

# Recent developments in the physics of the light quarks

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# Motivation

- QCD with massless quarks is the ideal of a theory: no dimensionless free parameters
- Gives rise to a rich structure at low energies
- High energy side looks like what we are used to: relevant degrees of freedom visible in the Lagrangian, can treat interaction as a perturbation
- Low energies are out of reach of perturbation theory  
⇒ Progress in understanding is slow
- $\exists$  many models that resemble QCD: instantons, monopoles, bags, superconductivity, gluonic strings, linear  $\sigma$  model, hidden gauge, NJL, AdS/CFT ...
- Discuss the model independent approach based on chiral perturbation theory + dispersion theory

# Le menu

~ *L' hors d'oeuvre* ~

## Energy gap of QCD

- Main characteristic of QCD at low energies:  
Energy gap is very small,  $M_\pi \simeq 140$  MeV
- In 1960, Nambu found out why that is so:  
Has to do with a hidden approximate symmetry
- Symmetry becomes exact for  $m_u, m_d \rightarrow 0$   
⇒ Energy gap disappears: pions become massless
- In reality  $m_u, m_d \neq 0$ , but very small  
⇒ Symmetry is not perfect, but nearly so

~ *L'entrée* ~

## Lattice results for light dynamical quarks

- Progress achieved on the lattice allows to simulate quarks with sufficiently small masses
- Can establish contact with physics, using  $\chi$ PT
- First meaningful determinations of effective coupling constants
- Lattice results for some of the CKM-matrix elements now reach an accuracy comparable to the results obtained from nuclear  $\beta$  decay

~ *La spécialité du patron* ~

Interaction among the pions

- Significant progress in understanding the interaction among the pions by means of dispersion theory
- Roy equations for the  $\pi\pi$  scattering amplitude are particularly well suited
- Two subtraction constants  $\leftrightarrow$  S-wave scattering lengths
- Use low energy theorems for the scattering lengths
- $\Rightarrow$  Scattering amplitude is fixed to an incredible precision
- $\Rightarrow$  Currently, theory is ahead of experiment ...

~ *Le dessert* ~

## Lowest resonance of QCD

- Masses and widths of the lowest resonances can now be calculated in a reliable manner
- ⇒ The lightest resonance of QCD has the quantum numbers of the vacuum and sits at

$$M_\sigma - \frac{i}{2}\Gamma_\sigma \simeq 441 - i 272 \text{ MeV}$$

~ *L' hors d'oeuvre* ~

Energy gap of QCD

## Massless quarks

- The size of the energy gap is controlled by  $m_u, m_d$   
The remaining flavours do not play an essential role

- Theoretical paradise: only two flavours, no mass

$$\text{QCD with } N_f = 2, m_u = m_d = 0$$

- Interactions of  $u$  and  $d$  are identical  
If the masses are the same, there is no difference at all

- More symmetry: for massless fermions,  
right and left do not communicate

- ⇒ Lagrangian is invariant under independent rotations of  
the right- and left-handed quark fields  
 $SU(2)_R \times SU(2)_L$  chiral symmetry

- ⇒ 6 conserved “charges”

$$Q_1^V, Q_2^V, Q_3^V \quad (\text{vector currents})$$

$$Q_1^A, Q_2^A, Q_3^A \quad (\text{axial currents})$$



# Asymmetry of the ground state

- Charges commute with the Hamiltonian:

$$[Q_i^V, H_0] = 0 \quad [Q_i^A, H_0] = 0$$

- The state of lowest energy is asymmetric with respect to chiral rotations:  $Q_i^A |0\rangle \neq 0$  Nambu 1960

⇒ Chiral symmetry is hidden, “spontaneously broken”

- Very strong experimental evidence ✓
- Very strong evidence from lattice calculations ✓
- Analytic understanding of the ground state still poor

# Goldstone bosons

- Consequence of  $Q_i^A |0\rangle \neq 0$  :

$$H_0 Q_i^A |0\rangle = Q_i^A H_0 |0\rangle = 0$$

- ⇒ Spectrum of QCD with two massless flavours must contain 3 states  $Q_1^A |0\rangle$ ,  $Q_2^A |0\rangle$ ,  $Q_3^A |0\rangle$  with  $E = 0$

“Goldstone bosons”

- spin 0, negative parity
- Indeed, the 3 lightest hadrons do have these quantum numbers:  $\pi^+$ ,  $\pi^0$ ,  $\pi^-$

But massless they are not

## Back to earth

- world  $\neq$  paradise

Light quark masses break chiral symmetry,  
allow the left to talk to the right

- Chiral symmetry broken in two ways:

spontaneously

$$Q_i^A |0\rangle \neq 0$$

explicitly

$$m_u, m_d \neq 0$$

- Only the diagonal vector currents are strictly conserved  
in QCD:  $N_u, N_d, N_s, N_c, N_b, N_t \rightarrow$  baryon number,  
electric charge, strangeness, charm, ...

- It so happens that  $m_u$  and  $m_d$  are small

$\Rightarrow H_{\text{QCD}}$  has an approximate  $SU(2)_L \times SU(2)_R$  symmetry

# Quark masses as perturbations

- Masses of  $u, d$  enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$

$$H_1 = \int d^3x \{m_u \bar{u}u + m_d \bar{d}d\}$$

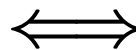
$H_0$  describes  $u, d$  as massless,  $s, c, b, t$  as massive

$H_0$  is invariant under  $SU(2)_L \times SU(2)_R$

- $H_0$  treats the pions as massless particles

$H_1$  gives them a mass

Expansion in  
powers of  $m_u, m_d$



Perturbation series  
in powers of  $H_1$

# Gell-Mann-Oakes-Renner formula

- First order perturbation theory in  $H_1$  yields:

$$M_\pi^2 = (m_u + m_d) \times |\langle 0 | \bar{u}u | 0 \rangle| \times \frac{1}{F_\pi^2}$$

$\uparrow$  explicit                       $\uparrow$  spontaneous

Gell-Mann, Oakes & Renner 1968

Coefficient: decay constant  $F_\pi$

$$\langle 0 | \bar{d} \gamma^\mu \gamma_5 u | \pi^+ \rangle = i p^\mu \sqrt{2} F_\pi$$

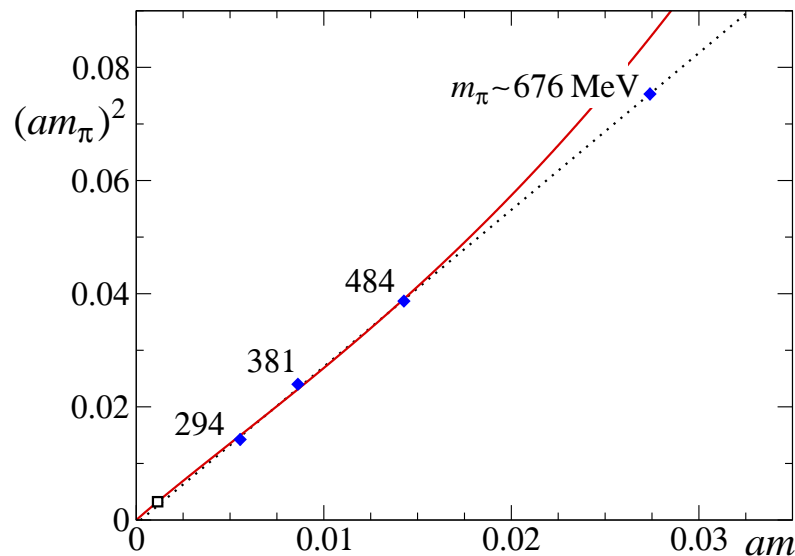
Value of  $F_\pi$  is known from  $\pi^+ \rightarrow \mu^+ \nu$

~ *L' hors d'oeuvre* ~

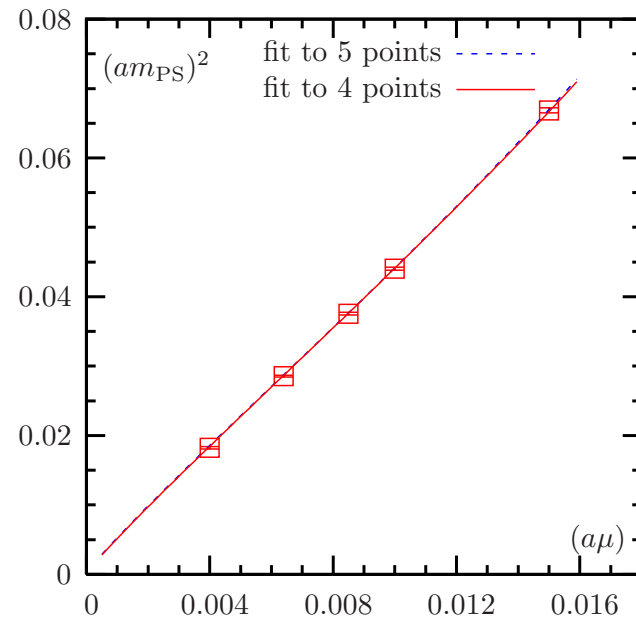
# Lattice results for light dynamical quarks

## Lattice results for $M_\pi$

- GMOR formula can now be checked on the lattice:  
determine  $M_\pi$  as a function of  $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- No quenching, quark masses are sufficiently light  
 $\Rightarrow$  legitimate to use  $\chi$ PT for the extrapolation to the physical values of  $m_u, m_d$

# Lattice

- Quality of data is impressive
- Proportionality of  $M_\pi^2$  to the quark mass appears to hold out to values of  $m_u, m_d$  that are an order of magnitude larger than in nature
- Main limitation: systematic uncertainties in particular:  $N_f = 2 \rightarrow N_f = 3$



# Chiral perturbation theory

- Consequences of hidden, approximate symmetry can be worked out by means of an effective field theory

Weinberg 1979

- Hidden symmetry controls energy gap of QCD
- ⇒ Can calculate how gap grows as the symmetry breaking parameters  $m_u, m_d$  are turned on
- Hidden symmetry also determines the interaction of the Goldstone bosons at low energies, among themselves, as well as with other hadrons

# Expansion of $M_\pi^2$ in powers of the quark mass

- GMOR formula represents leading term of  $\chi$ PT
- At NLO, the expansion contains a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

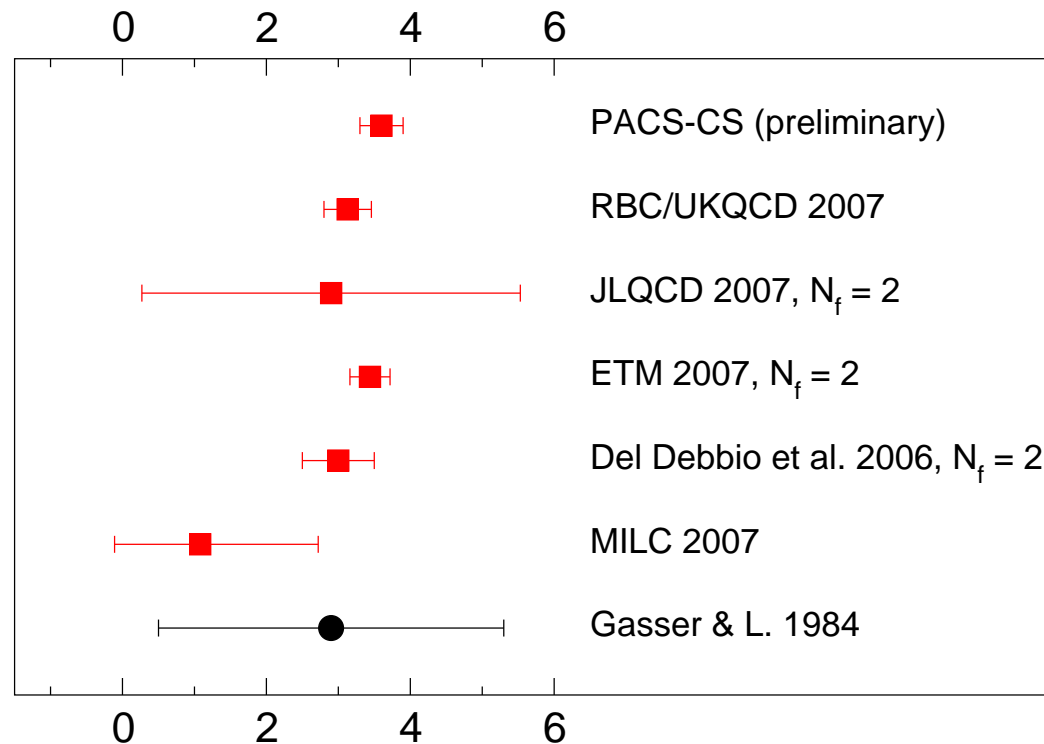
$$M^2 \equiv B(m_u + m_d)$$

Langacker & Pagels 1973, Gasser and Zepeda 1980, Gasser 1981

- Coefficient is determined by pion decay constant  
Symmetry does not determine the scale  $\Lambda_3$
- Crude result, based on  $SU(3) \times SU(3)$ :  
 $0.2 \text{ GeV} \lesssim \Lambda_3 \lesssim 2 \text{ GeV}$

