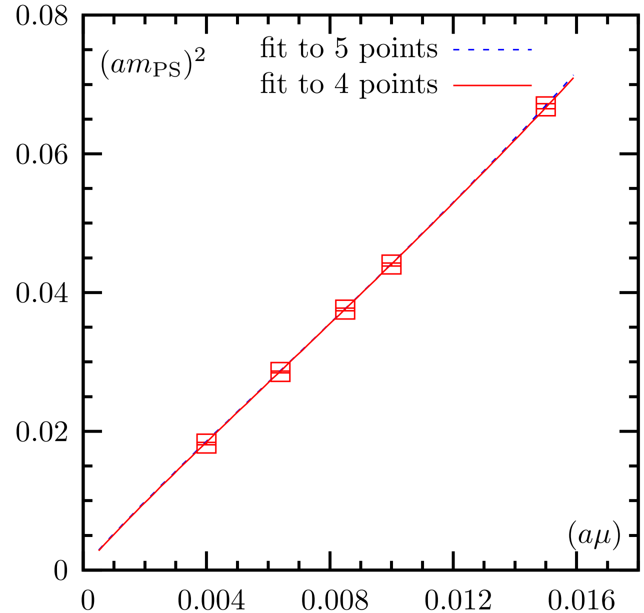
- Can now be checked on the lattice:

  set $m_u = m_d = m$, determine M_π as a function of m

M_π^2 versus quark mass m



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- No quenching, quark masses are sufficiently light

$\Rightarrow$ legitimate to use χPT for the extrapolation to the physical values of m_u, m_d

- Quality of data is impressive
- Main limitation: systematic uncertainties in particular: $N_f = 2 \rightarrow N_f = 3$

Expansion of M_π^2 in powers of the quark mass

- GMOR formula represents leading term
- At NLO, the expansion contains a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

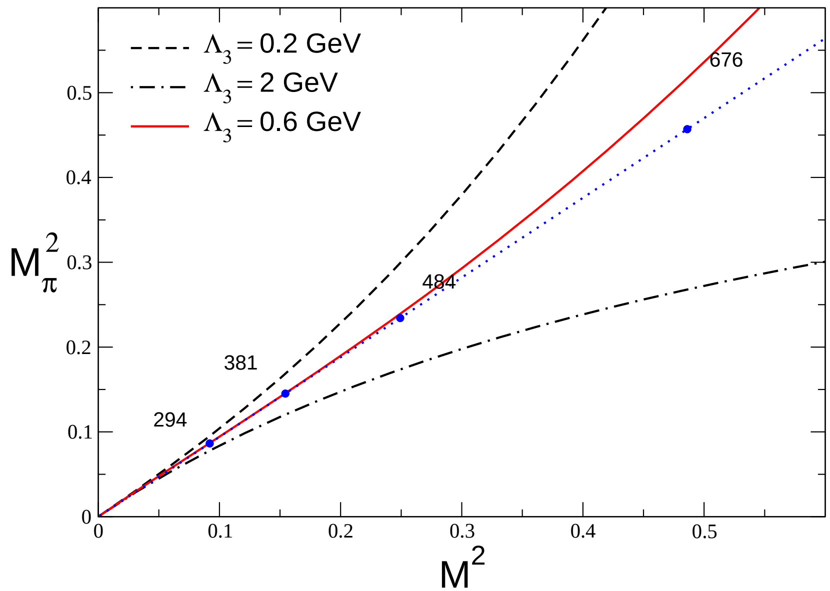
$$M^2 \equiv B(m_u + m_d)$$

- Coefficient is determined by pion decay constant F
  Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3) mass formulae:

$0.2\,\text{GeV} < \Lambda_3 < 2\,\text{GeV}$ Gasser & L. 1984

- Lattice allows more accurate determination of the scale

Lattice results for Λ_3



Express the result for Λ_3 in terms of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

$\bar{\ell}_3 = 2.9 \pm 2.4 \;\leftrightarrow\; 0.2\,\text{GeV} < \Lambda_3 < 2\,\text{GeV}$ Gasser & L. 1984

$\bar{\ell}_3 = 0.6 \pm 1.2$ MILC 2004, 2006

$\bar{\ell}_3 = 3.0 \pm 0.5$ Del Debbio et al. 2006

$\bar{\ell}_3 = 3.62 \pm 0.12$ ETM Collaboration 2007

$\pi\pi$ interaction

- Plays a crucial role whenever the strong interaction is involved at low energies

  Example: Standard model prediction for muon magnetic moment

- Significant theoretical progress, based on χPT + dispersion theory $\Rightarrow$ short break: commercial for theory

Commercial for theory of $\pi\pi$ interaction

- $\pi\pi$ scattering is special: crossed channels are identical

$\Rightarrow$ Re $T(s,t)$ can be represented as a twice subtracted dispersion integral over Im $T(s,t)$ in physical region

S.M. Roy 1971

- The 2 subtraction constants can be identified with the S-wave scattering lengths:

$$a_0^0 \;,\; a_0^2 \begin{array}{l} \leftarrow \text{isospin} \\ \leftarrow \text{angular momentum} \end{array}$$

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*
- Pioneering work on the physics of the Roy equations was done around the time when QCD was discovered

Pennington & Protopopescu 1973, Basdevant, Froggatt & Petersen 1974

Roy equations

- Dispersion integrals converge rapidly (2 subtractions)

$\Rightarrow$ Crude phenomenological information on $\operatorname{Im} T(s,t)$ for energies above 800 MeV suffices

$\Rightarrow$ Given a_0^0, a_0^2, the scattering amplitude can be calculated quite accurately

Ananthanarayan, Colangelo, Gasser & L. 2001
Descotes, Fuchs, Girlanda & Stern 2002

$\Rightarrow$ a_0^0, a_0^2 are the essential parameters at low energy

- Main problem in early work: a_0^0, a_0^2 poorly known experimental information near threshold is meagre

Low energy theorems

- Chiral perturbation theory provides the missing piece: theoretical prediction for a_0^0, a_0^2

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}, \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}$$

