Model independent determination of the σ pole

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SCADRON 70

Workshop on Scalar Mesons and Related Topics

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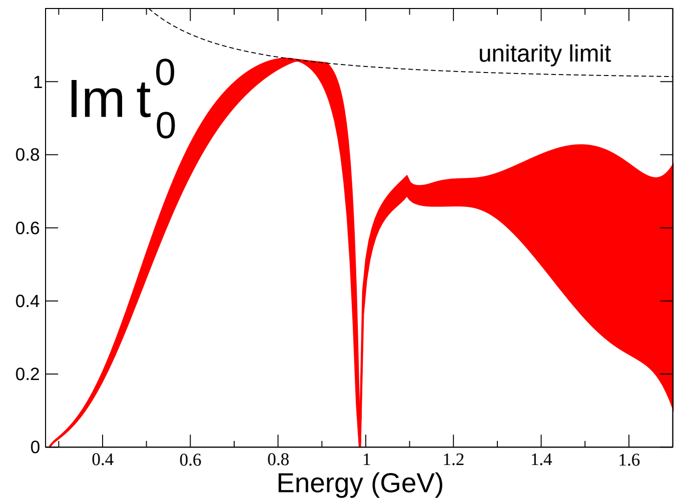
Motivation

- QCD with massless quarks is the ideal of a theory: no free parameters
- Gives rise to a very rich structure at low energies
- High energy side looks like what we are used to: relevant degrees of freedom visible in the Lagrangian, can treat interaction as a pertubation
- Low energies are out of reach of perturbation theory

$\Rightarrow$ Progress in understanding is slow

- $\exists$ many models that resemble QCD: instantons, monopoles, superconductivity, gluonic strings, linear σ model, hidden gauge, NJL, RSE, AdS/CFT ...
- Discuss a model independent approach
  Focus on the sector with the quantum numbers of the vacuum: $I = \ell = 0$

The red dragon



There is the broad object seen in $\pi\pi$ scattering, often called "background", which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance[2] *which we identify as the lightest glueball with quantum numbers* $J^{PC} = 0^{++} \ldots$

[2] *we refer to it as red dragon*

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

Play a variation of Mike's motto:

There's more than one way to talk about the red dragon

Energy gap of QCD

- Main characteristic of QCD at low energies:

  Energy gap is very small, $M_\pi \simeq$ 140 MeV

- In 1960, Nambu found out why that is so:
  - Has to do with a hidden approximate symmetry
  - Symmetry becomes exact for $m_u, m_d \longrightarrow 0$

  $\Rightarrow$ Energy gap disappears: pions become massless
  - In reality $m_u, m_d \neq 0$, but very small

  $\Rightarrow$ Symmetry is not perfect, but nearly so
  - The state of lowest energy is not symmetric

  $\Rightarrow$ Chiral symmetry is hidden, “spontaneously broken”
  - Very strong experimental evidence $\surd$

    Very strong evidence from lattice calculations $\surd$

    Analytic understanding of the ground state still poor

Chiral perturbation theory

- Consequences of hidden, approximate symmetry can be worked out by means of an effective field theory

  Weinberg 1979

- Hidden symmetry controls energy gap of QCD

$\Rightarrow$ Can calculate how gap grows as the symmetry breaking parameters m_u, m_d are turned on

- Hidden symmetry also determines the interaction of the Goldstone bosons at low energies, among themselves, as well as with other hadrons

Quark masses as perturbations

- Masses of u, d enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d\}$$

H_0 describes u, d as massless, s, c, b, t as massive

H_0 is invariant under $\text{SU}(2)_\text{L} \times \text{SU}(2)_\text{R}$

- H_0 treats the pions as massless particles

H_1 gives them a mass

Expansion in powers of m_u, m_d	$\Longleftrightarrow$	Perturbation series in powers of H_1

Gell-Mann-Oakes-Renner formula

- First order perturbation theory in H_1 yields:

$$M_\pi^2 = (m_u + m_d) \times |\langle 0|\, \bar{u}u\, |0\rangle| \times \frac{1}{F_\pi^2}$$

$$\Uparrow \qquad\qquad\qquad \Uparrow$$

explicit spontaneous

Gell-Mann, Oakes & Renner 1968

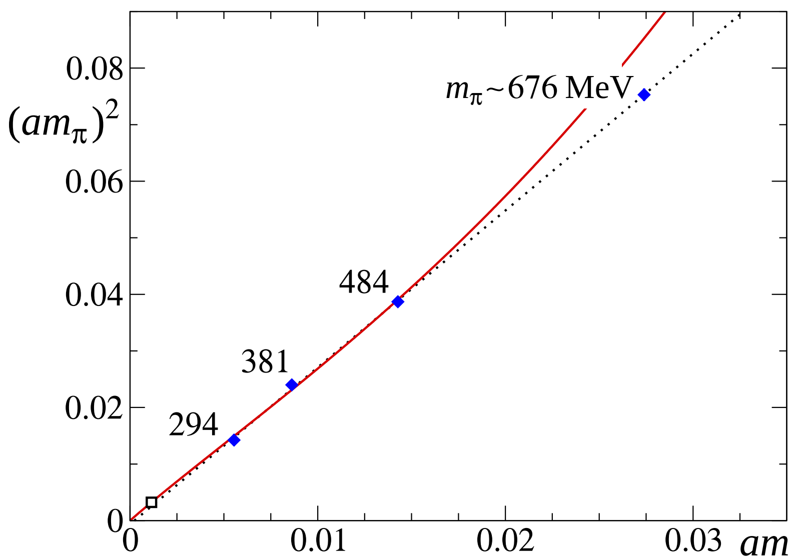
Coefficient: decay constant F_π

$$\langle 0|\, \bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\, p^\mu \sqrt{2}\, F_\pi$$

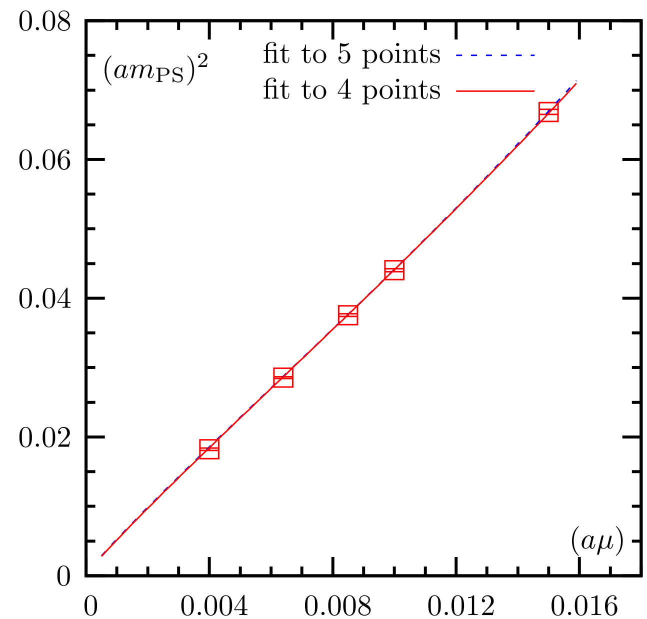
Value of F_π is known from $\pi^+ \to \mu^+\nu$

Lattice results for M_π

- GMOR formula can now be checked on the lattice: determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- No quenching, quark masses are sufficiently light

$\Rightarrow$ legitimate to use χPT for the extrapolation to the physical values of m_u, m_d

Lattice

- Quality of data is impressive
- Proportionality of M_π^2 to the quark mass appears to hold out to values of m_u, m_d that are an order of magnitude larger than in nature
- Main limitation: systematic uncertainties
  in particular: $N_f = 2 \rightarrow N_f = 3$

Expansion of M_π^2 in powers of the quark mass

- GMOR formula represents leading term of χPT
- At NLO, the expansion contains a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