Gasser & L. 1984

# Lattice allows more accurate determination of $\Lambda_3$



Horizontal axis shows the value of  $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

Range for  $\Lambda_3$  obtained in 1984 corresponds to  $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of RBC/UKQCD 2007, for instance, is  $\bar{\ell}_3 = 3.13 \pm 0.33$

## Expansion of $F_\pi$ in powers of the quark mass

- Also contains a logarithm at NLO:

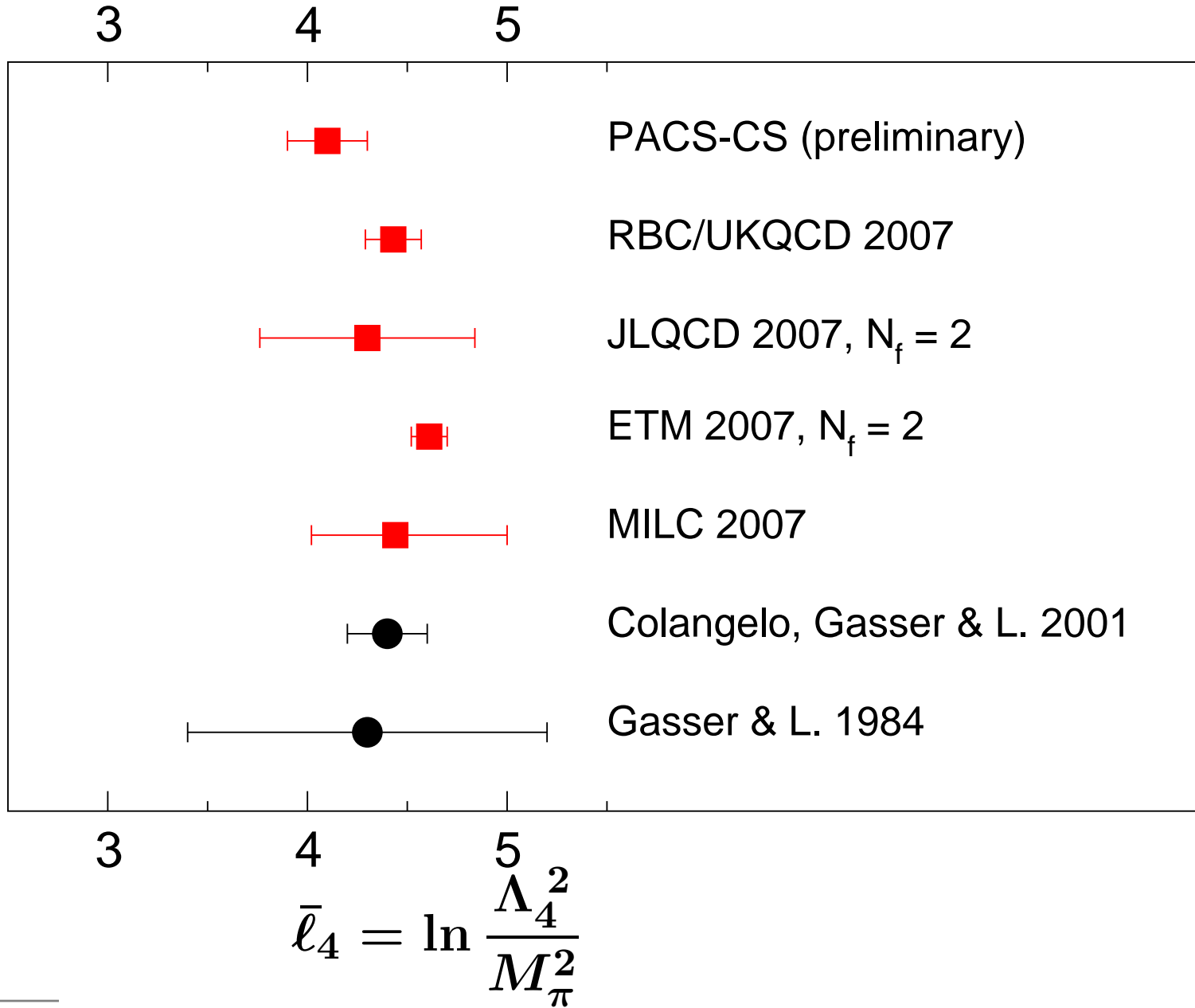
$$F_\pi = F \left\{ 1 - \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\Lambda_4^2} + O(M^4) \right\}$$

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

$F$  is value of pion decay constant in limit  $m_u, m_d \rightarrow 0$

- Structure is the same, coefficients and scale of logarithm are different
- Quark mass dependence of  $F_\pi$  can also be measured on the lattice  $\Rightarrow$  measurement of  $\Lambda_4$
- Alternative method: determine the scalar form factor of the pion, radius  $\langle r^2 \rangle_s \Leftrightarrow \bar{\ell}_4$

# Lattice results for $\Lambda_4$



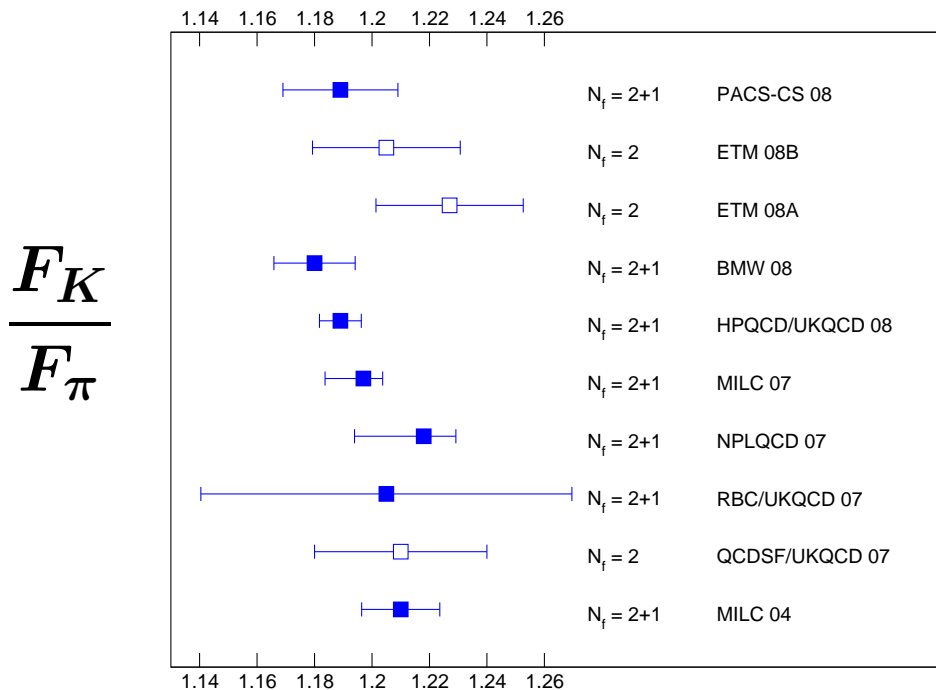
# Lattice results for $\pi \rightarrow \ell\nu, K \rightarrow \ell\nu$

- Relevant for determination of the CKM matrix

$$\frac{\Gamma_{K \rightarrow \ell\nu}}{\Gamma_{\pi \rightarrow \ell\nu}} = \left| \frac{V_{us} F_K}{V_{ud} F_\pi} \right|^2 = (0.2760 \pm 0.0006)^2$$

$$\langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi \rangle = i\sqrt{2} F_\pi p^\mu, \quad \langle 0 | \bar{u} \gamma^\mu \gamma_5 s | K \rangle = i\sqrt{2} F_K p^\mu$$

$\Rightarrow F_K / F_\pi$  determines  $V_{us} / V_{ud}$



Review of lattice data:  
FLAG, to appear soon

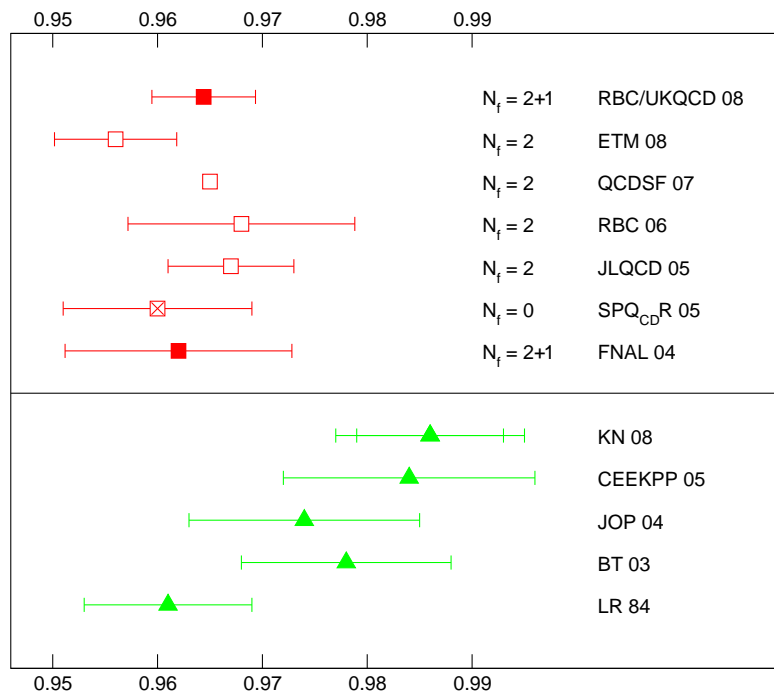
# Results for $K \rightarrow \pi \ell \nu$

●  $|V_{us}| f_+(0) = 0.21661 \pm 0.00047$

$$\langle K | \bar{u} \gamma^\mu s | \pi \rangle = (p_K + p_\pi) f_+(t) + (p_K - p_\pi) f_-(t)$$

$\Rightarrow f_+(0)$  determines  $V_{us}$

$f_+(0)$



lattice results

estimates based on ChPT

## Comment concerning estimates based on $\chi$ PT

- All of the  $\chi$ PT estimates rely on a model
- Recent work: saturation by resonances

Works very well in the vector channel  
vector meson dominance ✓

Possibly fails in other channels  
scalar meson dominance ?