Weinberg 1966

- Valid at LO, corrections known to NNLO

Gasser & L. 1983, Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

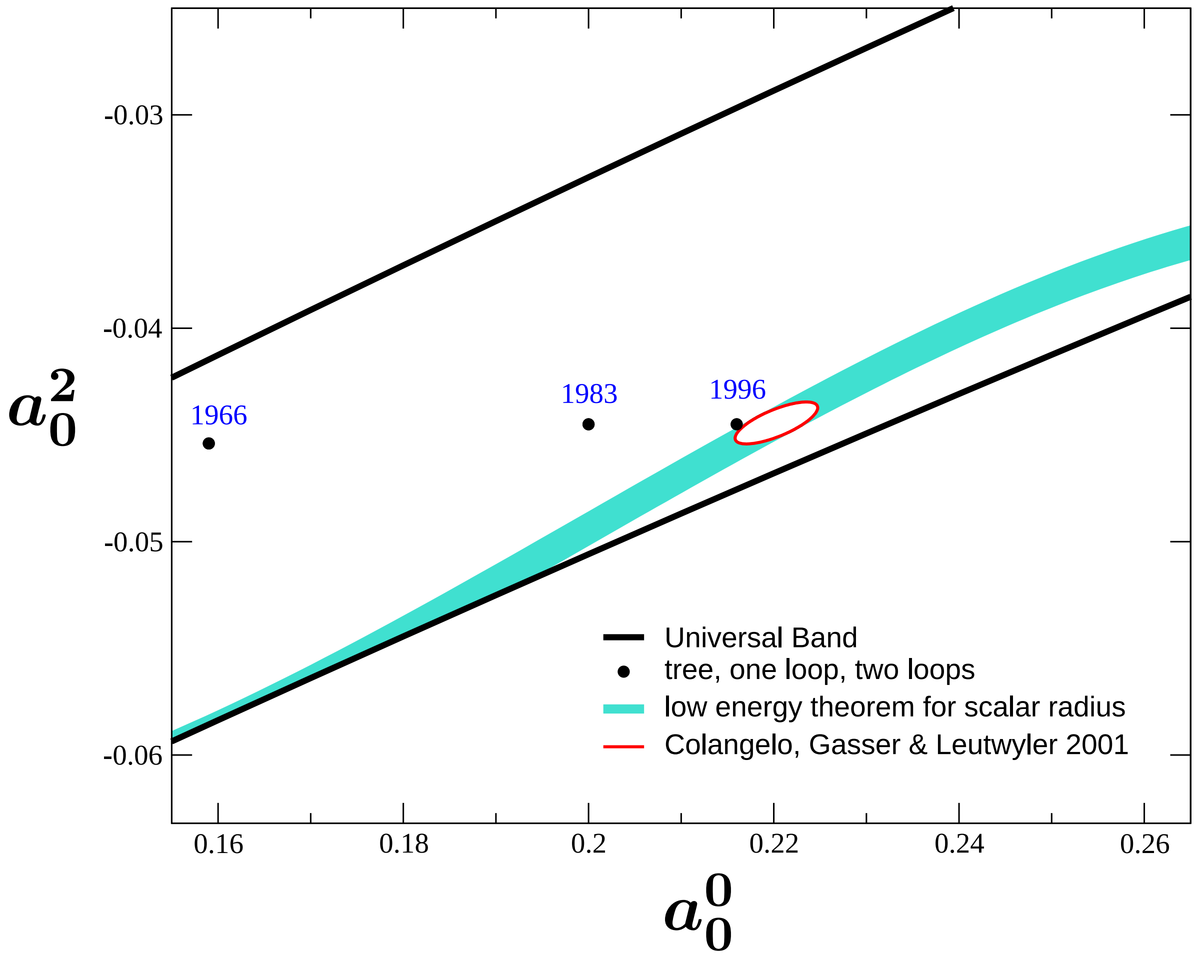
- Most accurate results for a_0^0, a_0^2 are obtained by matching the chiral and dispersive representations near the center of the Mandelstam triangle

Colangelo, Gasser & L. 2001

- In combination with the low energy theorems for a_0^0, a_0^2, the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to an incredible degree of accuracy

end of commercial

Predictions for the S-wave $\pi\pi$ scattering lengths

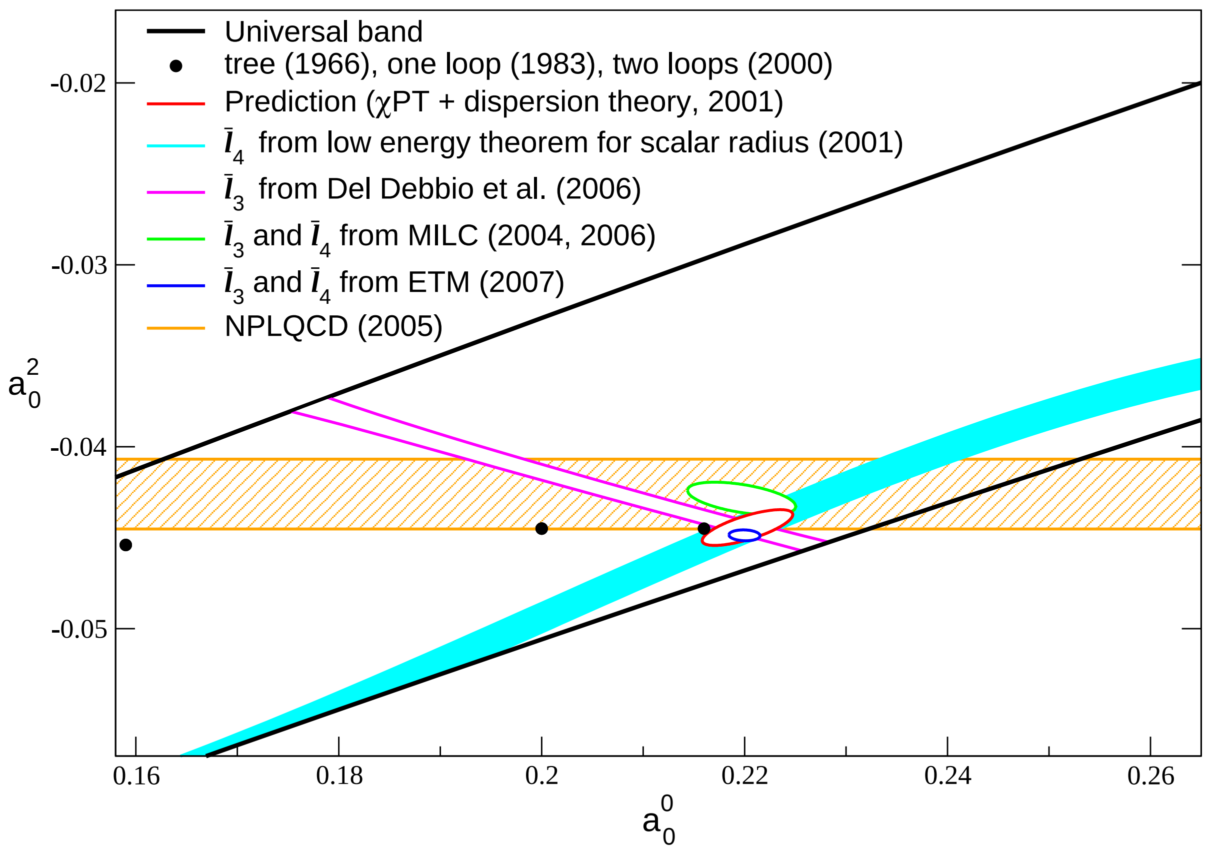


Sizeable corrections in a_0^0, while a_0^2 nearly stays put

Lattice results relevant for a_0^0, a_0^2

- Determine a_0^2 from volume dependence of energy levels
- Rely on χPT and dispersion theory Colangelo, Gasser & L. 2001

  Most important source of uncertainty in that analysis:
  $\bar{\ell}_3, \bar{\ell}_4 \Rightarrow$ take these from the lattice



Experiments on light flavours at low energy

- Production experiments $\pi N \to \pi\pi N$, $\psi \to \pi\pi\omega \ldots$
  problem: pions are not produced in vacuo

$\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple
  Accuracy rather limited