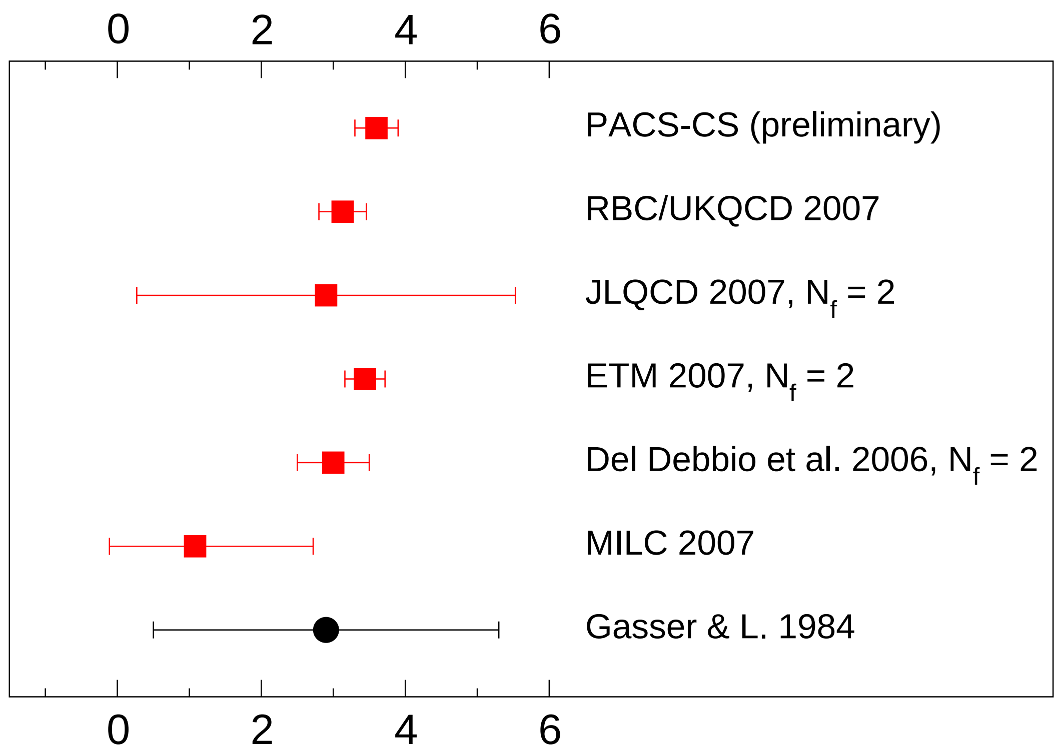
$$M^2 \equiv B(m_u + m_d)$$

Gasser & L. 1983

- Coefficient is determined by pion decay constant
  Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3)$\times$SU(3):
  0.2 GeV $\lesssim \Lambda_3 \lesssim 2$ GeV

Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



Horizontal axis shows the value of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

Range for Λ_3 obtained in 1984 corresponds to $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of RBC/UKQCD 2007, for instance, is $\bar{\ell}_3 = 3.13 \pm 0.33$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

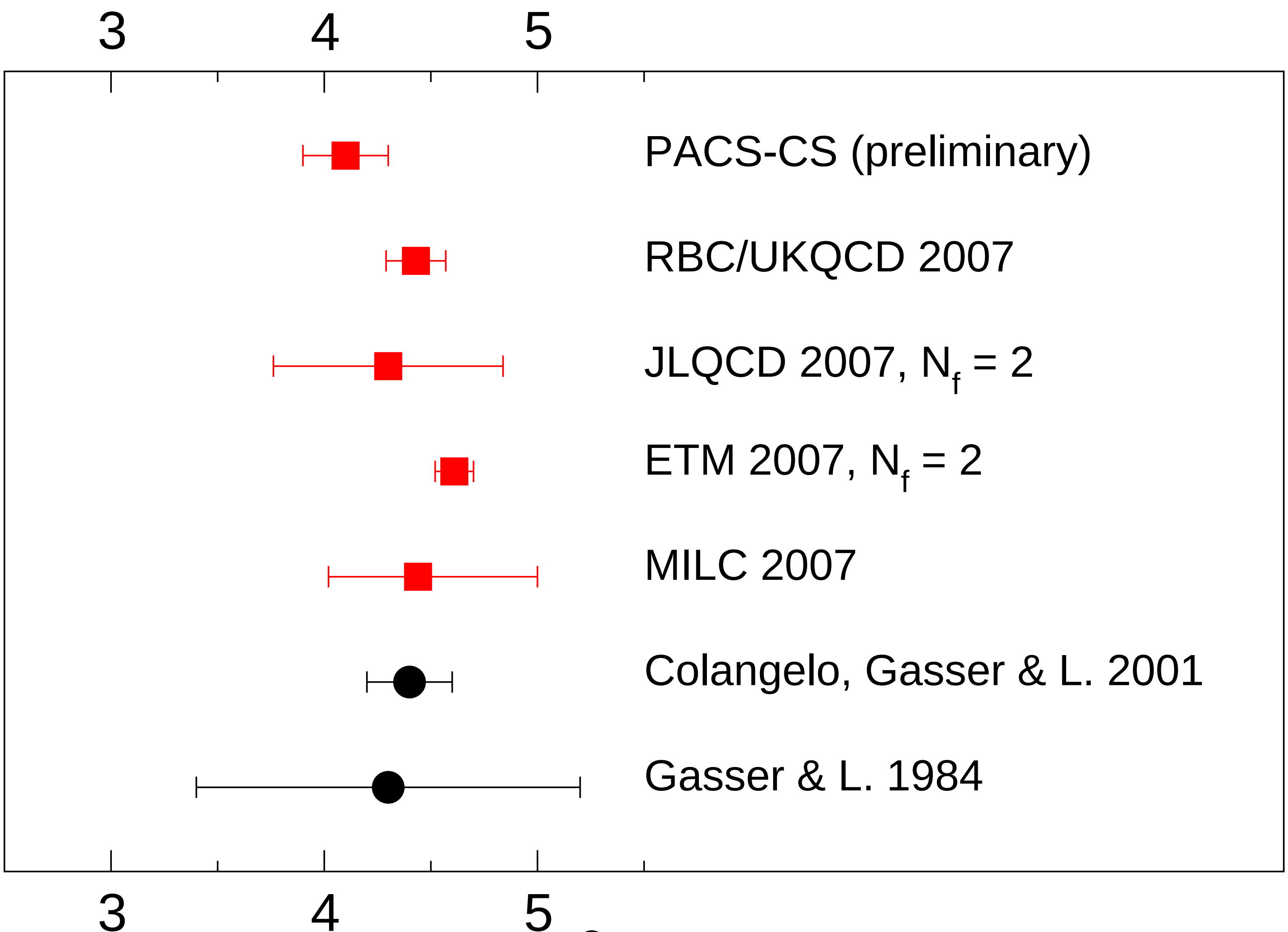
$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

  F is value of pion decay constant in limit $m_u, m_d \to 0$

- Structure is the same, coefficients and scale of logarithm are different
- Quark mass dependence of F_π can also be measured on the lattice $\Rightarrow$ measurement of Λ_4
- Alternative method: determine the scalar form factor of the pion, radius $\langle r^2\rangle_s \Leftrightarrow \bar{\ell}_4$

Lattice results for Λ_4



$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

Theory of $\pi\pi$ interaction

- $\pi\pi$ scattering is special: crossed channels are identical

$\Rightarrow$ Re $T(s,t)$ can be represented as a twice subtracted dispersion integral over Im $T(s,t)$ in physical region

S.M. Roy 1971

- The 2 subtraction constants can be identified with the S-wave scattering lengths:

$$a_0^0\;,\;a_0^2 \begin{array}{l}\leftarrow \text{isospin}\\ \leftarrow \text{angular momentum}\end{array}$$

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*

Roy equations

- Pioneering work on the physics of the Roy equations was done around the time when QCD was discovered

  Pennington & Protopopescu 1973, Basdevant, Froggatt & Petersen 1974

- Dispersion integrals converge rapidly (2 subtractions)

$\Rightarrow$ Crude phenomenological information on Im $T(s,t)$ for energies above 800 MeV suffices

$\Rightarrow$ Given a_0^0, a_0^2, the scattering amplitude can be calculated very accurately

Ananthanarayan, Colangelo, Gasser & L. 2001
Descotes, Fuchs, Girlanda & Stern 2002

$\Rightarrow$ a_0^0, a_0^2 are the essential parameters at low energy

- Main problem in early work: a_0^0, a_0^2 poorly known
  Experimental information near threshold is meagre

Low energy theorems

- Chiral perturbation theory provides the missing piece: theoretical prediction for a_0^0, a_0^2