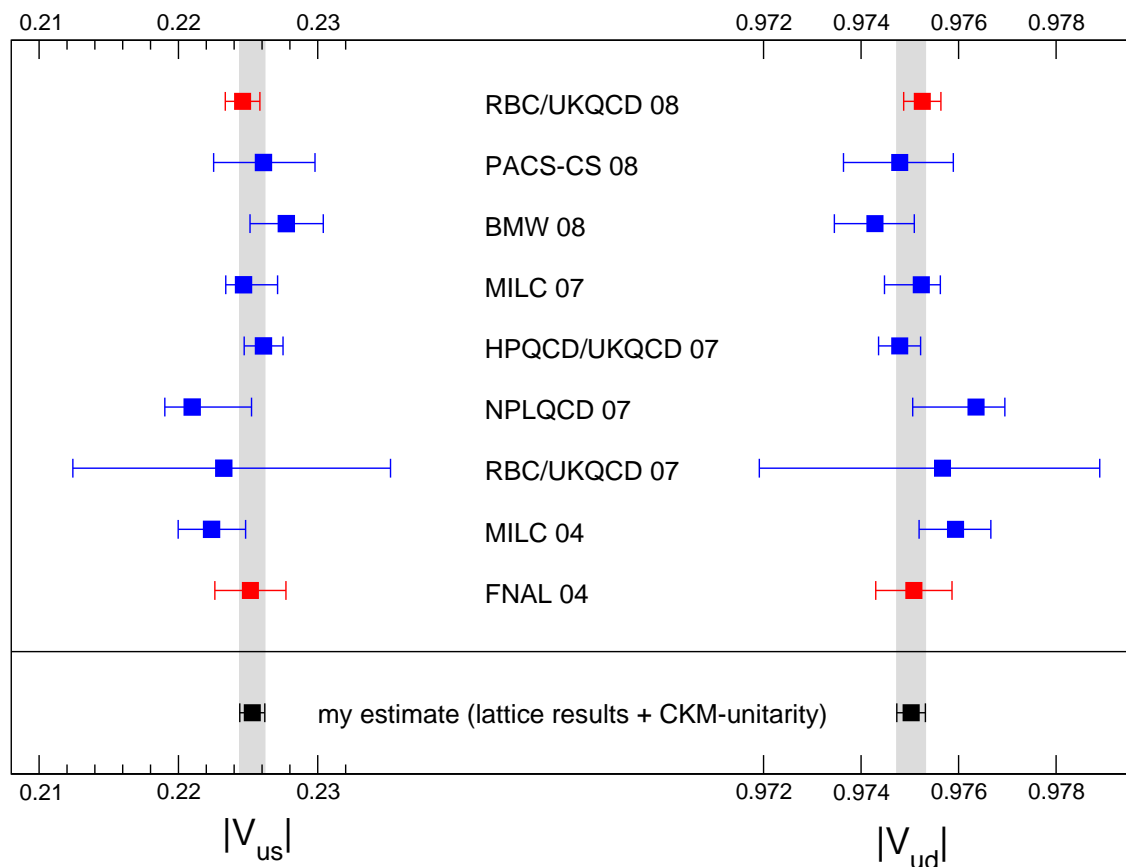
# Consequence of lattice results for $V_{us}$ and $V_{ud}$

● Use only unquenched data for  $N_f = 2 + 1$

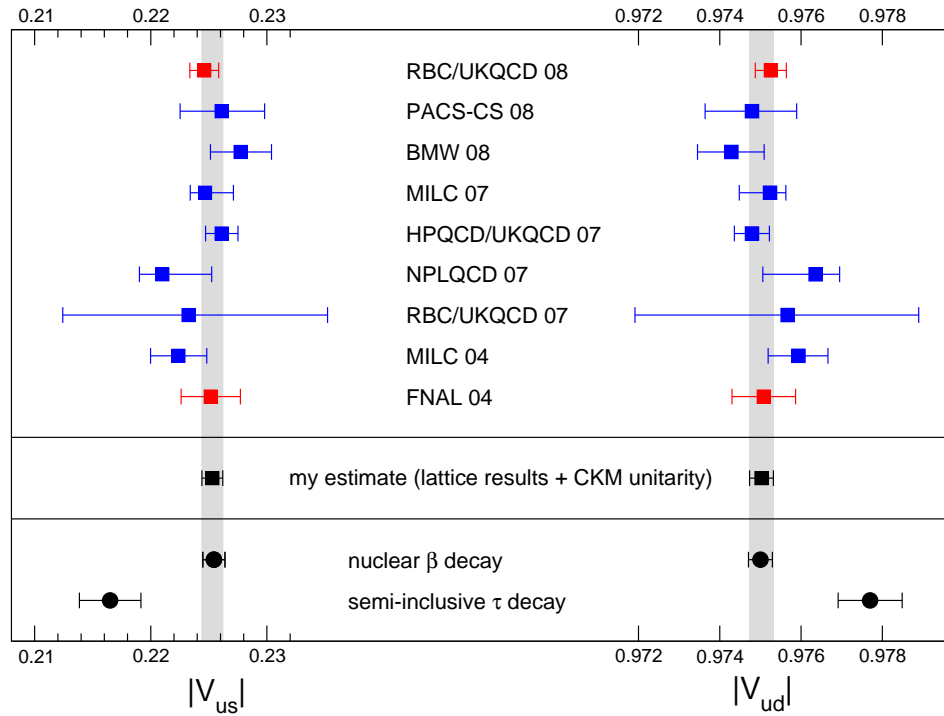
● Assume that CKM-matrix is unitary (SM)

$$\Rightarrow V_{us} = 0.2243 \pm 0.0004_{\text{stat}} \pm 0.0009_{\text{syst}}$$

$$\Rightarrow V_{ud} = 0.97427 \pm 0.0009_{\text{stat}} \pm 0.0020_{\text{syst}}$$



# Comparison with experiment



- Lattice data are consistent with CKM unitarity
- Confirm value of  $V_{ud}$  extracted from nuclear  $\beta$  decay
- $\tau$ -decay ? New physics ?

~ *La spécialité du patron* ~

Interaction among the pions

# Theory of $\pi\pi$ interaction

- $\pi\pi$  scattering is special: crossed channels are identical
- ⇒  $\text{Re } T(s, t)$  can be represented as a twice subtracted dispersion integral over  $\text{Im } T(s, t)$  in physical region

S.M. Roy 1971

- The 2 subtraction constants can be identified with the  $S$ -wave scattering lengths:

$$a_0^0, a_0^2 \begin{array}{l} \leftarrow \text{isospin} \\ \leftarrow \text{angular momentum} \end{array}$$

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*

# Roy equations

- Pioneering work on the physics of the Roy equations was done around the time when QCD was discovered  
Pennington & Protopopescu 1973, Basdevant, Froggatt & Petersen 1974
- Dispersion integrals converge rapidly (2 subtractions)  
⇒ Crude phenomenological information on  $\text{Im } T(s, t)$  for energies above 800 MeV suffices
- ⇒ Given  $a_0^0, a_0^2$ , the scattering amplitude can be calculated very accurately  
Ananthanarayan, Colangelo, Gasser & L. 2001  
Descotes, Fuchs, Girlanda & Stern 2002
- ⇒  $a_0^0, a_0^2$  are the essential parameters at low energy
- Main problem in early work:  $a_0^0, a_0^2$  poorly known  
Experimental information near threshold is meagre

# Low energy theorems

- Chiral perturbation theory provides the missing piece: theoretical prediction for  $a_0^0, a_0^2$

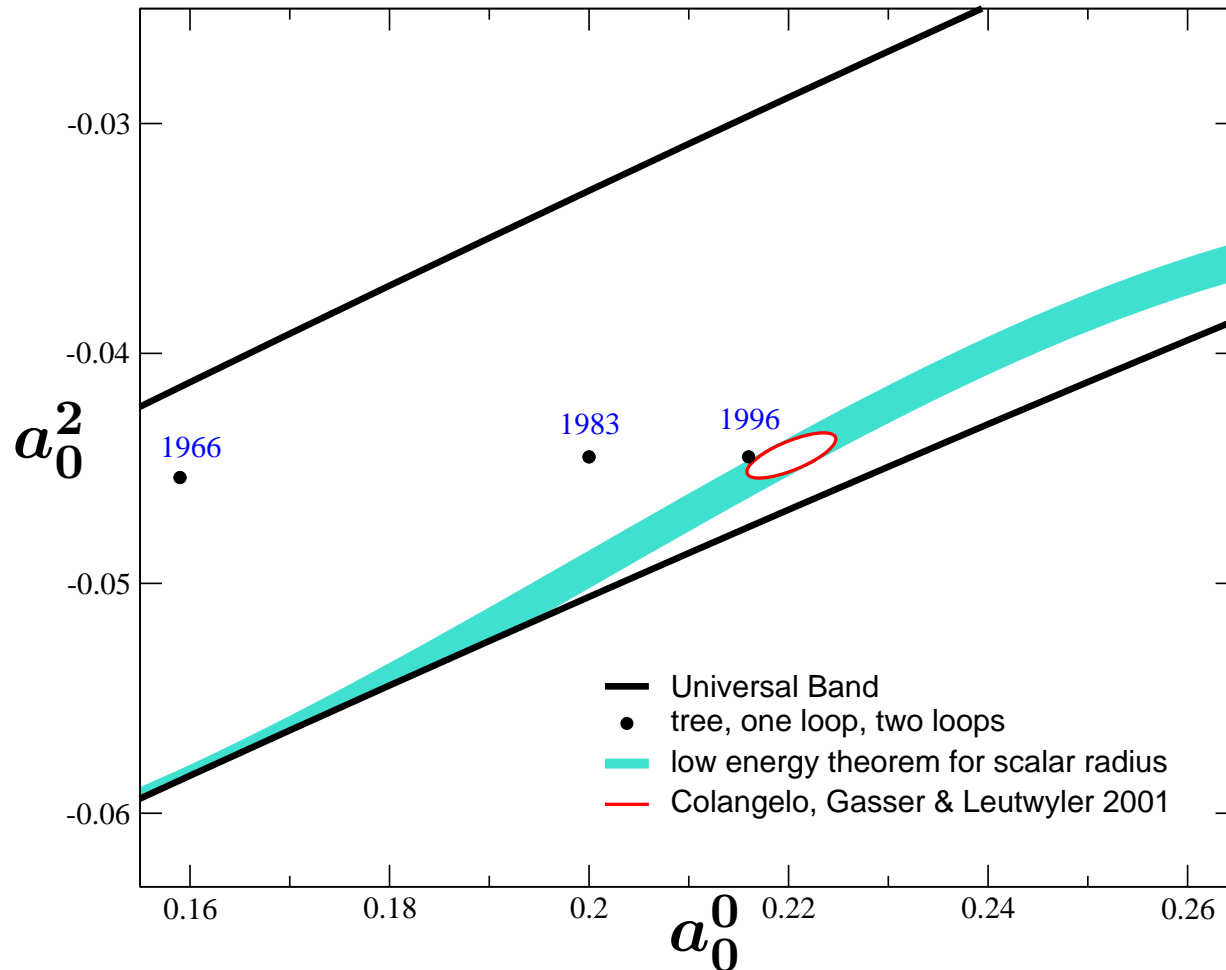
Weinberg 1966, Gasser & L. 1983, Bijens, Colangelo, Ecker, Gasser & Sainio 1996

- Most accurate results for  $a_0^0, a_0^2$  are obtained by matching the chiral and dispersive representations in the unphysical region below threshold

Colangelo, Gasser & L. 2001

- In combination with the low energy theorems for  $a_0^0, a_0^2$ , the dispersion relations for the partial waves fix the  $\pi\pi$  scattering amplitude to an incredible degree of accuracy