- $\pi^+\pi^-$ atoms, DIRAC
- $K^\pm \to \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $K^\pm \to \pi^+\pi^- e^\pm \nu$ precision data from E865, NA48/2

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> Puzzle with $K_L \to \pi^\pm \mu^\mp \nu$
>
> Data on the scalar form factor are in conflict with the Callan-Treiman-relation
>
> NA48 (hep-ex/0703002) confirms KTeV

Pionic atoms

- $\pi^+\pi^-$ atoms provide an ideal laboratory
- Decay through the strong interaction $\pi^+\pi^- \to \pi^0\pi^0$
  Decay rate $\propto (a_0^0 - a_0^2)^2$
- Interference of e.m. and strong interactions in bound state and decay is well understood

$\Rightarrow$ Can reliably measure low energy properties of the $\pi\pi$ scattering amplitude in this way

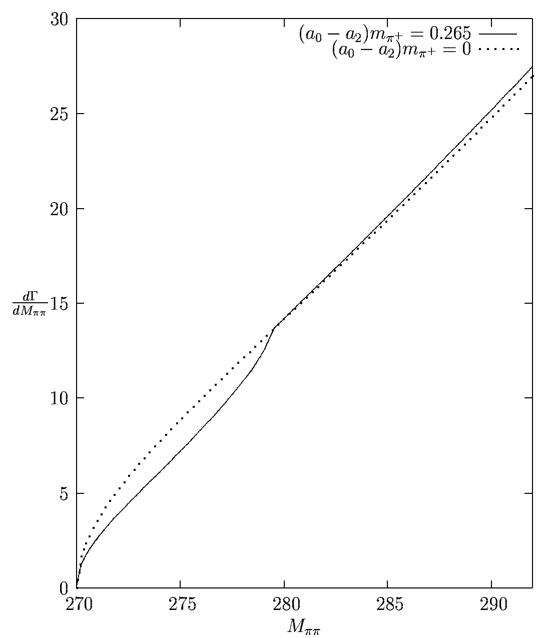
- Prediction for the lifetime: $\tau = 2.9 \pm 0.1$ fs

Gasser, Lyubovitskij, Rusetsky & Gall 2001

- Experimental result: $\tau = 2.91\,^{+0.49}_{-0.62}$ fs DIRAC 2005
- Experiment on πK-atoms is under way $\Rightarrow$ fabulous tool to explore strange quarks at low energy

Cusp in $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$

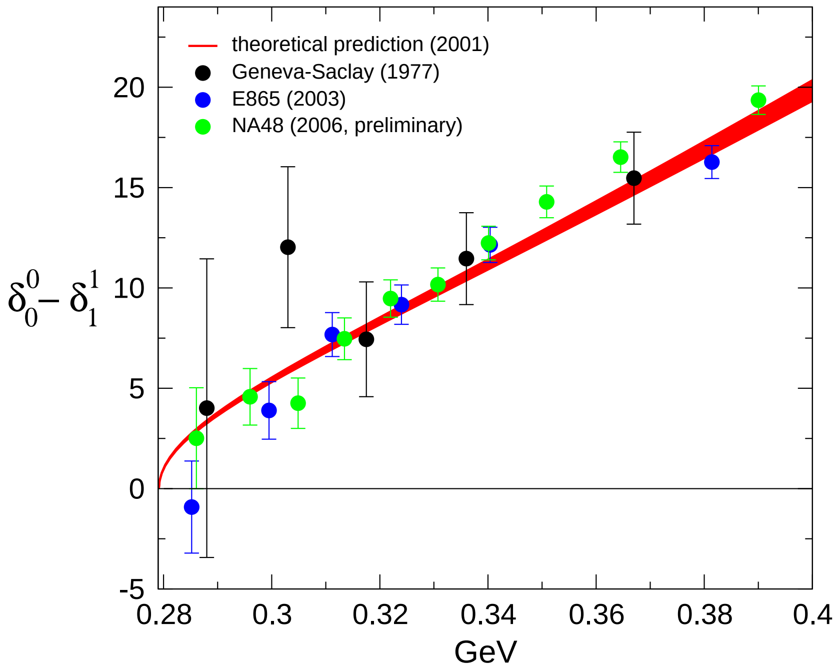
- Accurate data in the threshold region of the decay $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ allow a determination of $a_0^0 - a_0^2$
- NA48/2 has collected $\sim 10^8$ decays in this channel !



taken from N. Cabibbo, hep-ph/0405001

K_{e4} decay

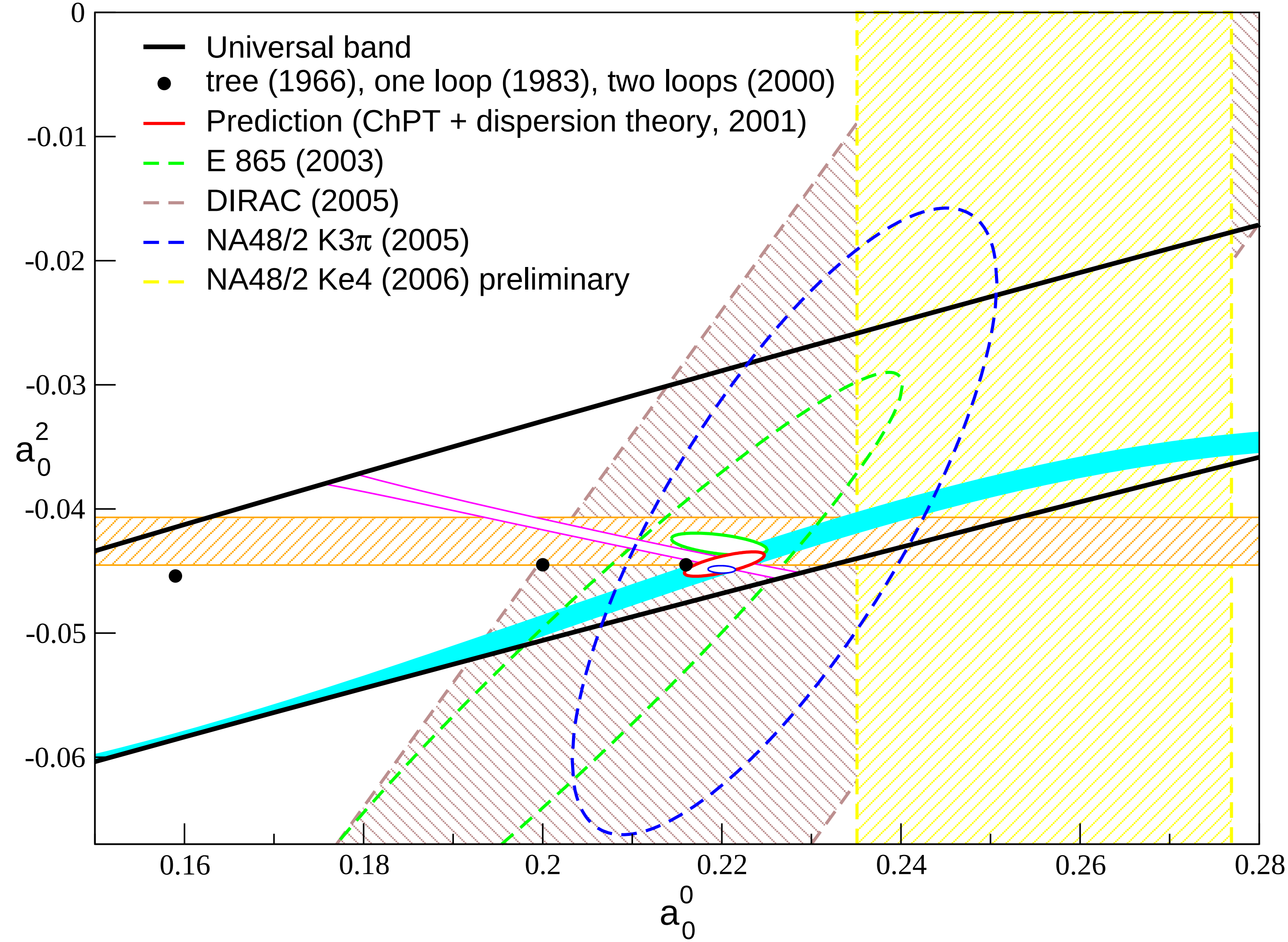
- $K \to \pi\pi\ell\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- Experimental situation is not conclusive:
E865 confirms the prediction, NA48/2 disagrees with it