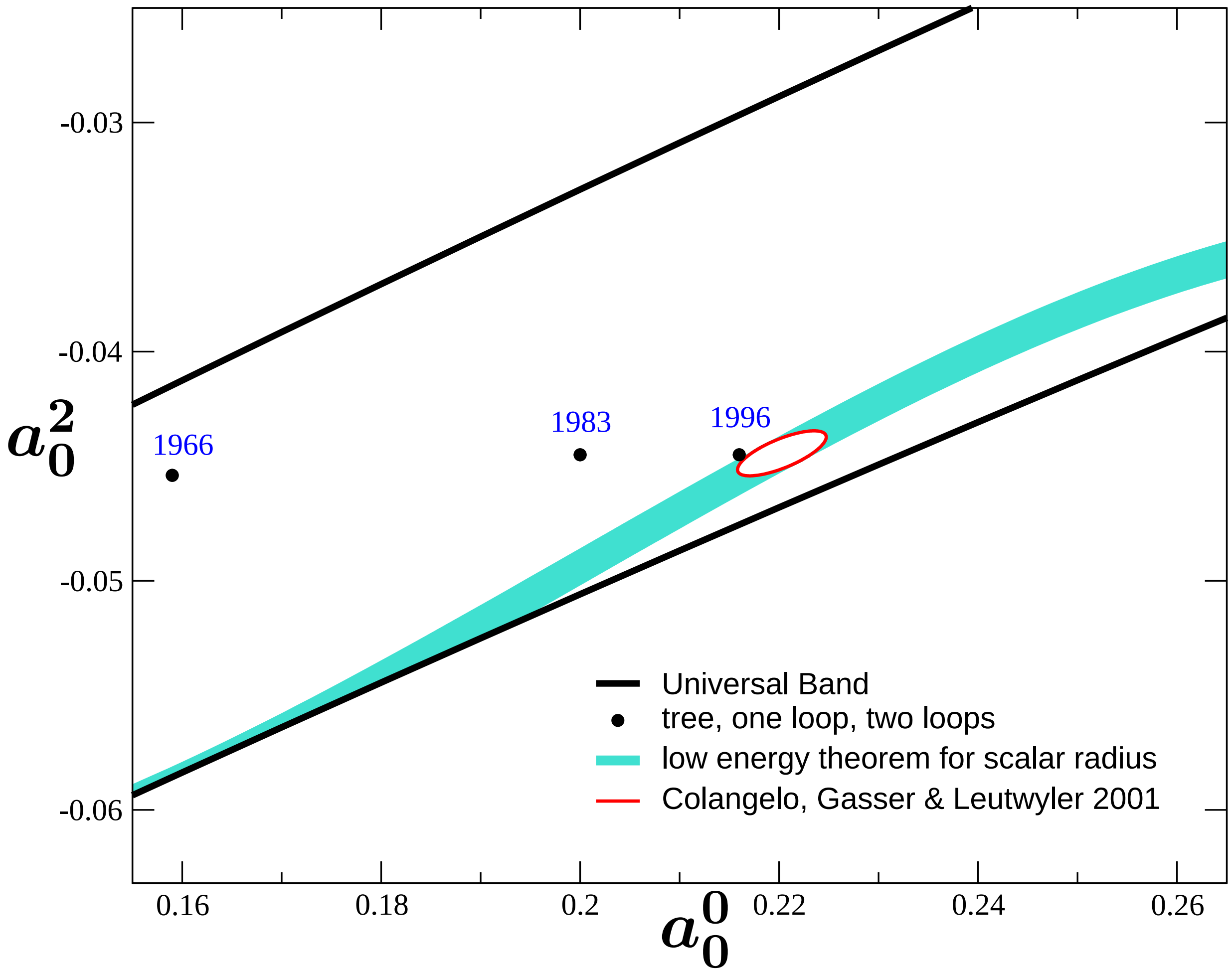
  Weinberg 1966, Gasser & L. 1983, Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

- Most accurate results for a_0^0, a_0^2 are obtained by matching the chiral and dispersive representations in the unphysical region below threshold

  Colangelo, Gasser & L. 2001

- In combination with the low energy theorems for a_0^0, a_0^2, the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to an incredible degree of accuracy

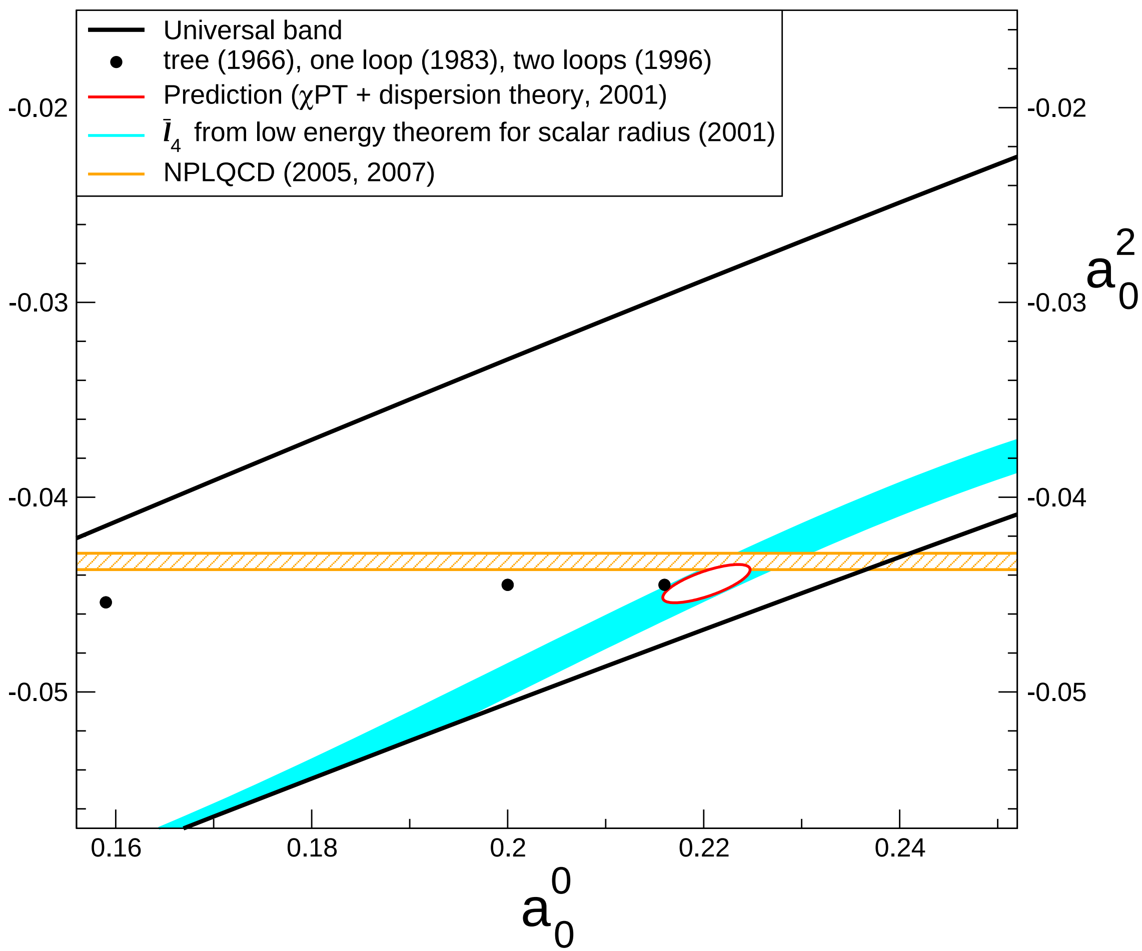
Predictions for the S-wave $\pi\pi$ scattering lengths