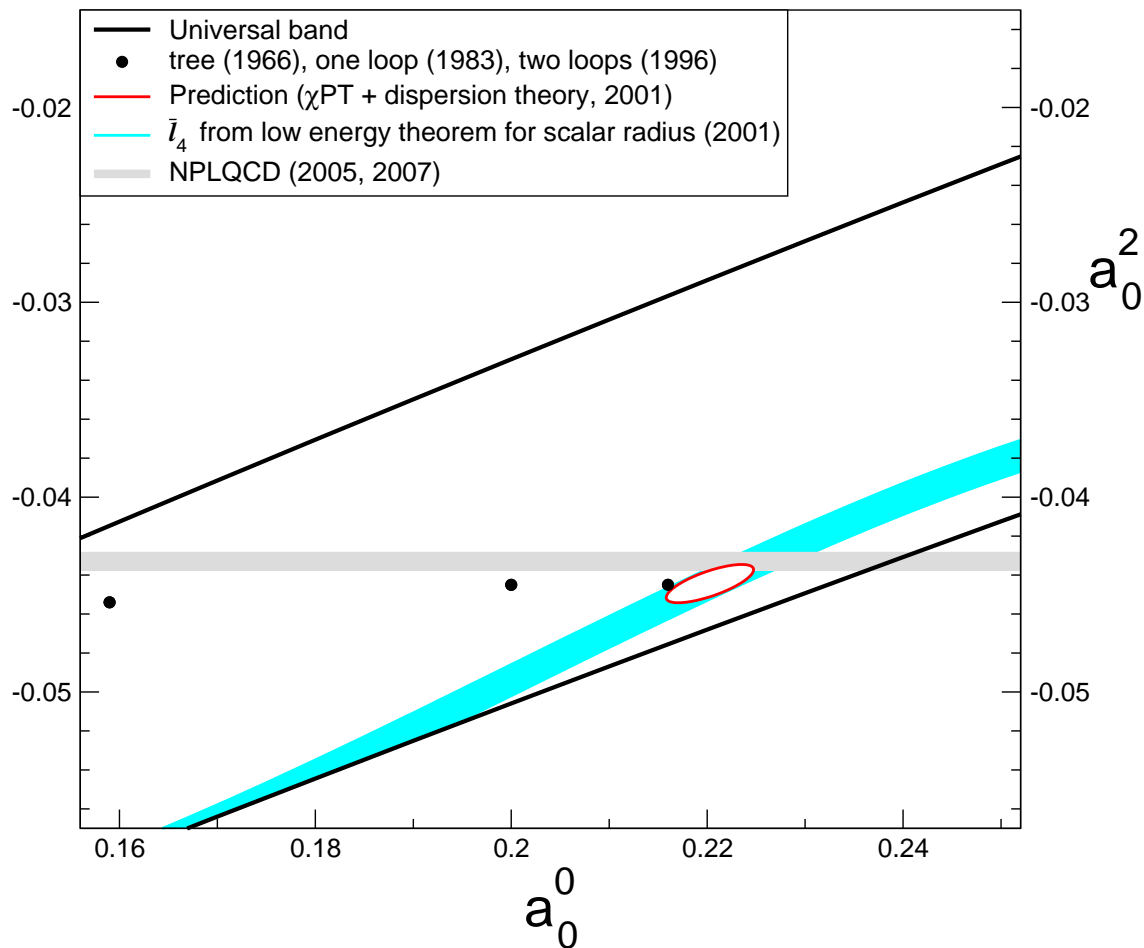
# Predictions for the S-wave $\pi\pi$ scattering lengths



Sizable corrections in  $a_0^0$ , while  $a_0^2$  nearly stays put

# Lattice result for $a_0^2$

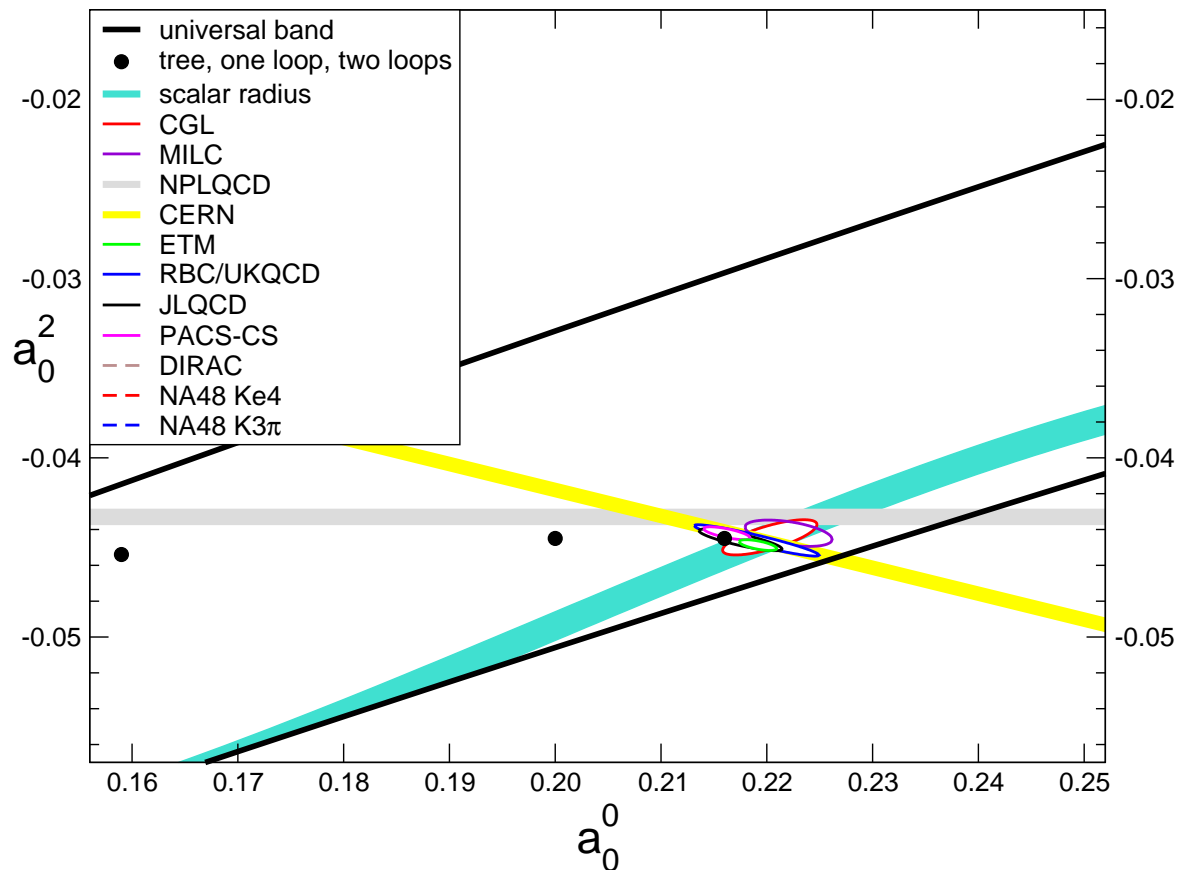
- Lattice allows direct measurement of  $a_0^2$  via volume dependence of energy levels





## Consequence of lattice results for $l_3, l_4$

- Uncertainty in prediction for  $a_0^0, a_0^2$  is dominated by the uncertainty in the effective coupling constants  $l_3, l_4$   
Can make use of the lattice results for these

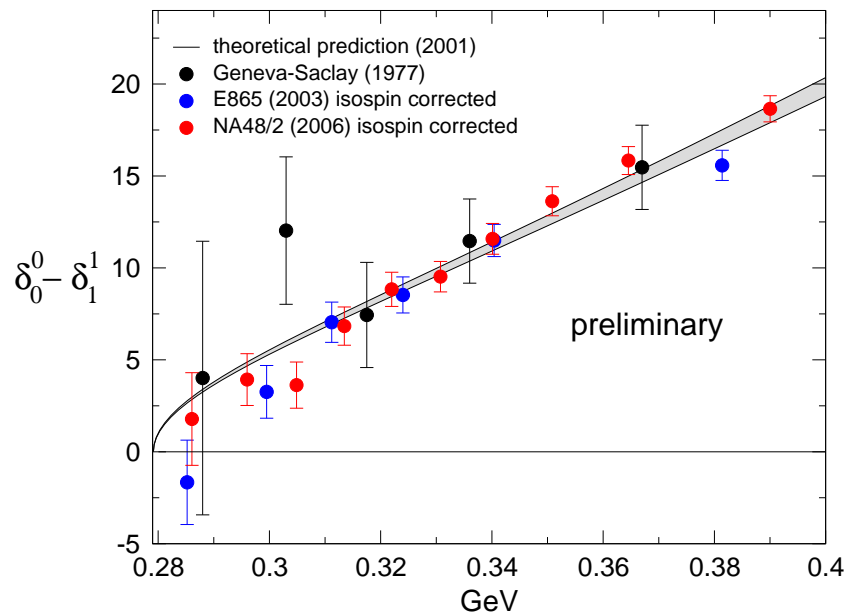


# Experiments on light flavours at low energy

- Production experiments  $\pi N \rightarrow \pi\pi N$ ,  $\psi \rightarrow \pi\pi\omega \dots$   
Problem: pions are not produced in vacuo  
⇒ Extraction of  $\pi\pi$  scattering amplitude not simple  
Accuracy rather limited
- $\pi^+\pi^-$  atoms, DIRAC
- $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$  cusp near threshold: NA48/2
- $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$  precision data from E865, NA48/2

# $K_{e4}$ decay

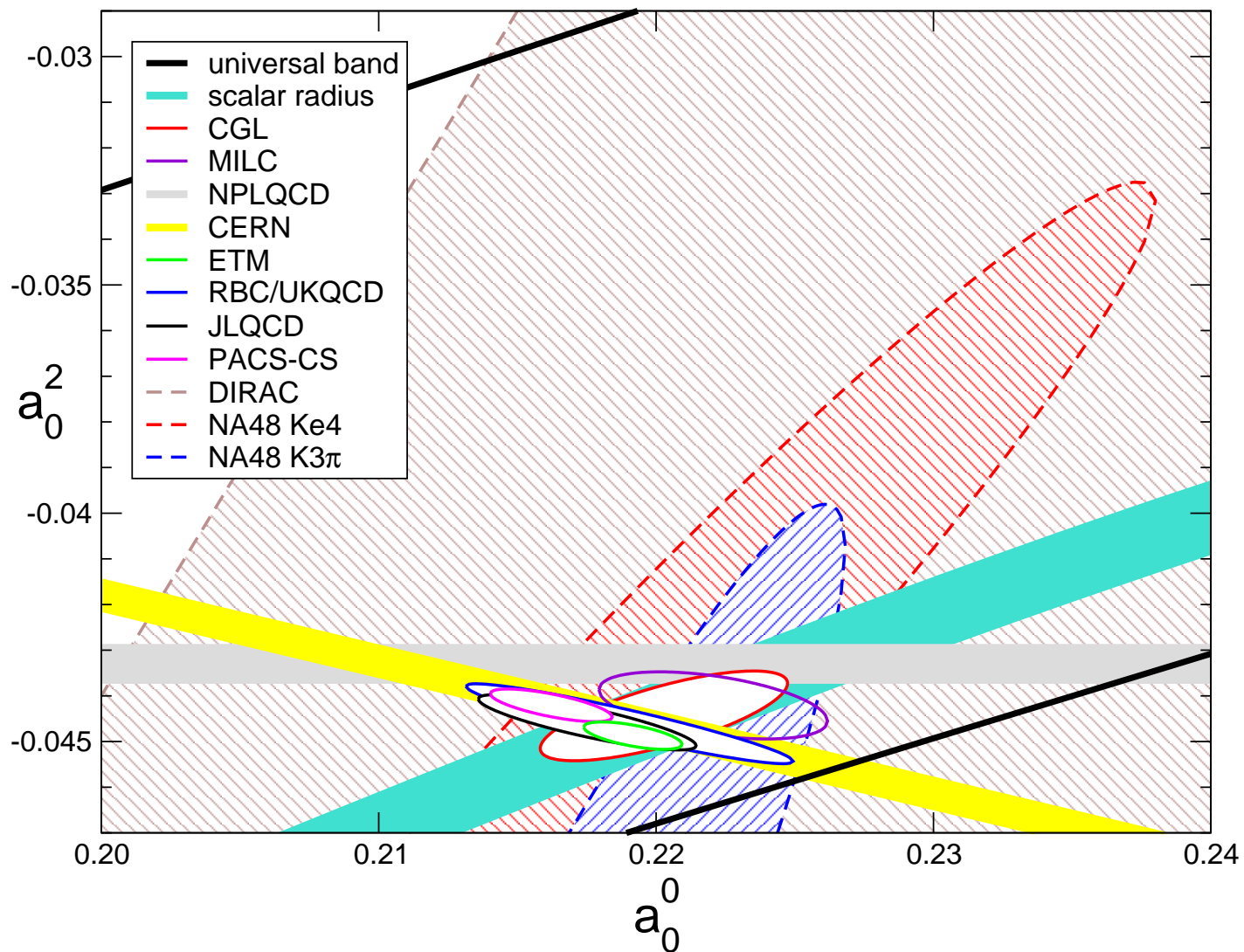
- $K \rightarrow \pi\pi e\nu$  allows clean measurement of  $\delta_0^0 - \delta_1^1$
- Theory predicts  $\delta_0^0 - \delta_1^1$  as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from  $K \rightarrow \pi^0\pi^0 e\nu \rightarrow \pi^+\pi^- e\nu$  had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007