$\Rightarrow$ talk by Brigitte Bloch-Devaux

Summary of experimental situation for a_0^0, a_0^2



Puzzling results for $K_L \to \pi^\pm \mu^\mp \nu$

- Hadronic matrix element of weak current:

$$\langle K^0 | \bar{u} \gamma^\mu s | \pi^- \rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

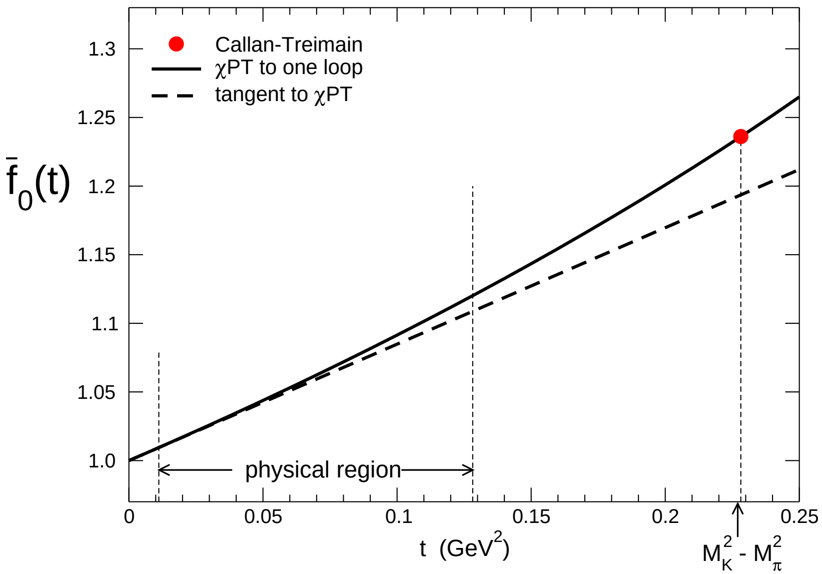
- Scalar form factor

$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- Slope parameter: $f_0(t) = f_0(0) \left\{ 1 + \frac{\lambda_0 \, t}{M_\pi^2} + O(t^2) \right\}$
- Low energy theorem of Callan and Treiman (1966):

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi} \left\{ 1 + O(m_u, m_d) \right\}$$

Scalar $K\pi$ form factor



Plot shows normalized scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- NLO corrections to Callan-Treiman-relation are tiny
  Early prediction for slope: $\lambda_0 = 1.7 \pm 0.4 \times 10^{-2}$

  Gasser & L. 1985

- Form factor now known to NNLO

  Post & Schilcher, Bijnens & Talavera

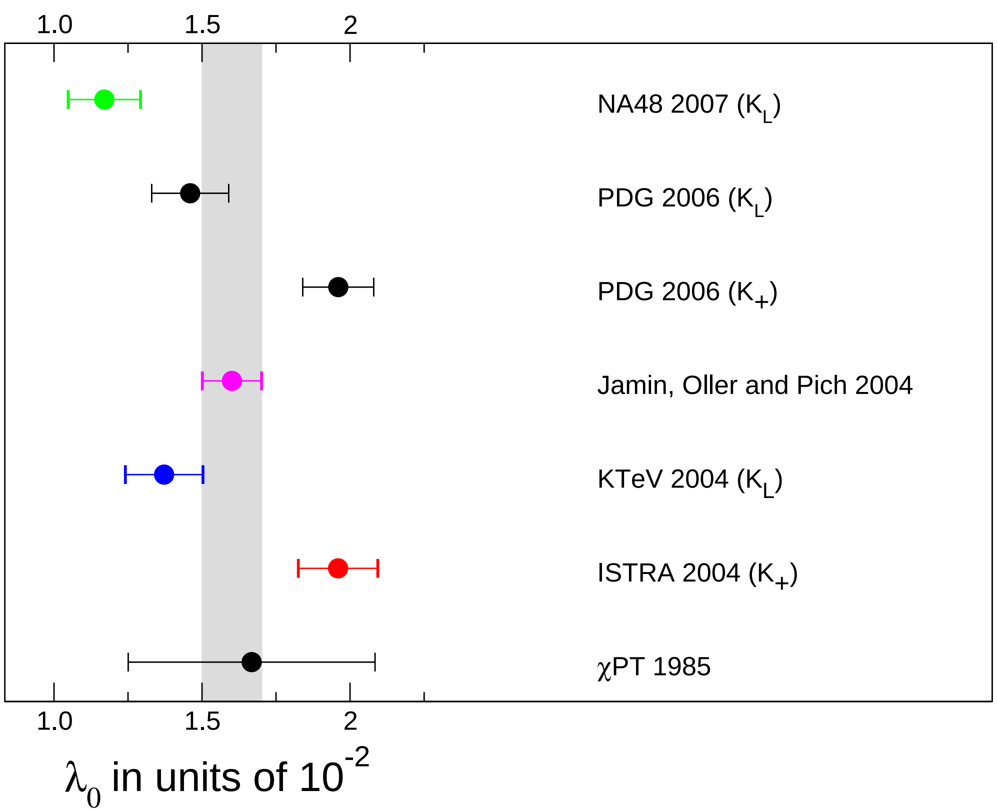
- Curvature not negligible at this precision
  is due to $I = \frac{1}{2}$ $K\pi$ final state interaction