Sizable corrections in a_0^0, while a_0^2 nearly stays put

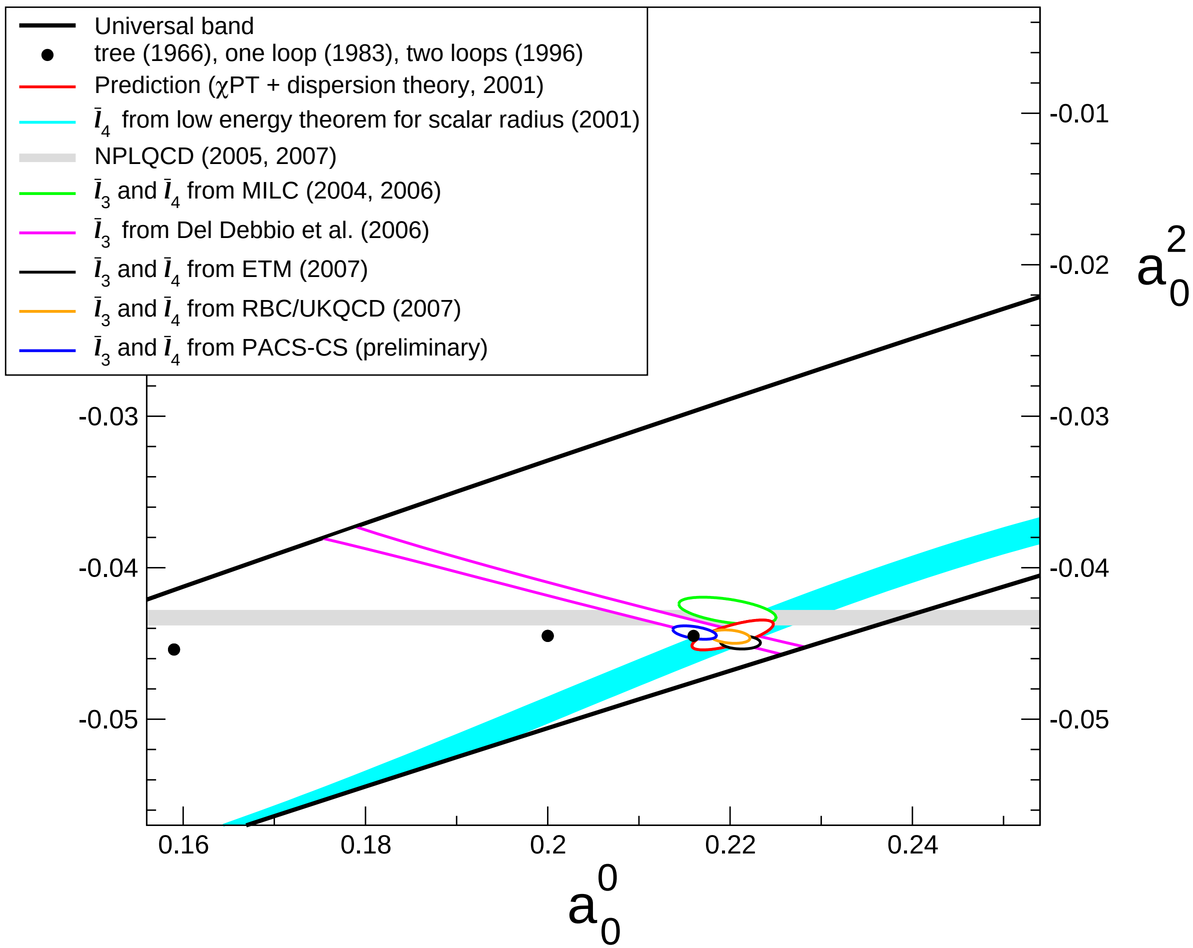
Lattice result for a_0^2

- Lattice allows direct measurement of a_0^2 via volume dependence of energy levels



Consequence of lattice results for ℓ_3, ℓ_4

- Uncertainty in prediction for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4
  Can make use of the lattice results for these



Experiments on light flavours at low energy

- Production experiments $\pi N \to \pi\pi N$, $\psi \to \pi\pi\omega \ldots$

  Problem: pions are not produced in vacuo

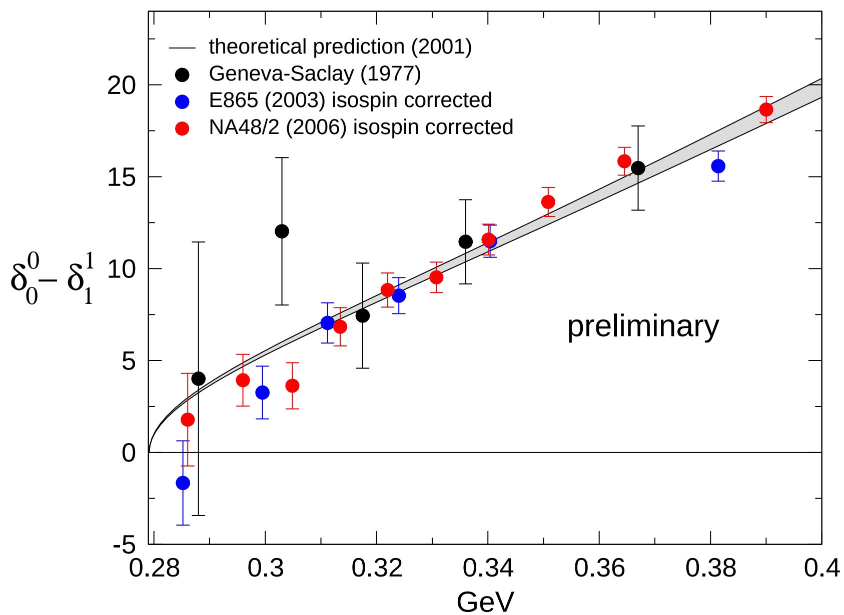
$\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple

  Accuracy rather limited

- $\pi^+\pi^-$ atoms, DIRAC
- $K^\pm \to \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $K^\pm \to \pi^+\pi^- e^\pm \nu$ precision data from E865, NA48/2

K_{e4} decay

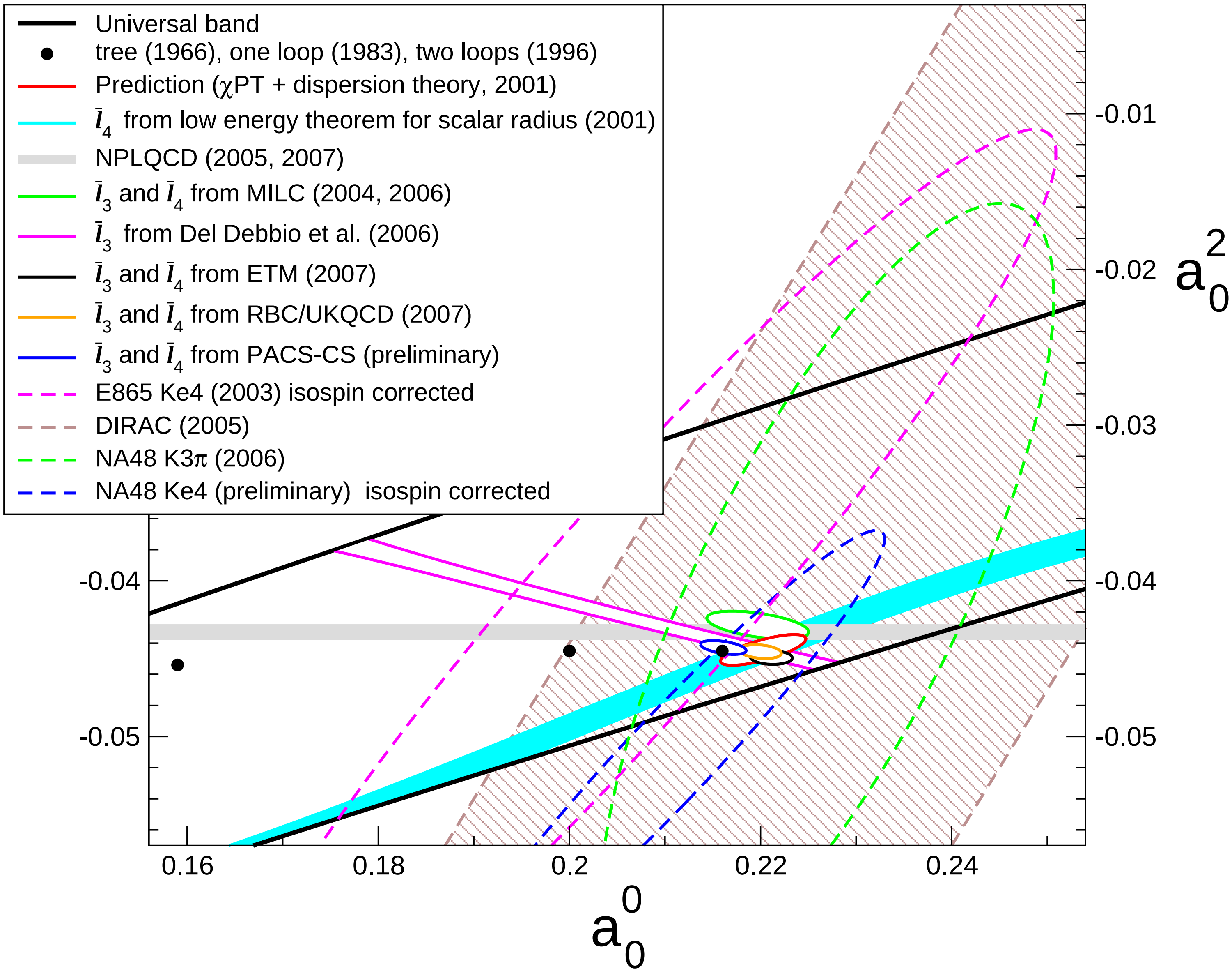
- $K \to \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007

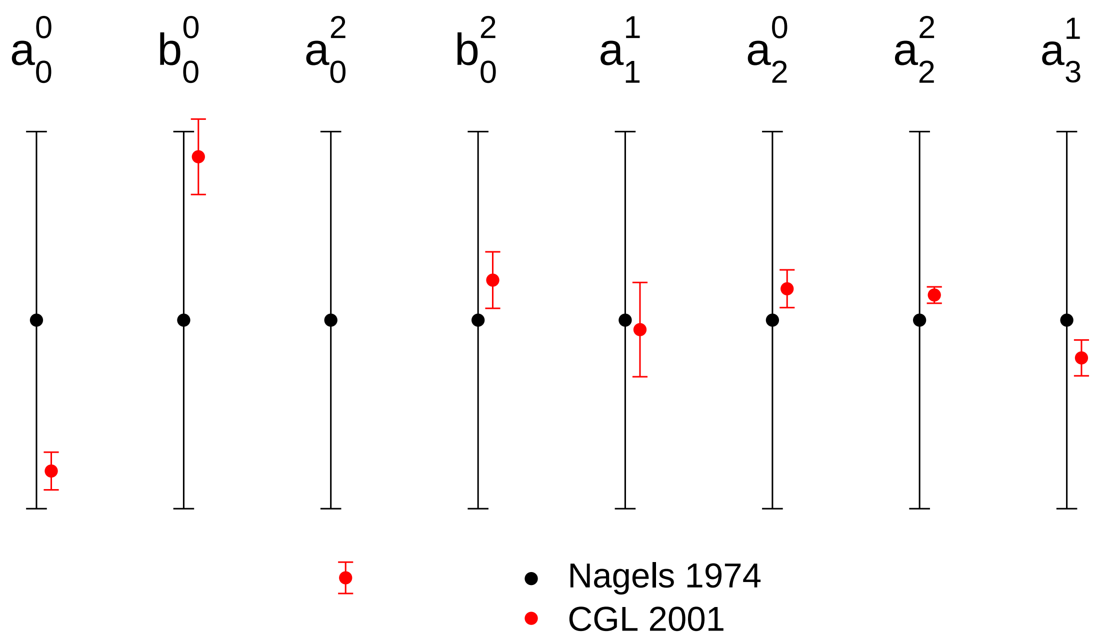
a_0^0, a_0^2 : prediction, lattice & experiment



Theory is ahead of experiment . . .