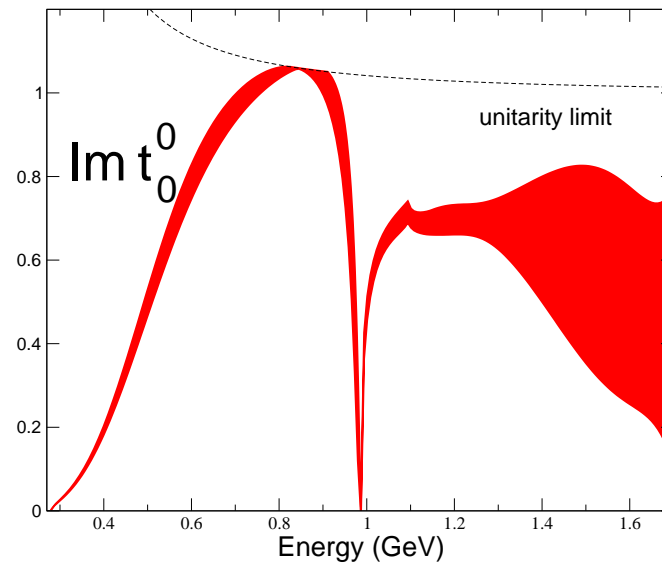
# $a_0^0, a_0^2$ : prediction, lattice & experiment



~ *Le dessert* ~

Lowest resonance of QCD

# The red dragon



*There is the broad object seen in  $\pi\pi$  scattering, often called “background”, which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance<sup>2</sup> which we identify as the lightest glueball with quantum numbers  $J^{PC} = 0^{++} \dots$*

<sup>2</sup> we refer to it as *red dragon*

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

# Where is the lowest resonance of QCD ?

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001

- Does QCD have a resonance near threshold ?
  - Concerns the nonperturbative domain of QCD
  - Quark and gluon degrees of freedom useless there
  - ⇒ Understanding very poor, pattern of energy levels ?
  - Lowest resonance:  $\sigma$  ?  $\rho$  ?
- Resonance  $\leftrightarrow$  pole on second sheet
  - Poles are universal
  - Pole position is unambiguous, even if width is large
  - Where is the pole closest to the origin ?

# $f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that  $\Gamma \approx 2 \text{Im}(\sqrt{s_{\text{pole}}})$ .

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
<b>(400–1200)–i(250–500) OUR ESTIMATE</b>			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(552^{+84}_{-106})-i(232^{+81}_{-72})$	1 ABLIKIM	07A	BES2 $\psi(2S) \rightarrow \pi^+ \pi^- J/\psi$
$(441^{+16}_{-8})-i(272^{+9}_{-12.5})$	2 CAPRINI	06	RVUE $\pi\pi \rightarrow \pi\pi$
$(470 \pm 50)-i(285 \pm 25)$	3 ZHOU	05	RVUE
$(541 \pm 39)-i(252 \pm 42)$	4 ABLIKIM	04A	BES2 $J/\psi \rightarrow \omega\pi^+\pi^-$
$(528 \pm 32)-i(207 \pm 23)$	5 GALLEGOS	04	RVUE Compilation
$(440 \pm 8)-i(212 \pm 15)$	6 PELAEZ	04A	RVUE $\pi\pi \rightarrow \pi\pi$
$(533 \pm 25)-i(247 \pm 25)$	7 BUGG	03	RVUE
$532 - i272$	BLACK	01	RVUE $\pi^0\pi^0 \rightarrow \pi^0\pi^0$
$(470 \pm 30)-i(295 \pm 20)$	2 COLANGELO	01	RVUE $\pi\pi \rightarrow \pi\pi$
$(535^{+48}_{-36})-i(155^{+76}_{-53})$	8 ISHIDA	01	$\Upsilon(3S) \rightarrow \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	9 SUROVTSEV	01	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$(558^{+34}_{-27})-i(196^{+32}_{-41})$	ISHIDA	00B	$\rho\bar{\rho} \rightarrow \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH	99	RVUE $\pi$ scalar form factor
$(523 \pm 12)-i(259 \pm 7)$	KAMINSKI	99	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}, \sigma\sigma$
$442 - i 227$	OLLER	99	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$469 - i203$	OLLER	99B	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$445 - i221$	OLLER	99C	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$
$(1530^{+90}_{-250})-i(560 \pm 40)$	ANISOVICH	98B	RVUE Compilation
$420 - i 212$	LOCHER	98	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$(602 \pm 26)-i(196 \pm 27)$	10 ISHIDA	97	$\pi\pi \rightarrow \pi\pi$
$(537 \pm 20)-i(250 \pm 17)$	11 KAMINSKI	97B	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}, 4\pi$
$470 - i250$	12,13 TORNVIST	96	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}, K\pi,$ $\eta\pi$
$\sim (1100 - i300)$	AMSLER	95B	CBAR $\bar{\rho}\rho \rightarrow 3\pi^0$
$400 - i500$	13,14 AMSLER	95D	CBAR $\bar{\rho}\rho \rightarrow 3\pi^0$
$1100 - i137$	13,15 AMSLER	95D	CBAR $\bar{\rho}\rho \rightarrow 3\pi^0$
$387 - i305$	13,16 JANSSEN	95	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$525 - i269$	17 ACHASOV	94	RVUE $\pi\pi \rightarrow \pi\pi$
$(506 \pm 10)-i(247 \pm 3)$	KAMINSKI	94	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$370 - i356$	18 ZOU	94B	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$408 - i342$	13,18 ZOU	93	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$870 - i370$	13,19 AU	87	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$470 - i208$	20 VANBEVEREN	86	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta,$
$(750 \pm 50)-i(450 \pm 50)$	21 ESTABROOKS	79	RVUE $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$(660 \pm 100)-i(320 \pm 70)$	PROTOPOP...	73	HBC $\pi\pi \rightarrow \pi\pi, K\bar{K}$
$650 - i370$	22 BASDEVANT	72	RVUE $\pi\pi \rightarrow \pi\pi$

PDG tables,  
edition 2007



# Model independent determination of the pole

- Most of the results quoted by the PDG are obtained by
  - (a) parametrizing the data for real values of  $s$
  - (b) continuing this parametrization analytically in  $s$

⇒ Result is sensitive to the parametrization used
- We found a model independent method:
  1. Poles on second sheet are zeros on first sheet
  2. The Roy equations are valid for complex values of  $s$ , in a limited region of the first sheet

⇒ Exact representation of the partial waves in terms of observable quantities, valid for complex values of  $s$

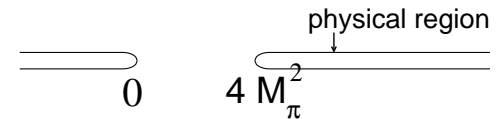
  3. Can evaluate this representation to good precision and determine the zeros numerically

## Pole on second sheet $\leftrightarrow$ zero on first sheet

s-plane

- $S(s) = \eta(s) \exp 2i\delta(s)$

$S(s)$  is analytic in the cut plane



- For  $0 < s < 4M_\pi^2$ ,  $S(s)$  is real

$\Rightarrow S(s^*) = S(s)^*$

$x$  in elastic interval:  $S(x \pm i\epsilon) = \exp \pm 2i\delta(x)$

- Second sheet is reached by continuation across the elastic interval of the right-hand cut

$$S(x - i\epsilon)^{II} = S(x + i\epsilon)^I = 1/S(x - i\epsilon)^I$$

Analyticity  $\Rightarrow$   $S(s)^{II} = 1/S(s)^I$  valid  $\forall s$

Pole in  $S(s)^{II} \iff$  zero in  $S(s)^I$

# Roy equation for the isoscalar S-wave

Two S-waves:  $I = 0$  and  $I = 2$

$$S_0^0(s) = \eta_0^0(s) \exp 2i\delta_0^0(s) \quad \text{S-matrix element}$$

$$S_0^0(s) = 1 + 2i\rho t_0^0(s) \quad \rho = \sqrt{1 - 4M_\pi^2/s}$$

↑  
partial wave amplitude

$$t_0^0(s) = a + (s - 4M_\pi^2)b + \sum_{I=0}^2 \sum_{\ell=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' K_{0\ell}^{0I}(s, s') \text{Im} t_\ell^I(s')$$

Roy equation

- The subtraction constants are determined by  $a_0^0, a_0^2$ :

$$a = a_0^0, \quad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$$

## Roy equation for the isoscalar S-wave

$$t_0^0(s) = a + (s - 4M_\pi^2) b + \sum_{I=0}^2 \sum_{\ell=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' K_{0\ell}^{0I}(s, s') \text{Im} t_\ell^I(s')$$

- The kernels are elementary functions, e.g.

$$K_{00}^{00}(s, s') = \underbrace{\frac{1}{\pi(s' - s)}}_{r.h.cut} + \underbrace{\frac{2 \ln\{(s + s' - 4M_\pi^2)/s'\}}{3\pi(s - 4M_\pi^2)} - \frac{5s' + 2s - 16M_\pi^2}{3\pi s'(s' - 4M_\pi^2)}}_{l.h.cut}$$

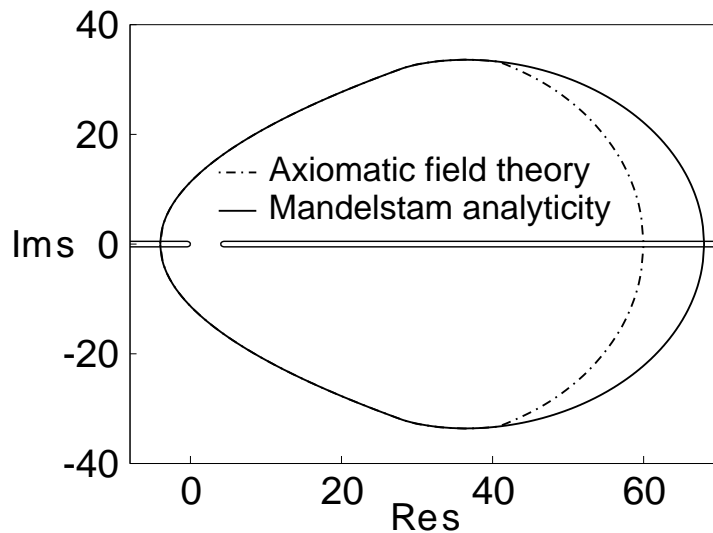
- Left hand cut is essential for rapid convergence:

$$K_{00}^{00}(s, s') \sim 1/s'^3 \text{ for large } s'$$

# Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval  $-4M_\pi^2 < s < 60M_\pi^2$
- Equations are valid for complex  $s$  in a limited region of the first sheet

I. Caprini, G. Colangelo and H. Leutwyler,  
Phys. Rev. Lett. 96 (2006) 132001



- Proof is based on first principles, general quantum field theory

A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing*, Lecture Notes in Physics, vol. 3, 1969.