Slope of the scalar $K\pi$ form factor

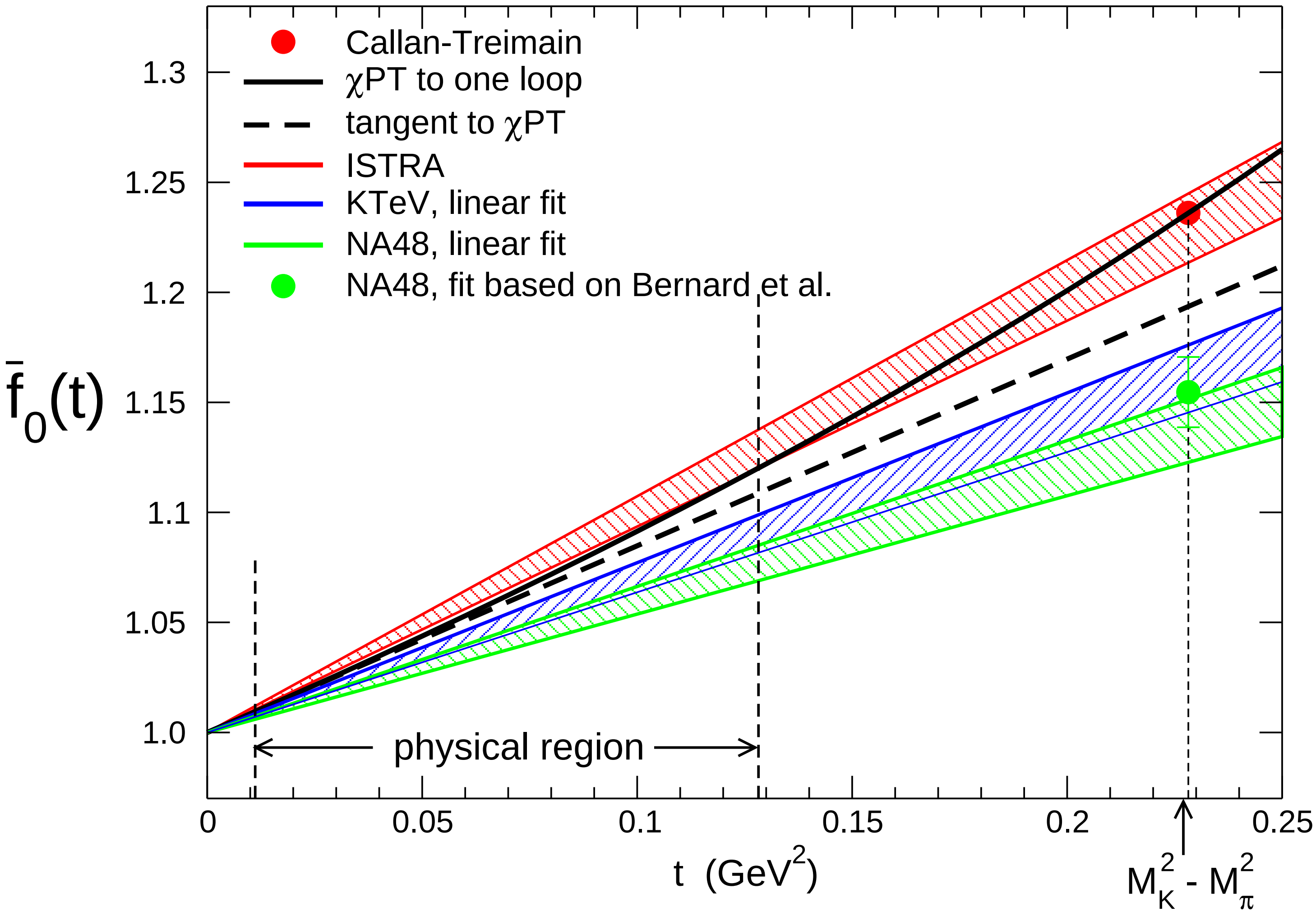
- Curvature can be calculated with dispersion theory

Jamin, Oller & Pich, Bernard, Oertl, Passemar & Stern

$\Rightarrow$ Callan-Treiman relation implies sharp prediction for the slope: $\lambda_0 = 1.60 \pm 0.10 \times 10^{-2}$ Jamin, Oller & Pich 2004



Scalar $K\pi$ form factor



Conclusions for scalar form factor ?

- KTeV and NA48 agree (K_L)
- KTeV and NA48 disagree with Callan-Treiman

$\Rightarrow$ Physics beyond the Standard Model

Righthanded currents ? Bernard, Oertl, Passemar & Stern

- KTeV and NA48 disagree with ISTRA ($K^{\pm}$)

$\Rightarrow$ Strong isospin breaking, also beyond Standard Model

Conclusions for scalar form factor ?

- Déja vu ?

  Donaldson 1974, 1.6×10^6 events

  $\lambda_0 = 1.9 \pm 0.4 \times 10^{-2} \Rightarrow$ Callan-Treiman $\surd$

  ISTRA: 0.54×10^6 events
  KTeV: 1.9×10^6 events
  NA48: 2.3×10^6 events

Conclusion for scalar form factor

a suivre !

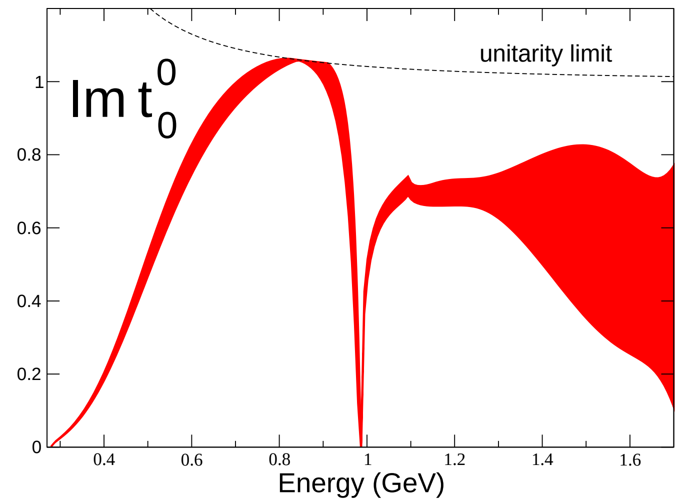
Where is the lowest resonance of QCD ?

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001

- Does QCD have a resonance near threshold ?
- Why care ?
  - Concerns the nonperturbative domain of QCD
  - Quark and gluon degrees of freedom useless there

  $\Rightarrow$ Understanding very poor, pattern of energy levels ?
  - Lowest resonance: σ ? ρ ?
- Resonance $\leftrightarrow$ pole on second sheet
  - Poles are universal
  - Pole position is unambiguous, even if width is large
  - Where is the pole closest to the origin ?

The red dragon



There is the broad object seen in $\pi\pi$ *scattering, often called "background", which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance*[2] *which we identify as the lightest glueball with quantum numbers* $J^{PC} = 0^{++}$ ...