What difference does it make whether or not the subtraction constants are known accurately ?

- Results obtained on the basis of the experimental information about a_0^0 and a_0^2 — Nagels et al. 1979
- Results obtained with the chiral predictions for a_0^0 and a_0^2 — Colangelo, Gasser & L. 2001



Where is the lowest resonance of QCD ?

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001

- Does QCD have a resonance near threshold ?
  - Concerns the nonperturbative domain of QCD
  - Quark and gluon degrees of freedom useless there

  $\Rightarrow$ Understanding very poor, pattern of energy levels ?
  - Lowest resonance: σ ? ρ ?
- Resonance $\leftrightarrow$ pole on second sheet
  - Poles are universal
  - Pole position is unambiguous, even if width is large
  - Where is the pole closest to the origin ?

$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(400–1200)−i(250–500) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(552^{+84}_{-106})-i(232^{+81}_{-72})$	[1] ABLIKIM	07A	BES2	$\psi(2S) \to \pi^+\pi^- J/\psi$
$(441^{+16}_{-8})-i(272^{+9}_{-12.5})$	[2] CAPRINI	06	RVUE	$\pi\pi \to \pi\pi$
$(470 \pm 50)-i(285 \pm 25)$	[3] ZHOU	05	RVUE	
$(541 \pm 39)-i(252 \pm 42)$	[4] ABLIKIM	04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528 \pm 32)-i(207 \pm 23)$	[5] GALLEGOS	04	RVUE	Compilation
$(440 \pm 8)-i(212 \pm 15)$	[6] PELAEZ	04A	RVUE	$\pi\pi \to \pi\pi$
$(533 \pm 25)-i(247 \pm 25)$	[7] BUGG	03	RVUE	
$532 - i272$	BLACK	01	RVUE	$\pi^0\pi^0 \to \pi^0\pi^0$
$(470 \pm 30)-i(295 \pm 20)$	[2] COLANGELO	01	RVUE	$\pi\pi \to \pi\pi$
$(535^{+48}_{-36})-i(155^{+76}_{-53})$	[8] ISHIDA	01		$\Upsilon(3S) \to \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	[9] SUROVTSEV	01	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(558^{+34}_{-27})-i(196^{+32}_{-41})$	ISHIDA	00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH	99	RVUE	π scalar form factor
$(523 \pm 12)-i(259 \pm 7)$	KAMINSKI	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\sigma\sigma$
$442 - i\ 227$	OLLER	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$469 - i203$	OLLER	99B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$445 - i221$	OLLER	99C	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$
$(1530^{+90}_{-250})-i(560 \pm 40)$	ANISOVICH	98B	RVUE	Compilation
$420 - i\ 212$	LOCHER	98	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(602 \pm 26)-i(196 \pm 27)$	[10] ISHIDA	97		$\pi\pi \to \pi\pi$
$(537 \pm 20)-i(250 \pm 17)$	[11] KAMINSKI	97B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, 4π
$470 - i250$	[12,13] TORNQVIST	96	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
$\sim (1100 - i300)$	AMSLER	95B	CBAR	$\overline{p}p \to 3\pi^0$
$400 - i500$	[13,14] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$1100 - i137$	[13,15] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$387 - i305$	[13,16] JANSSEN	95	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$525 - i269$	[17] ACHASOV	94	RVUE	$\pi\pi \to \pi\pi$
$(506 \pm 10)-i(247 \pm 3)$	KAMINSKI	94	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$370 - i356$	[18] ZOU	94B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$408 - i342$	[13,18] ZOU	93	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$870 - i370$	[13,19] AU	87	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$470 - i208$	[20] VANBEVEREN	86	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, ...
$(750 \pm 50)-i(450 \pm 50)$	[21] ESTABROOKS	79	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(660 \pm 100)-i(320 \pm 70)$	PROTOPOP...	73	HBC	$\pi\pi \to \pi\pi$, $K\overline{K}$
$650 - i370$	[22] BASDEVANT	72	RVUE	$\pi\pi \to \pi\pi$

HTTP://PDG.LBL.GOV   Page 1   Created: 6/7/2007 11:24

PDG tables, edition 2007

Model independent determination of the pole

- Most of the results quoted by the PDG are obtained by
  (a) parametrizing the data for real values of s
  (b) continuing this parametrization analytically in s

  $\Rightarrow$ Result is sensitive to the parametrization used

- We found a model independent method:
  1. Poles on second sheet are zeros on first sheet
  2. The Roy equations are valid for complex values of s, in a limited region of the first sheet

  $\Rightarrow$ Exact representation of the partial waves in terms of observable quantities, valid for complex values of s

  3. Can evaluate this representation to good precision and determine the zeros numerically

Roy equation for the isoscalar S-wave

$$S_0^0(s) = 1 + 2\,i\rho\,t_0^0(s) \qquad\qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2)\,b + \int_{4M_\pi^2}^\infty ds'\,\{K_0(s,s')\,\mathrm{Im}\,t_0^0(s')$$

$$+ K_1(s,s')\,\mathrm{Im}\,t_1^1(s') + K_2(s,s')\,\mathrm{Im}\,t_0^2(s')\}$$

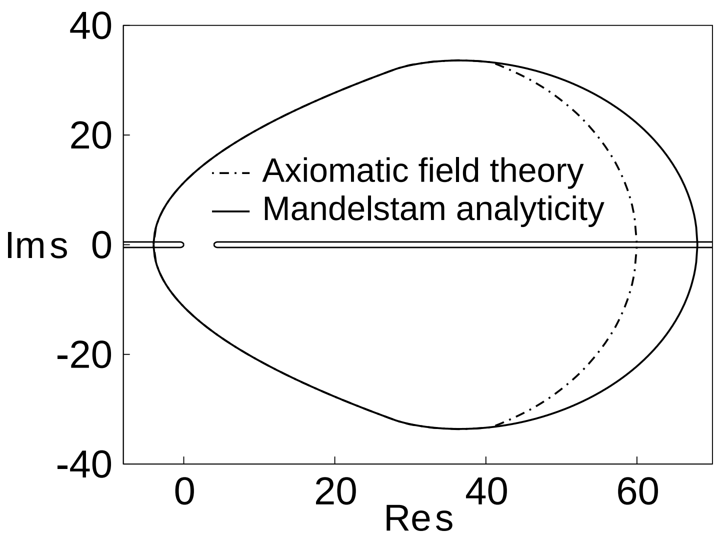
$+$ higher partial waves

- The subtraction constants are determined by a_0^0, a_0^2:
  $$a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$$
- The kernels are elementary functions, e.g.
  $$K_0(s,s') = \underbrace{\frac{1}{\pi(s'-s)}}_{r.h.cut} + \underbrace{\frac{2\,ln\{(s+s'-4M_\pi^2)/s'\}}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}}_{l.h.cut}$$
- Left hand cut is essential for convergence:
  $K_0(s,s') \sim 1/{s'}^3$ for large s'

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$
- Equations are valid for complex s in a limited region of the first sheet

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001



s in units of M_π^2

- Proof is based on first principles, general quantum field theory

A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing*, Lecture Notes in Physics, vol. 3, 1969.