G. Mahoux, S. M. Roy and G. Wanders,  
Nucl. Phys. B70 (1974) 297.

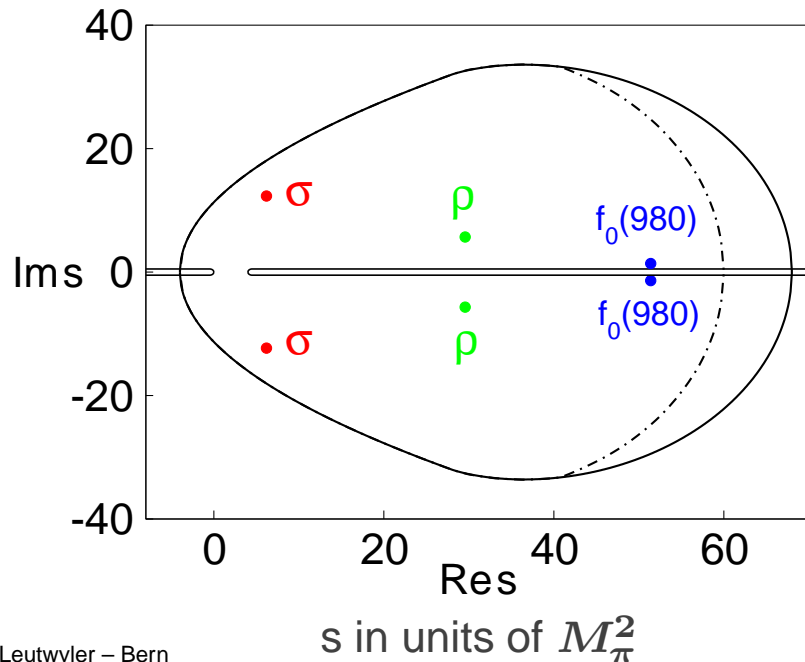
⇒ Exact representation for  $S_0^0(s)$  in this region  
Do not need to parametrize the amplitude

## Evaluation of the pole position

- Have an exact formula for the pole position in terms of physical quantities:  $S_0^0(s) = 0$
- For the central solution of the Roy equations,  $S_0^0(s)$  has two pairs of zeros in the region where the formula holds:

$$s = (6.2 \pm i 12.3) M_\pi^2 \quad \sigma$$

$$s = (51.4 \pm i 1.4) M_\pi^2 \quad f_0(980)$$



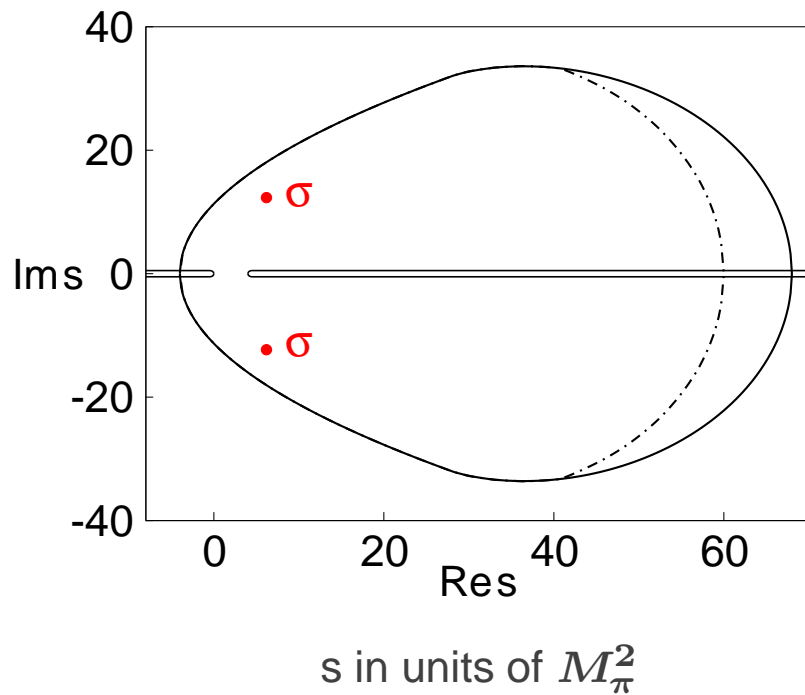
The eyes of the red dragon

Tail at 1.7 GeV:  $s \simeq 150 M_\pi^2$

# Result

- Lowest resonance of QCD has vacuum quantum numbers
- Pole on lower half of sheet II occurs in vicinity of

$$m_\sigma = 441 - i 272 \text{ MeV} = M_\sigma - \frac{i}{2}\Gamma_\sigma$$



## Loci Oculorum Draconis Rutili

T. Barnes, Theory summary, MESON 2006

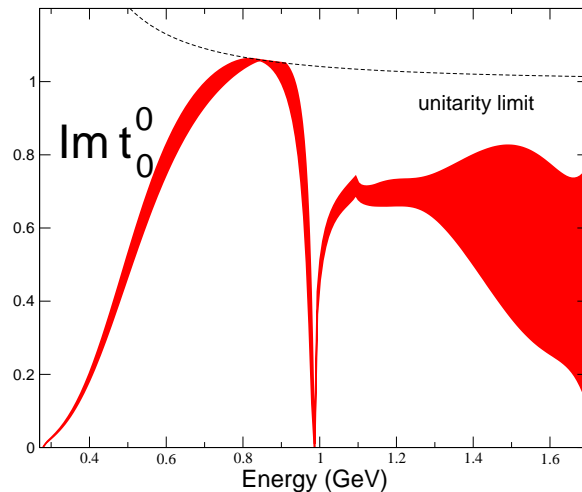
# Error analysis

- Formula is exact, evaluation is approximate
- Key point: can follow error propagation explicitly
- Split the formula into 3 pieces:
  1. Subtraction terms
  2. Contribution from  $\text{Im } t_0^0(s)$  below  $K\bar{K}$  threshold
  3. Higher energies and other partial waves
- Subtraction terms are determined by  $a_0^0, a_0^2$
- ⇒ Errors from there are very small
- Slowly varying background from 3. is dominated by the  $\rho$ , also accurately known
- ⇒ Uncertainty in result for  $\sigma$  pole is dominated by 2.



# Behaviour of $\text{Im } t_0^0(s)$ below $K\bar{K}$ threshold

↓  $K\bar{K}$  threshold

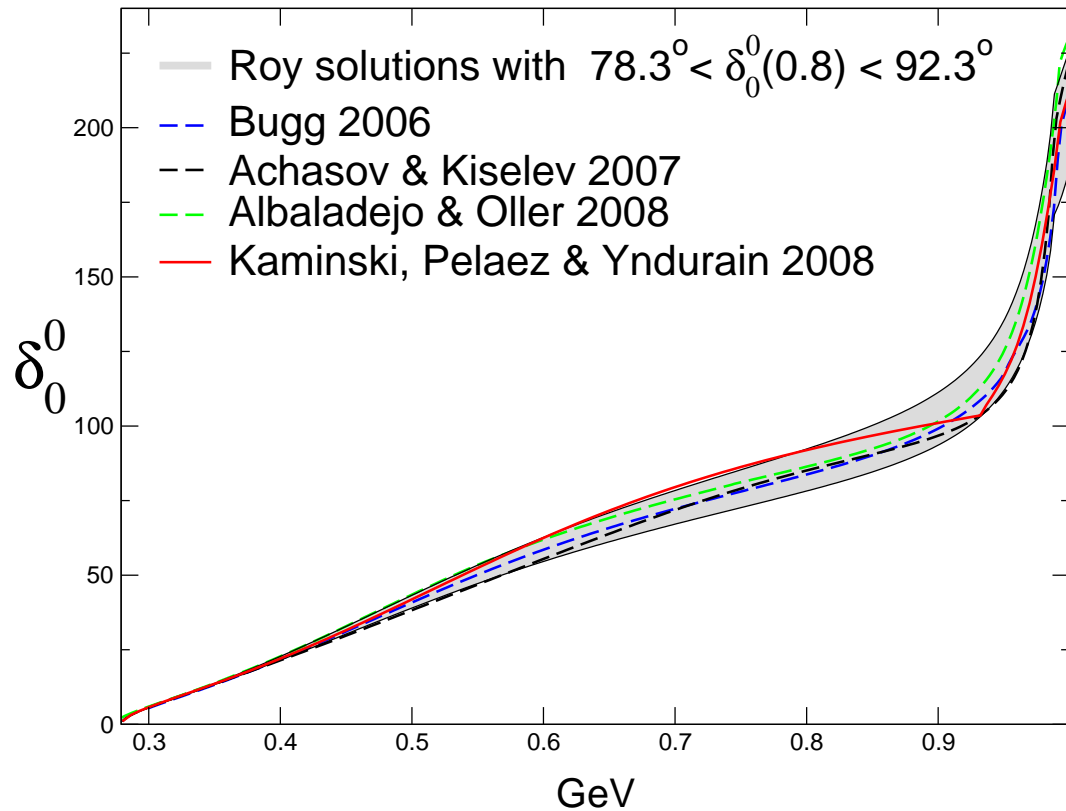


- ⇒ Head of dragon determines position of the  $\sigma$  pole
- Unitarity and dip leave little room above 0.8 GeV
- ⇒ Only the region below 0.8 GeV really matters
- There, the partial waves are nearly elastic:

$$\text{Im } t_0^0(s) \simeq \sin^2 \delta_0^0(s) / \sqrt{1 - 4M_\pi^2/s}$$

- ⇒ Need to know the phase  $\delta_0^0(s)$  below 0.8 GeV

# How well do we know $\delta_0^0(s)$ below $K\bar{K}$ threshold ?



Bugg uses BES data for  $J/\psi \rightarrow \omega\pi\pi$

Achasov & Kiselev use KLOE data on  $\phi \rightarrow \gamma\pi\pi$

Albaladejo & Oller: N/D fit to several data sets

Kaminski, Pelaez and Yndurain:  $\pi\pi$ ,  $K_{e4}$ ,  $K \rightarrow \pi\pi$

Thorough discussion of the phenomenological uncertainties in  $\delta_0^0(s)$ :  
Caprini, arXiv:0804.3504

## Illustration

- Replace the central Roy solution below  $2M_K$  by the phase representation of Bugg 2006  $\Rightarrow$  pole moves from  $441 - i 272$  to  $444 - i 267$  MeV
- Ditto with Achasov & Kiselev 2007:  
 $438 - i 274$  MeV
- Ditto with Albaladejo & Oller 2008:  
 $451 - i 257$  MeV
- Ditto with Kamiński, Peláez and Ynduráin 2008:  
 $458 - i 253$  MeV