[2] *we refer to it as red dragon*

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

Model independent determination of the pole

- All of the results quoted by the PDG are obtained by
  (a) parametrizing the data for real values of s
  (b) continuing this parametrization analytically in s

  $\Rightarrow$ Result is sensitive to the parametrization used
- We found a model independent method:
  1. Poles on second sheet are zeros on first sheet
  2. The Roy equations are valid for complex values of s, in a limited region of the first sheet

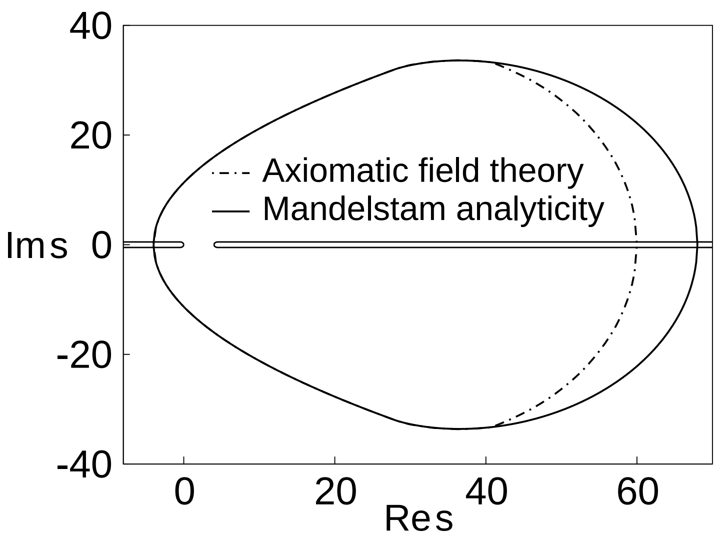
  $\Rightarrow$ Exact representation of the partial waves in terms of observable quantities, valid for complex values of s

  3. Can evaluate this representation to good precision and determine the zeros numerically

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$
- Equations are valid for complex s in a limited region of the first sheet

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001



s in units of M_π^2

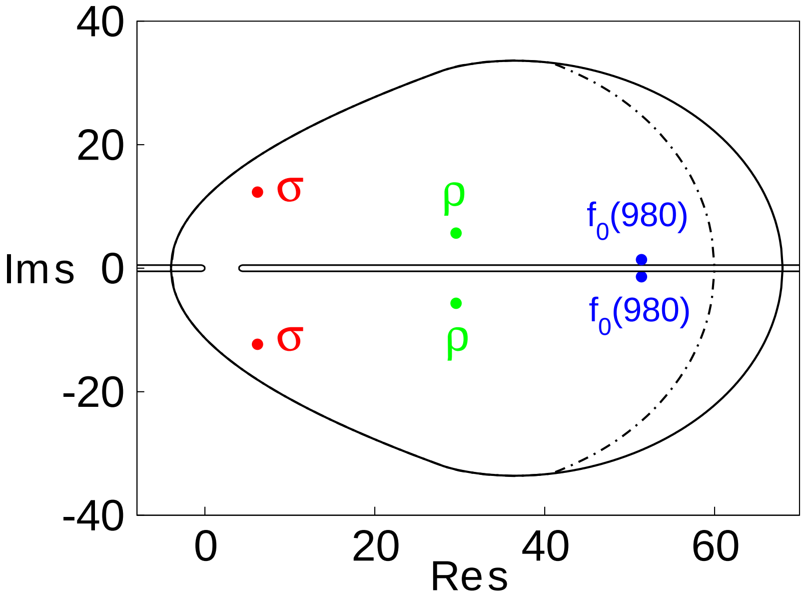
- Proof is based on first principles, general quantum field theory

A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing*, Lecture Notes in Physics, vol. 3, 1969.

G. Mahoux, S. M. Roy and G. Wanders, Nucl. Phys. B70 (1974) 297.

$\Rightarrow$ Exact representation for $S_0^0(s)$ in this region
Do not need to parametrize the amplitude