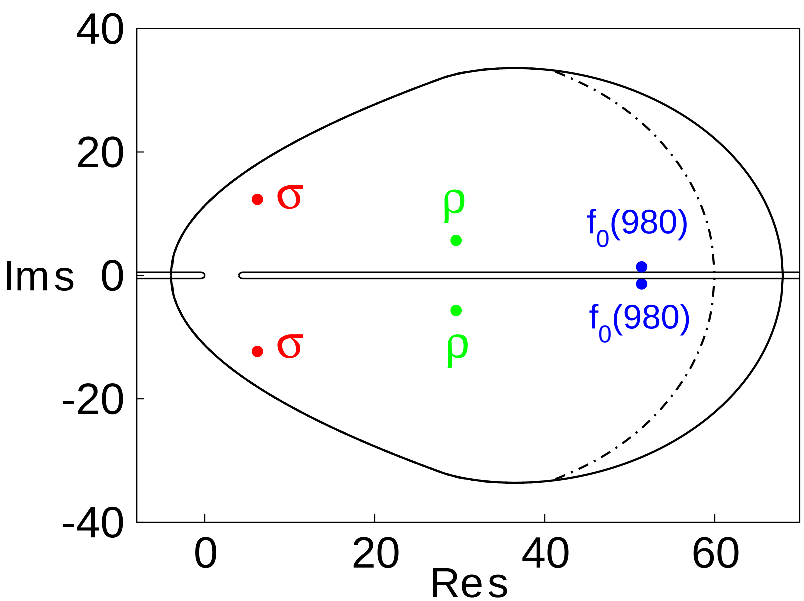
G. Mahoux, S. M. Roy and G. Wanders, Nucl. Phys. B70 (1974) 297.

$\Rightarrow$ Exact representation for $S_0^0(s)$ in this region
Do not need to parametrize the amplitude

Evaluation of the pole position

- Have an exact formula for the pole position in terms of physical quantities: $S_0^0(s) = 0$
- For the central solution of the Roy equations, $S_0^0(s)$ has two pairs of zeros in the region where the formula holds:

$$s = (6.2 \pm i\,12.3)\,M_\pi^2 \quad \sigma$$
$$s = (51.4 \pm i\,1.4)\,M_\pi^2 \quad f_0(980)$$



s in units of M_π^2

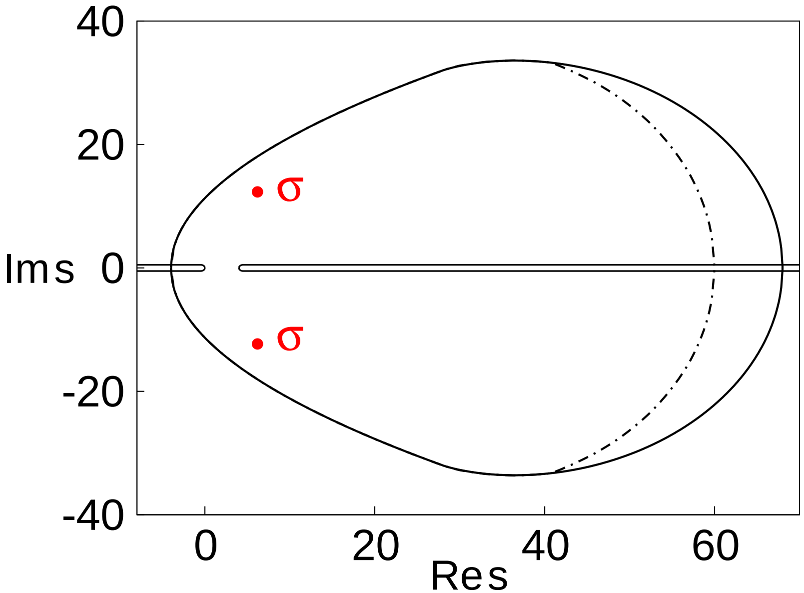
The eyes of the red dragon

Tail at 1.7 GeV: $s \simeq 150\,M_\pi^2$

Result

- Lowest resonance of QCD has vacuum quantum numbers
- Pole on lower half of sheet II occurs in vicinity of

$$m_\sigma = 441 - i\,272\,\text{MeV} = M_\sigma - \tfrac{i}{2}\Gamma_\sigma$$



s in units of M_π^2

Loci Oculorum Draconis Rutili

T. Barnes, Theory summary, MESON 2006

Exploring the vicinity of the σ pole

- Pole not far from boundary of domain where Roy equations are valid $\Rightarrow$ pole formula shaky there ?

  $\Rightarrow$ talk by Peter Minkowski

- Domain can be extended: modified version of the Roy equations

  Roy & Wanders (1978)

Error analysis

- Formula is exact, evaluation is approximate
- Key point: can follow error propagation explicitly
- Split the formula into 3 pieces:
  1. Subtraction terms
  2. Contribution from Im $t_0^0(s)$ below $K\bar{K}$ threshold
  3. Higher energies and other partial waves
- Abstract of the following discussion:

  Uncertainty in result for σ pole is dominated by 2.

  Start with 3.

1. Subtraction terms
2. Contribution from Im $t_0^0(s)$ below $K\bar{K}$ threshold
3. Higher energies and other partial waves

- Contributions from S^2, P^1, D^0, . . . are dominated by ρ
  Excellent experimental information from e^+e^-, τ
- Contributions from Im $t_0^0(s)$ above $2M_K$ are tiny (2 subtractions)

$\Rightarrow$ uncertainties from 3. $\ll$ uncertainties from 2.

- Illustration: Take only 1. and 2. from CCL
  Take 3. from KPY III

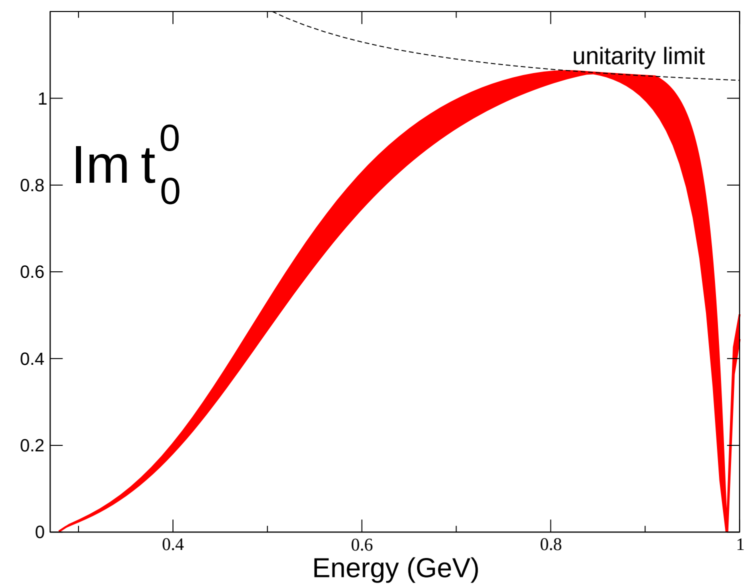
$\Rightarrow$ Pole moves by $-0.6 - i\,1.2$ MeV

1. Subtraction terms
2. Contribution from Im $t_0^0(s)$ below $K\bar{K}$ threshold
3. Higher energies and other partial waves

Unitarity and dip leave little room between 800 MeV and $2M_K$

Only the region below 800 MeV really matters

There, S^0 is nearly elastic



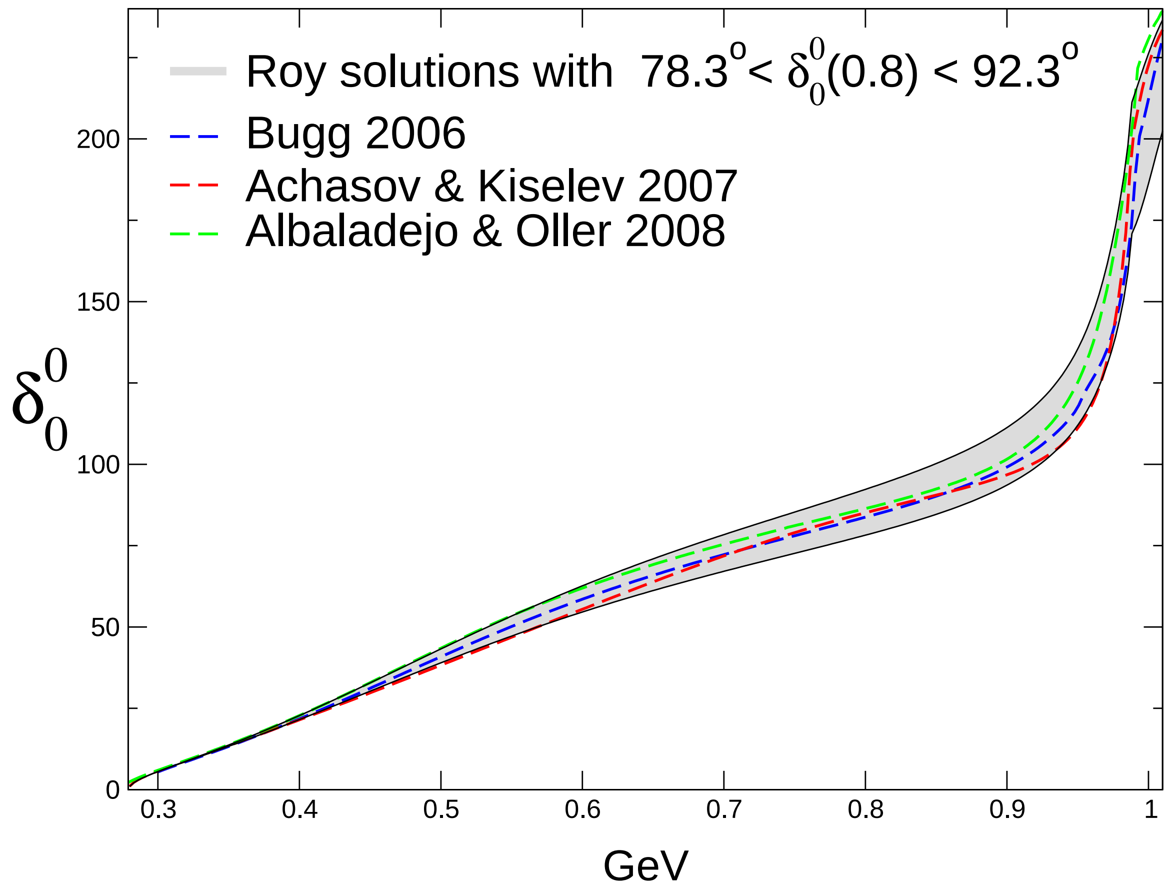
$$\text{Im}\, t_0^0(s) \simeq \sin^2\delta_0^0(s)/\sqrt{1-4M_\pi^2/s}$$