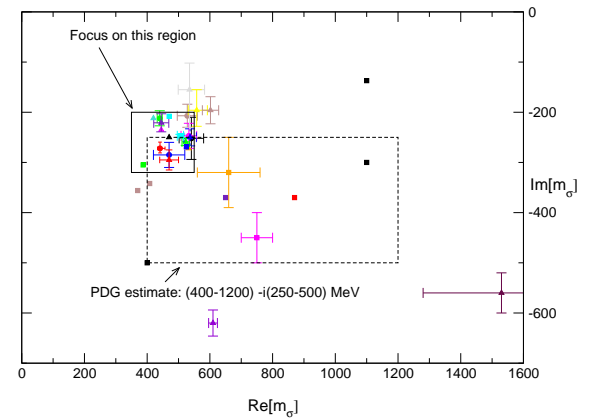
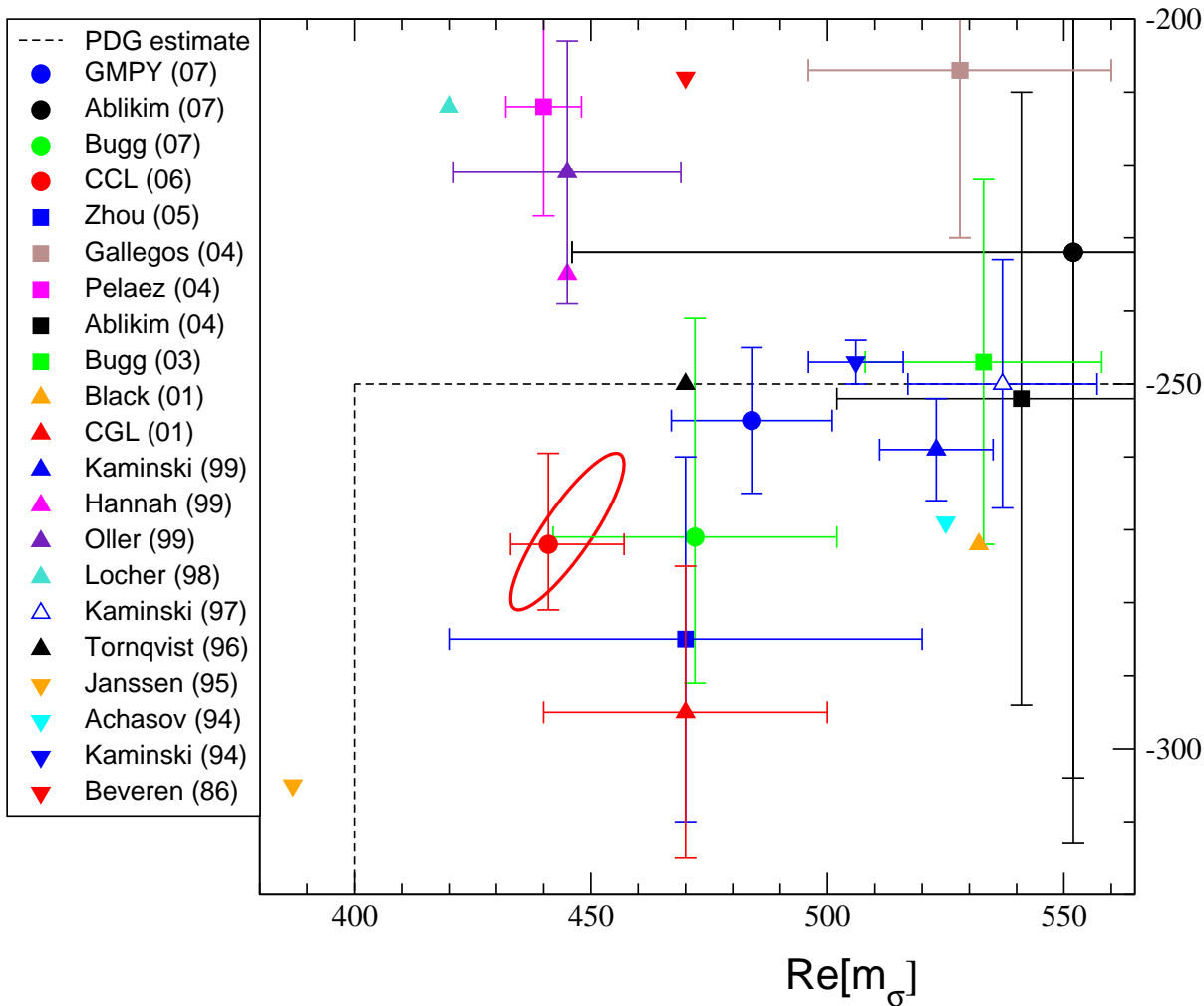
## Final result for the pole position

1. Subtraction terms
  2. Contribution from  $\text{Im} t_0^0(s)$  below  $K\bar{K}$  threshold
  3. Higher energies and other partial waves
- Vary these contributions within their uncertainties, follow error propagation and sum the errors up

$$\Rightarrow m_\sigma = 441^{+16}_{-8} - i 272^{+9}_{-13} \text{ MeV}$$

Caprini, Colangelo and L. 2006

# Comparison with compilation in PDG 2007



$\text{Im}[m_\sigma]$

$\text{Re}[m_\sigma]$

## Physical interpretation of the $\sigma$

- Glueball ?  $\bar{q}q$  ?  $\bar{q}q\bar{q}q$  ?
- Hadrons in terms of quarks and gluons ?
- Fock space is basis of perturbation theory – not clear how to use it quantitatively in nonperturbative domain
- My qualitative picture of the  $\sigma$ :
  - Can understand position of  $\sigma$  pole in terms of  $F_\pi$
  - ⇒ Low energy properties of the pions are relevant
  - ⇒ Physics of the  $\sigma \in$  Goldstone boson dynamics
  - ⇒ Head of the dragon contains only little glue
  - ⇒ Wave function has large tetra-quark component
- This picture is by no means commonly accepted

Törnqvist, Ishida, Jaffe, Minkowski, Ochs, Bugg, Pennington, Peláez, Oller, Hannah, Guo, Su, Xiao, Zheng, Zhou, Chen, Hosaka, Zhu, Liu, Maiani, Polosa, Piccinini, Riquer, Isidori, Nicolaci, Pacetti, Menessier, Narison, Fariborz, Jora, Schechter, van Beveren, Kleefeld, Rupp, Scadron, Ynduráin, García-Martín, . . .

⇒ Comprehensive review : Klempt & Zaitsev, arXiv:0708.4016

## Conclusion

- Low energy pion physics: theory ahead of experiment
  - Precision experiments carried out and under way
  - Lattice makes slow, but steady progress
  - Results for  $V_{us}$ ,  $V_{ud}$  consistent with unitarity  
Confirm nuclear  $\beta$ , at comparable precision
  - The lattice results for  $M_\pi$  and  $F_\pi$  can be used to calculate the corrections to the low energy theorems for the  $\pi\pi$  scattering lengths

# Conclusion

- Limitations of our approach:
  - Calculations cannot be done on back of an envelope
  - Analysis only covers low energies  
Extension to higher energies is under way
  - Only a few applications have been worked out:  
 $\pi\pi$  scattering, pion form factors, hadronic vacuum polarization in muon  $g - 2$
  - Much is yet to be done:  $J/\psi \rightarrow \omega\pi\pi$ ,  $D \rightarrow 3\pi$ ,  
 $\gamma\gamma \rightarrow \pi\pi$ ,  $\pi K$ ,  $\pi N$ , ...



## Conclusion

- Model independent method for analytic continuation
  - The lowest resonance of QCD occurs at
$$M_\sigma = 441^{+16}_{-8} \text{ MeV} \quad \Gamma_\sigma = 544^{+18}_{-25} \text{ MeV}$$
and carries vacuum quantum numbers
  - Crossing symmetry plays an essential role:  
Fixes contributions from left-hand cut  
Ensures fast convergence, low energy dominance
  - Pole occurs at low value of  $s$ , closer to left-hand cut than to singularities from  $K\bar{K}$ ,  $f_0(980)$
  - Result for  $\Gamma_\sigma$  relies on theory for  $a_0^2$   
Experiments concerning  $a_0^2$  would be most welcome



# VISIT THE RED DRAGON

GENTLE ANIMAL  
LOOK IN HIS EYES FROM CLOSE  
SMELL HIS GOOD BREATH  
BRING YOUR PIONS ALONG AND  
FEED HIM YOURSELF

The management denies responsibility for incidents involving the dragon's tail

# 6<sup>th</sup> International Workshop on Chiral Dynamics

University of Bern, Switzerland, July 6-10, 2009

$u^b$



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Goldstone Boson Dynamics  
Meson-Baryon Interactions  
Few-Body Physics  
Lattice QCD and ChPT

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C. Bernard (St. Louis)  
J.P. Chen (JLab)  
S. Dürr (NIC)  
E. Epelbaum (Jülich / Bonn)  
S. Giovannella (Frascati)  
B. Kubis (Bonn)  
J. Portolés (IFIC)  
H.R. Weller (Duke / TJNL)

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S. Aoki (Tsukuba)  
S.R. Beane (New Hampshire)  
A.M. Bernstein (MIT)  
J. Bijnens (Lund)  
M.C. Birse (Manchester)  
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FlaviA  
net



# SPARES

# NNLO

- The next order contains the square of a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{x}{2} \ln \frac{M^2}{\Lambda_3^2} + \frac{17x^2}{8} \left( \ln \frac{M^2}{\Lambda_M^2} \right)^2 + x^2 k_M + O(M^6) \right\}$$

$$F_\pi = F \left\{ 1 - x \ln \frac{M^2}{\Lambda_4^2} - \frac{5x^2}{4} \left( \ln \frac{M^2}{\Lambda_F^2} \right)^2 + x^2 k_F + O(M^6) \right\}$$

$$x \equiv \left( \frac{M}{4\pi F} \right)^2$$

Colangelo 1995, Bijnens et al. 1996, Bürgi 1996

- For physical value of  $m_u, m_d$ , the NNLO terms are tiny  
⇒ Size of  $\Lambda_M, k_M, \Lambda_F, k_F$  barely known
- Must become clearly visible if  $m_u, m_d$  are made larger

## The $\kappa$

- $K\pi$  scattering amplitude obeys an analog of the Roy equations. Pole from  $\kappa$  can be calculated on this basis

$$m_{\kappa} = (658 \pm 13) - i(278.5 \pm 12) \text{ MeV}$$

Descotes-Genon and Moussallam 2006

- Confirms an earlier calculation, where the l.h. cut was estimated with  $\chi$ PT

Zhou and Zheng 2006

- Back-of-the-envelope calculation for  $K\pi$  gives

$$m_{\kappa} = 671 - i 262 \text{ MeV}$$

⇒ Physics of  $\sigma$  and  $\kappa$  is very similar

## Qualitative picture for $\kappa$ , $f_0(980)$ , ...

- Can also understand the  $\kappa$  pole in terms of  $F_\pi$
- $f_0(980)$  and  $a_0(980)$ 
  - Suspiciously close to  $K\bar{K}$  threshold - an accident ?
  - Interaction among two kaons plays important role
- Multiplet pattern ?
  - The  $\pi\pi$ ,  $\pi K$ ,  $K\bar{K}$  thresholds strongly break SU(3)
  - ⇒ Expect strong symmetry breaking in the masses and widths of the lowest  $0^+$  resonances
  - Do these form complete SU(3) multiplets at all ?

## Puzzling results on $K_L \rightarrow \pi \mu \nu$

- NA48 Data are in conflict with a venerable low energy theorem: Callan-Treiman-relation
- Physics beyond the Standard Model or incorrect measurement ?