Result



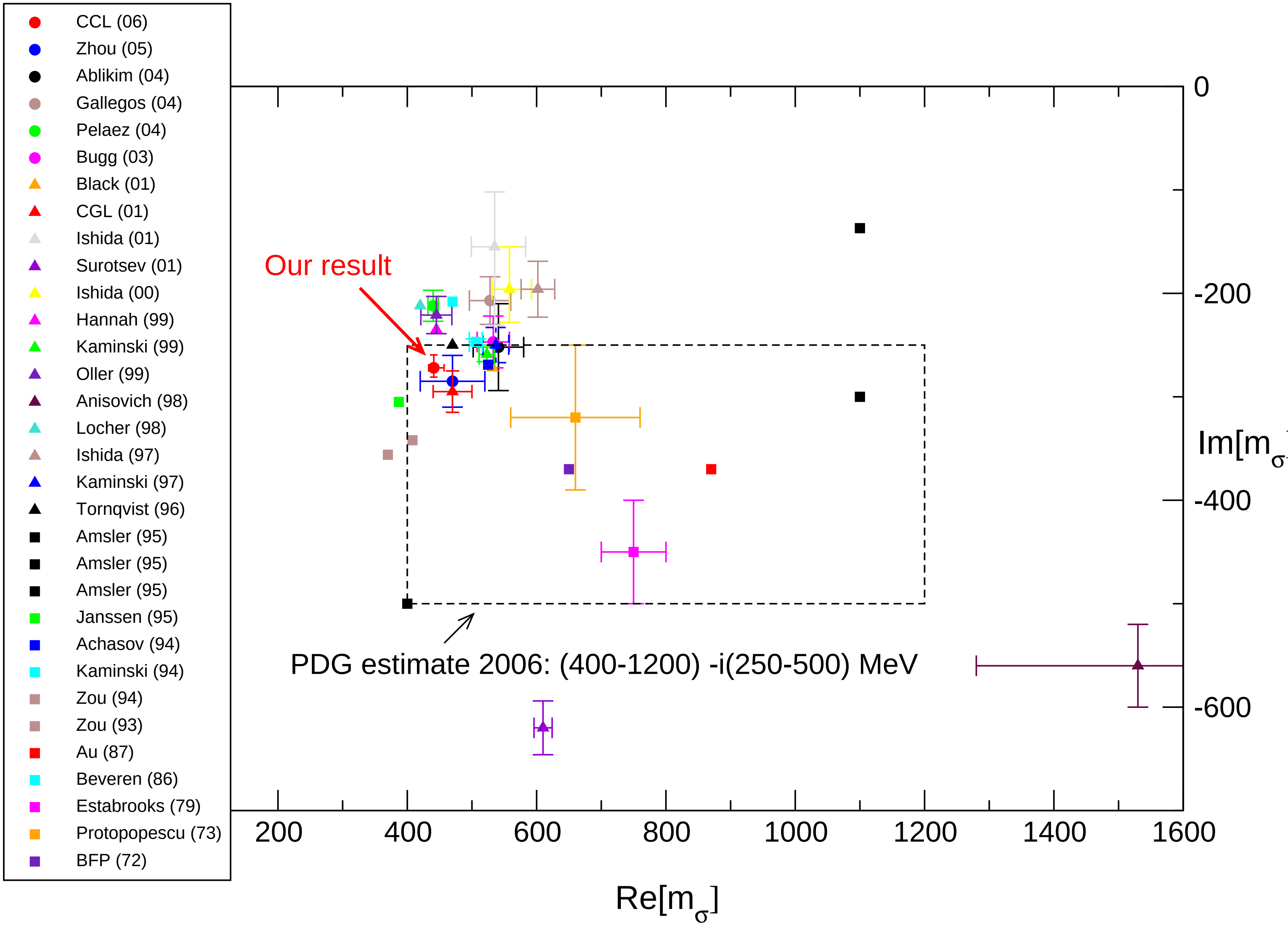
Loci Oculorum Draconis Rutili

T. Barnes, Theory summary, MESON 2006

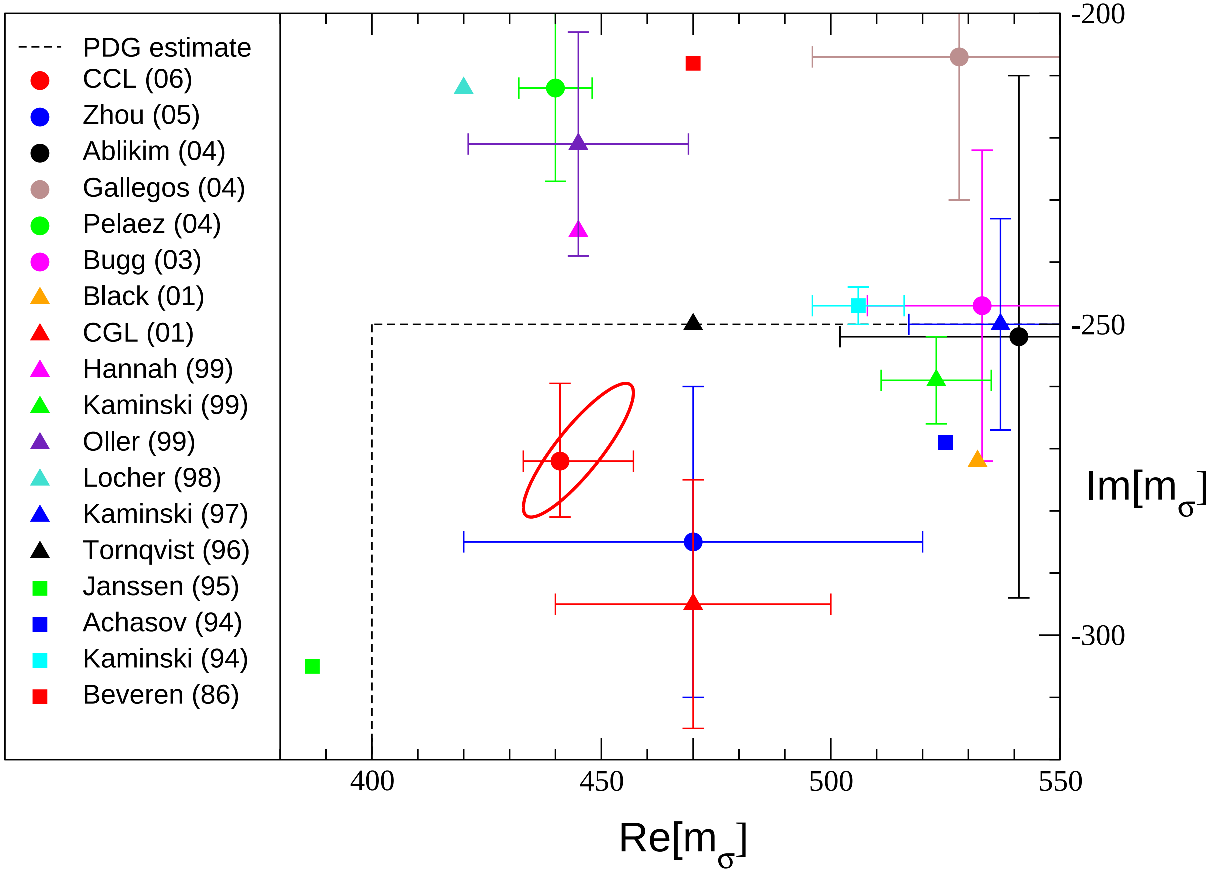
- Lowest resonance of QCD has vacuum quantum numbers
- Pole on lower half of second sheet occurs at

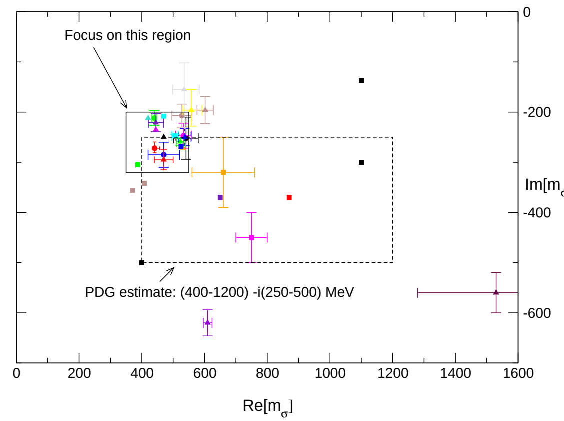
$$\sqrt{s} = 441\,^{+16}_{-8}\; - i\; 272\,^{+9}_{-13}\,\text{MeV} = M_\sigma - \frac{i}{2}\Gamma_\sigma$$

Comparison with compilation of PDG



Vicinity of the pole





Results for Re$[m_\sigma]$ and Im$[m_\sigma]$ are strongly correlated

Why are our errors so incredibly small ?

- The σ occurs at low energies
- At low energies, the subtraction term dominates

$$t_0^0(s) \simeq a_0^0 + (2a_0^0 - 5a_0^2)\frac{(s-4M_\pi^2)}{12M_\pi^2}$$

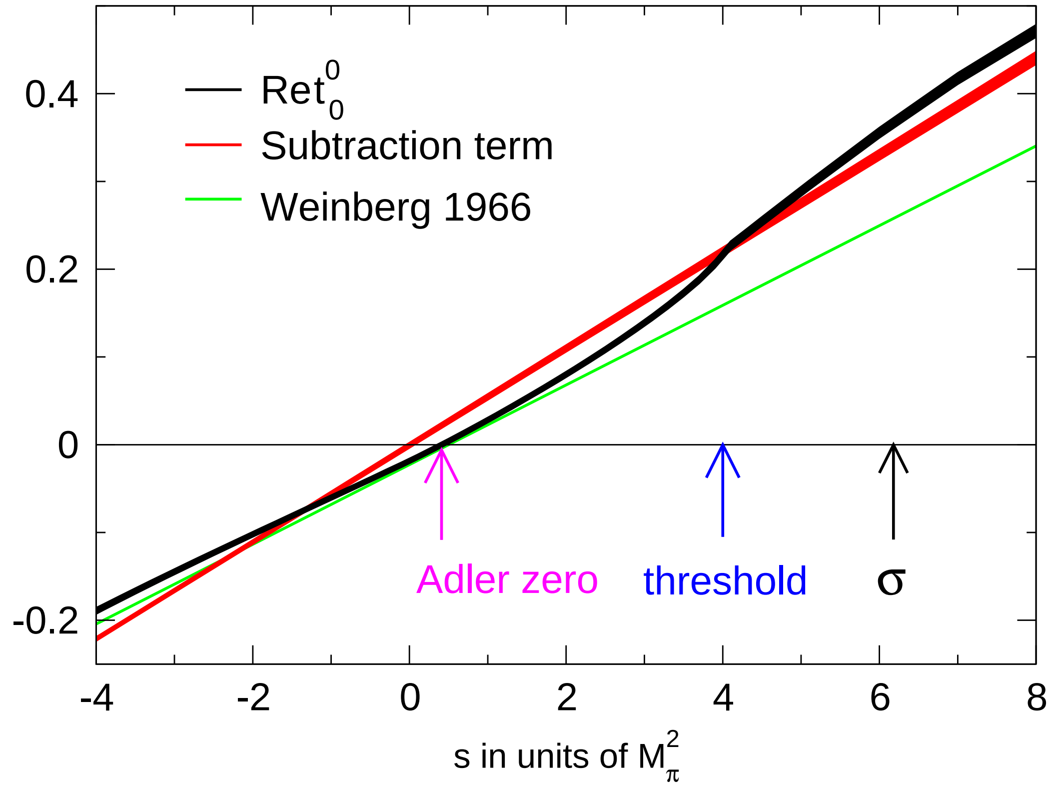
Insert low energy theorem for a_0^0, a_0^2