$\Rightarrow$ Need to know the phase $\delta_0^0(s)$ below 800 MeV

Solution of Roy equations is not unique

- On the interval between $2M_\pi$ and 1.15 GeV, the space of solutions is 2-dimensional
- Convenient to identify the two free parameters with the values of δ_0^0 and δ_1^1 at 800 MeV
- δ_1^1 is known very accurately
- δ_0^0 is not known well
- Range used in CCL: $\delta_0^0(0.8) = 82.3^{+10}_{-4}$

Behaviour of the phase below 800 MeV



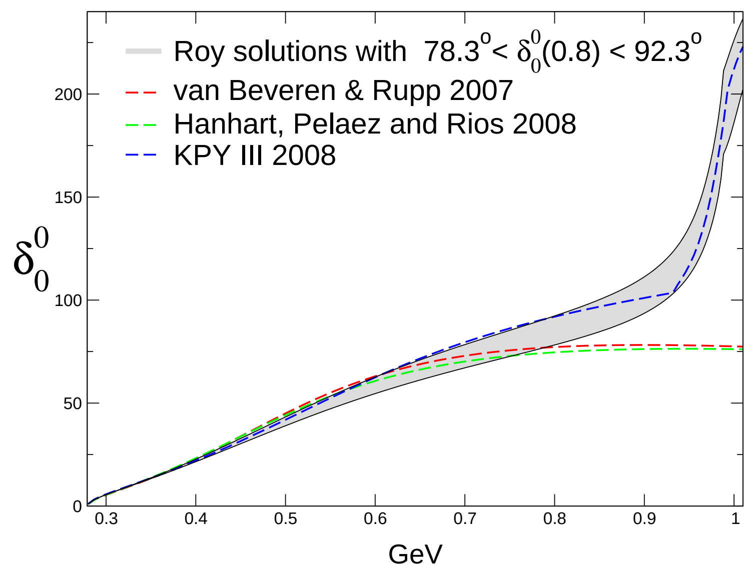
Bugg uses BES data for $J/\psi \longrightarrow \omega\pi\pi$

Achasov & Kiselev use KLOE data on $\phi \longrightarrow \gamma\pi\pi$

Albaladejo: N/D fi t to several data sets

Of humps and kinks

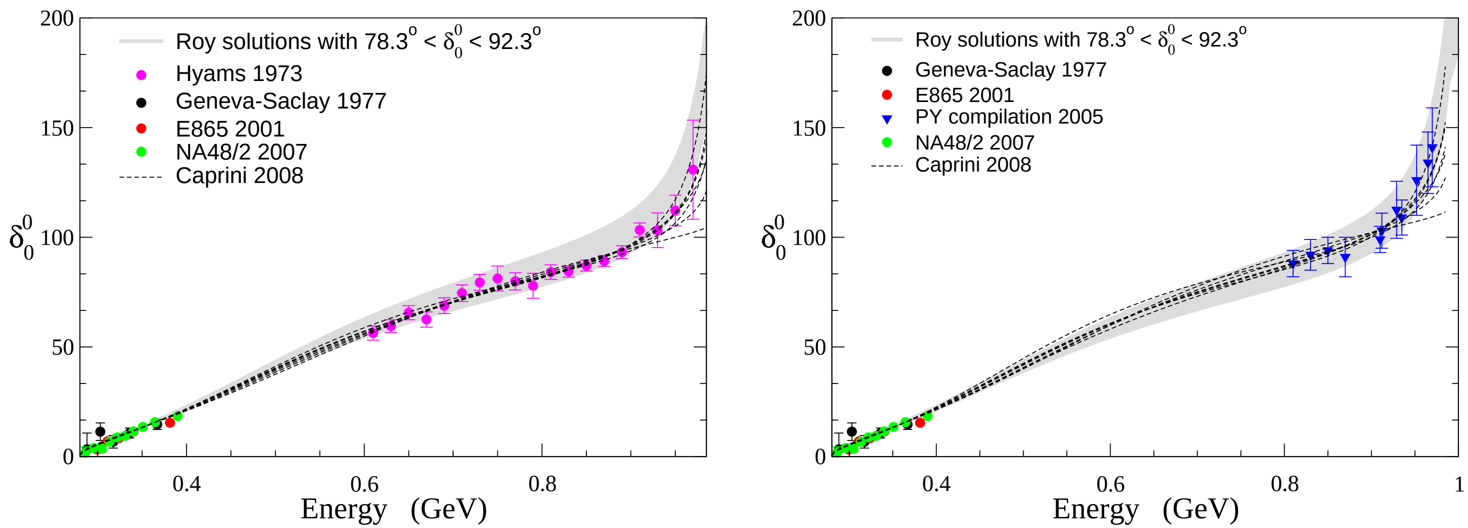
- The Roy band belonging to $\delta_0^0(0.8) = 82.3^{+10}_{-4}$ does not cover all parametrizations proposed in the literature
- IAM representation based on SU(2)×SU(2) and RSE model do have a hump, but these models fail to account for the rise of the phase due to the $f_0(980)$



- Parametrizations with a kink (discontinuity in the first derivative) violate causality

Pole position from analytic continuation

- Result for pole position depends on choice of parametrization

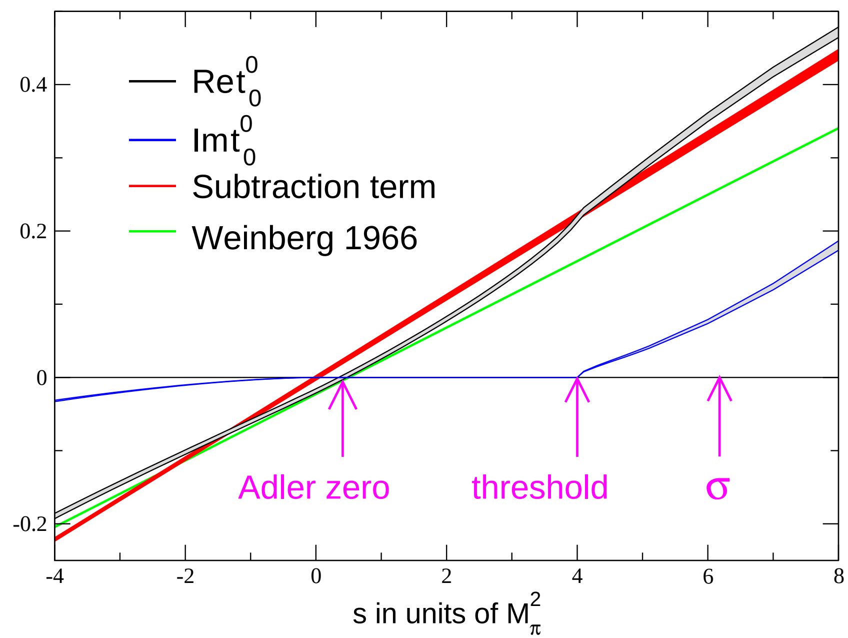


Plots kindly provided by Irinel Caprini ⇒ see her talk

- All parametrizations that are halfways acceptable above 900 MeV stay within the Roy band at lower energies

At low energies, the subtraction terms dominate

1. Subtraction terms
2. Contribution from Im $t_0^0(s)$ below $K\bar{K}$ threshold
3. Higher energies and other partial waves