## Callan-Treiman-relation

- Hadronic matrix element of weak current:

$$\langle K^0 | \bar{u} \gamma^\mu s | \pi^- \rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

- Scalar form factor  $\sim \langle K^0 | \partial_\mu (\bar{u} \gamma^\mu s) | \pi^- \rangle$

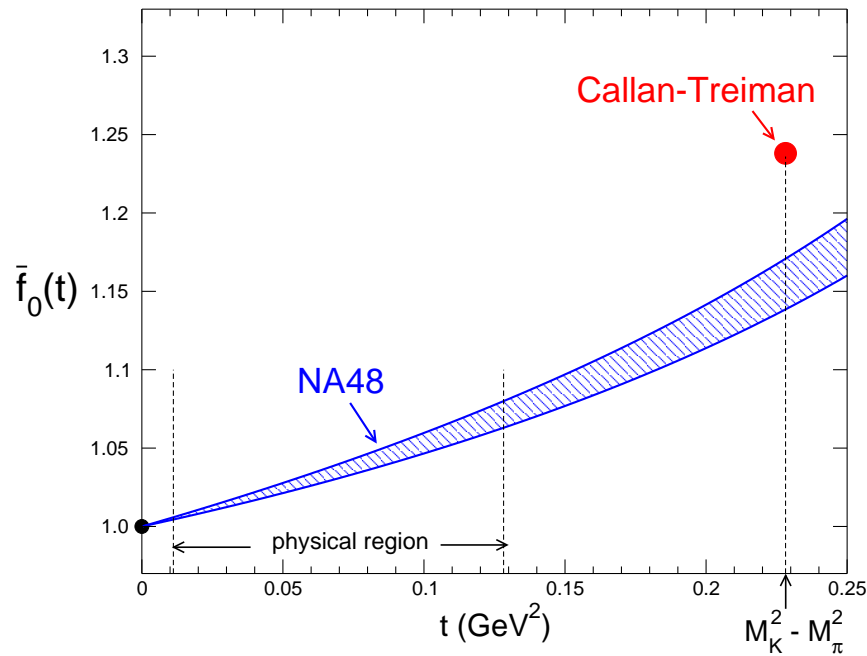
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- Low energy theorem of Callan and Treiman (1966):

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi} \left\{ 1 + O(m_u, m_d) \right\} \simeq 1.19$$

$$f_0(0) = f_+(0) \simeq 0.96 \text{ relevant for determination of } V_{us}$$

# Comparison with experiment



NA48 Phys. Lett. B 647 (2007) 341  
141 authors,  $2.3 \times 10^6$  events

Plot shows normalized  
scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

● Callan-Treiman relation in this normalization:

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)} \left\{ 1 + O(m_u, m_d) \right\}$$

● Experimental value:  $\frac{F_K}{F_\pi f_+(0)} = 1.2446 \pm 0.0041$

Bernard and Passemar 2008

## Uncertainty in prediction & extrapolation

- Callan-Treiman-relation is exact only for  $m_u, m_d \rightarrow 0$   
Corrections of NLO were worked out long ago, are tiny  
Gasser & L. 1985

Form factor now known to NNLO  
Post & Schilcher 2002,  
Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005

Including the uncertainties from  $m_u, m_d \neq 0$ :

$$\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$$

Bernard, Oertel, Passemar & Stern, preliminary

⇒ Cannot blame the discrepancy on the prediction

- CT-point not in physical region, extrapolation needed

Curvature can be calculated with dispersion theory

Jamin, Oller & Pich 2004, Bernard, Oertel, Passemar & Stern 2006

⇒ Cannot blame the discrepancy on the extrapolation

# Implications

- NA48 data on  $K_L \rightarrow \pi \mu \nu$  disagree with SM

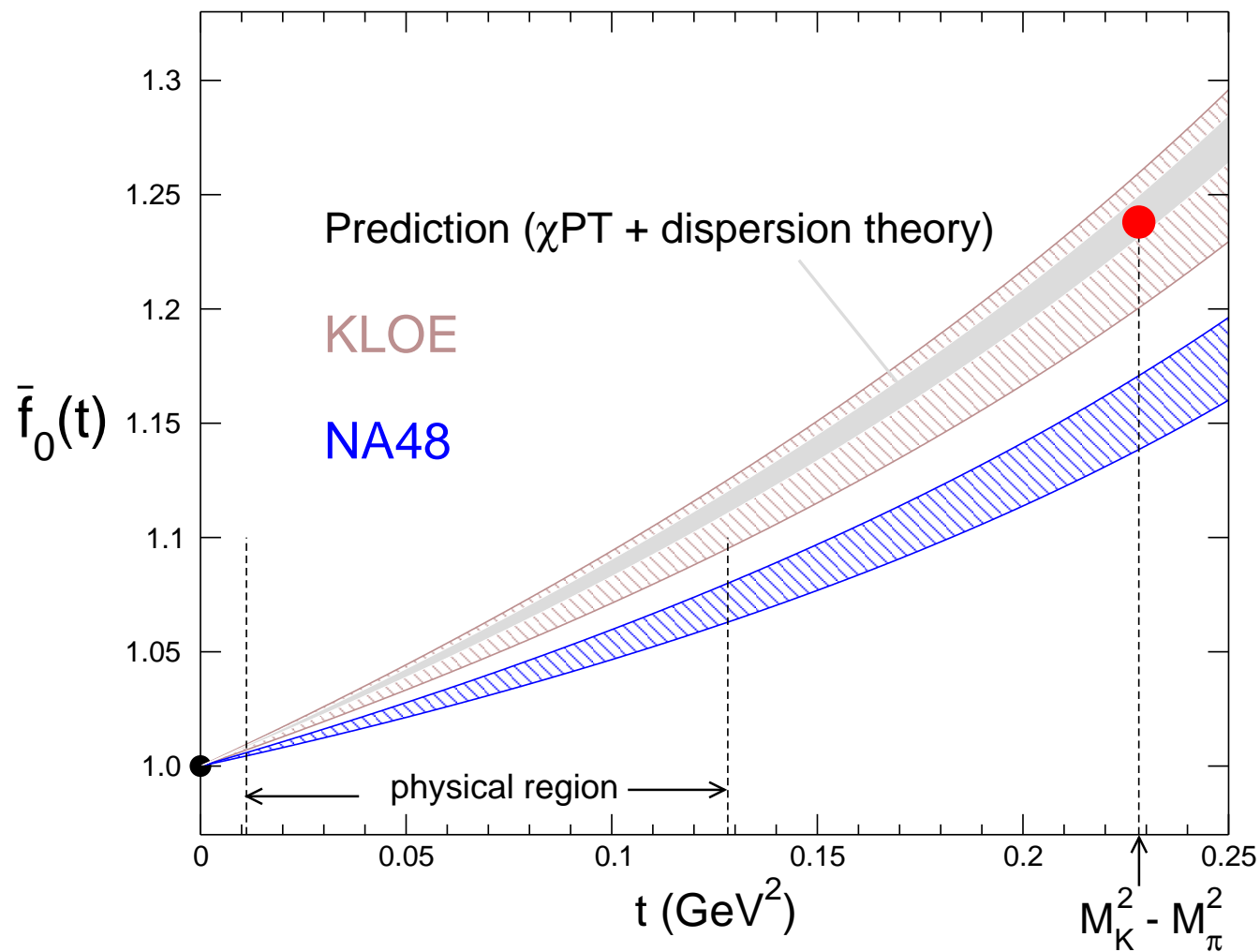
If confirmed, the implications are dramatic:

⇒ Right-handed currents ? Bernard, Oertel, Passemar & Stern 2006

- In the meantime, new data from KLOE

Results are consistent with Callan-Treiman-relation

# Comparison of KLOE and NA48



I thank Emilie Passemar for some of the material shown in this figure

# Slope of the scalar form factor

- Definition of the slope  $\bar{f}_0(t) = 1 + \frac{\lambda_0 t}{M_{\pi^+}^2} + O(t^2)$

- Callan-Treiman-relation implies sharp prediction:

$$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$

Jamin, Oller & Pich 2004

- Update with current experimental information

$$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

Bernard, Oertel, Passemar & Stern, preliminary

- To be compared with the result of NA48:

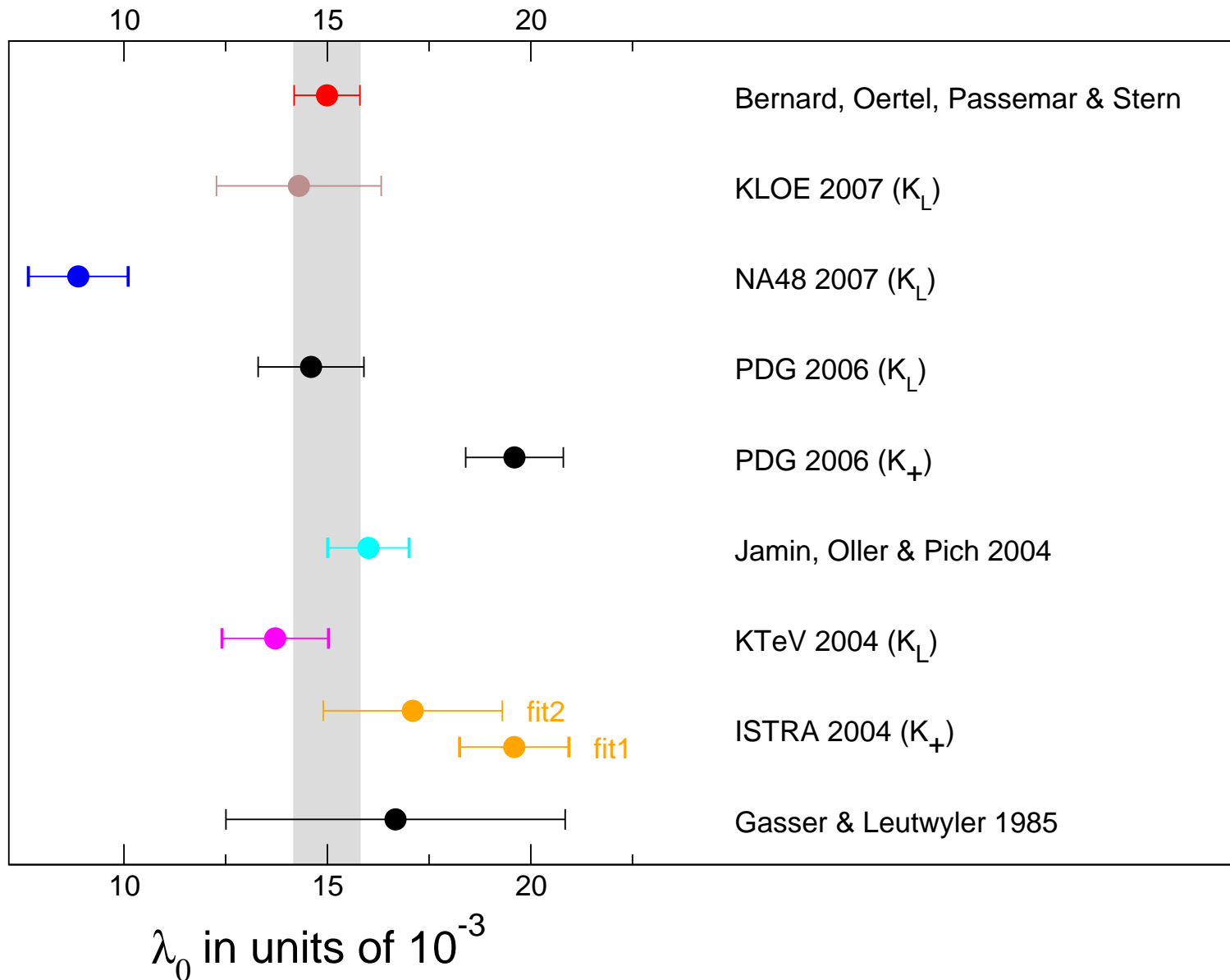
$$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$

Fit with dispersive representation of BOPS

$$\lambda_0 = (11.7 \pm 0.7_{\text{stat}} \pm 1.0_{\text{syst}}) \times 10^{-3}$$

Linear fit

# Comparison of results for the slope



# Conclusions for $K \rightarrow \mu\nu\pi$

- Experimental discrepancies need to be resolved
- Important to analyze existing data on charged K-decays (isospin breaking)
- Dispersion theory fixes the shape of the form factors

The most recent analyses properly account for the curvature  
Publishing linear fits is useless

- KTeV is being reanalyzed, ISTRA ?

ISTRA:  $0.54 \times 10^6$  events

KTeV:  $1.9 \times 10^6$  events

NA48:  $2.3 \times 10^6$  events

- Excellent overview of current experimental situation:  
Antonelli, talk at Lepton-Photon 07

<http://chep.knu.ac.kr/lp07>