$\Rightarrow$ Roy equation reduces to Weinberg formula

$$t_0^0(s) \simeq \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Dispersion integrals only represent a correction

At low energies, the subtraction term dominates



$$s = (0.41 \pm 0.06)\,M_\pi^2$$ Adler zero

$$s = 4\,M_\pi^2$$ threshold

$$s = (6.2 - i\,12.3)\,M_\pi^2$$ pole from σ

Goldstone bosons of low energy interact only weakly

Physical interpretation of the σ

- The head of the dragon is not made of glue
- The dragon likes flavoured food, pions in particular

$\Rightarrow$ Physics of the $\sigma \in$ Goldstone boson dynamics

$\Rightarrow$ Wave function has large tetra-quark component

- Physics of the $f_0(980) \in$ Goldstone boson dynamics
  Interaction among two kaons is relevant
- These states are very sensitive to SU(3) breaking
- Multiplet pattern ? $a_0(980)$?
- $K\pi$ scattering amplitude obeys an analog of the Roy equations. Pole from κ can be calculated on this basis

$$m_\kappa = (658 \pm 13) - i\,(278.5 \pm 12)\ \text{MeV}$$

Descotes-Genon and Moussallam 2006

$\Rightarrow$ Physics of σ and κ is very similar

Conclusion

- Low energy pion physics: theory ahead of experiment
  - Precision experiments carried out and under way
  - Lattice makes slow, but steady progress
- Limitations of our approach:
  - Analysis only covers low energies
  - Calculations cannot be done on back of an envelope
  - Only a few applications have been worked out: $\pi\pi$ scattering, pion form factors, hadronic vacuum polarization in muon $g-2$, $\gamma\gamma \to \pi^0\pi^0$
  - Much is yet to be done: $J/\psi \to \omega\pi\pi$, $D \to 3\pi$, ... πK, πN, ...

Conclusion

- Model independent method for analytic continuation
  - The lowest resonance of QCD occurs at

    $$M_\sigma = 441\,{}^{+16}_{-8}\ \text{MeV} \qquad \Gamma_\sigma = 544\,{}^{+18}_{-25}\ \text{MeV}$$

    and carries vacuum quantum numbers
  - Crossing symmetry plays an essential role:
    Fixes contributions from left hand cut
    Ensures fast convergence, low energy dominance
  - Pole occurs at low value of s, closer to left hand cut than to singularities from $K\bar{K}$, $f_0(980)$
- Almost all tests confirm the theory, two exceptions:

  K_{e4} from NA48/2
  $K_{\mu 3}$ from KTeV and NA48
  Physics beyond the Standard Model ?

VISIT THE RED DRAGON

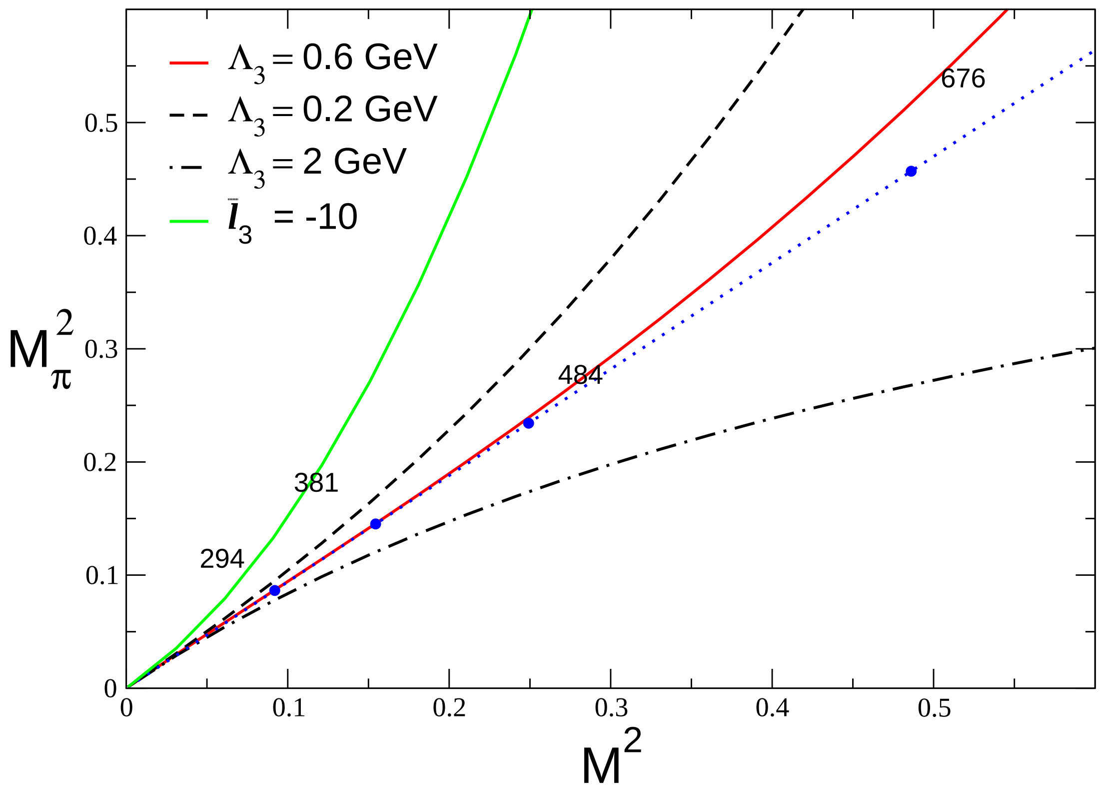
GENTLE ANIMAL
LOOK IN HIS EYES FROM CLOSE
SMELL HIS GOOD BREATH
BRING YOUR PIONS ALONG AND
FEED HIM YOURSELF

The management denies responsibility for incidents involving the dragon's tail

SPARES

K_{e4} decay

- fits to NA48/2 (Ke4) data require a negative value of $\bar{\ell}_3$
  $\Rightarrow$ M_π^2 cannot be linear in the quark mass



K_{e4} decay

- the weak current can produce a pair of neutral pions

$$K \rightarrow \pi^0 \pi^0 e\,\nu \quad \text{followed by} \quad \pi^0 \pi^0 \rightarrow \pi^+ \pi^-$$

$\Rightarrow$ phase of scalar form factor $\neq S$-wave phase shift

effect amounts to about half a degree, could remove the discrepancy with the prediction for $\delta_0^0 - \delta_1^1$

Gasser & Rusetsky 2007