Goldstone bosons of low energy interact only weakly

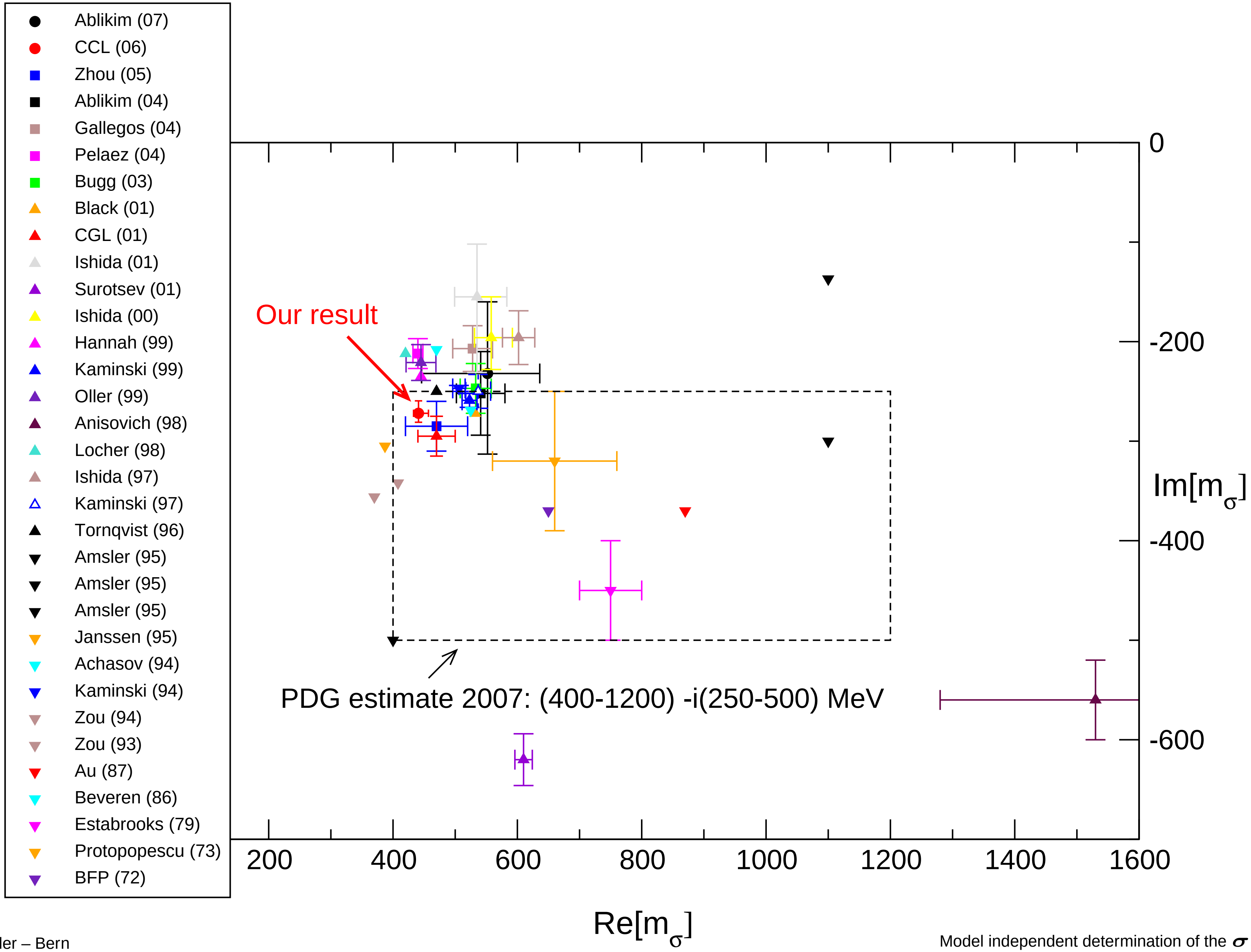
Result for pole position

- Final result: insert the predictions for a_0^0, a_0^2, use the phenomenological range for $\delta_0^0(0.8)$ and add errors up:

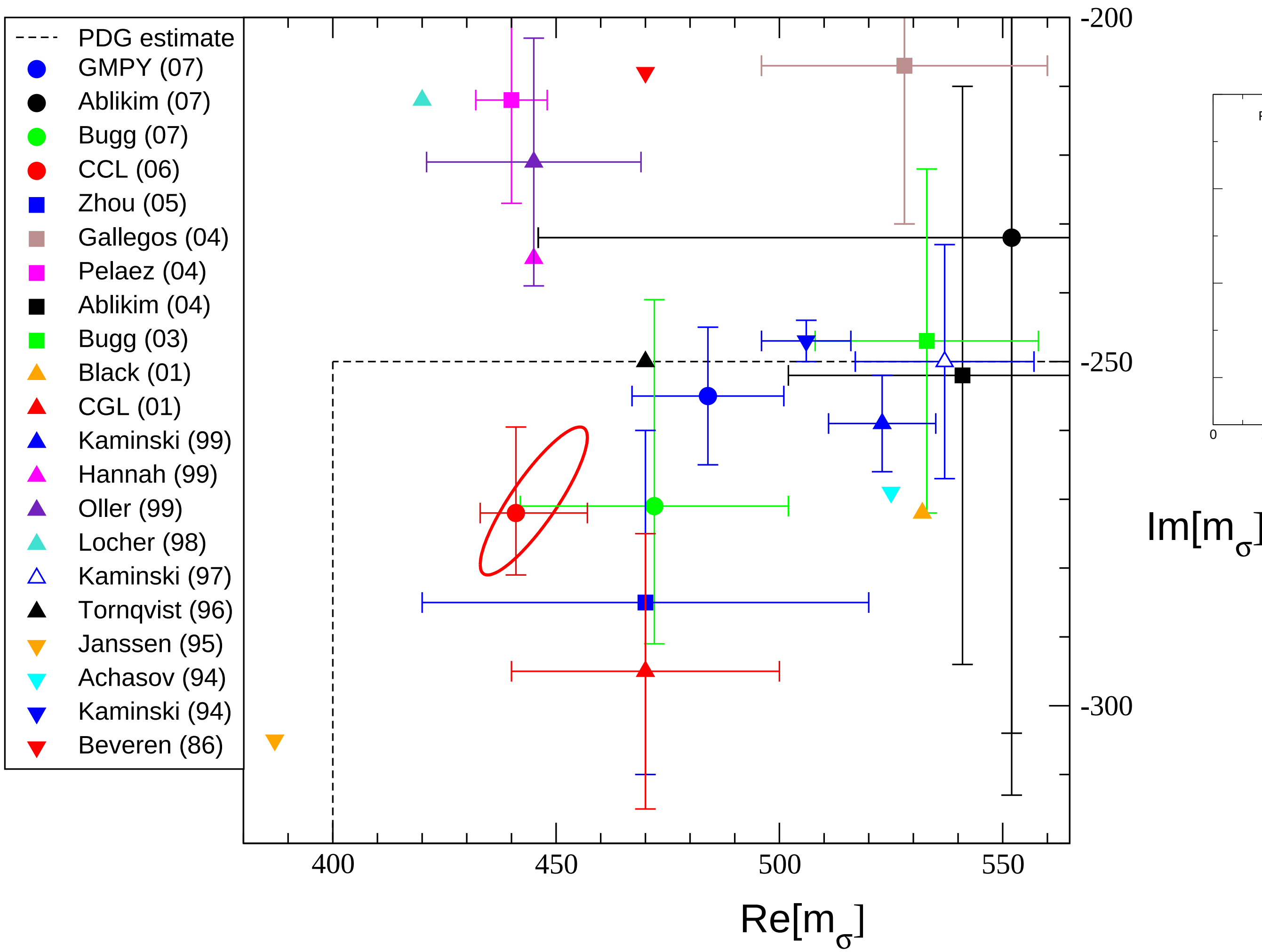
$$m_\sigma = 441\,{}^{+16}_{-8}\; - i\, 272\,{}^{+9}_{-13}\ \text{MeV}$$

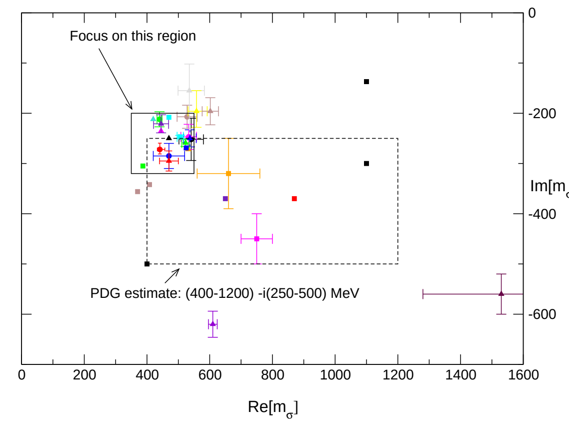
- Replace the central Roy solution below $2M_K$ by the phase representation of Bugg 2006 $\Rightarrow$ pole moves to $444 - i\,267$ MeV
- Ditto with Achasov & Kiselev 2007 gives $438 - i\,274$ MeV
- Ditto with van Beveren & Rupp 2007 gives $461 - i\,252$ MeV

Comparison with compilation in PDG 2007

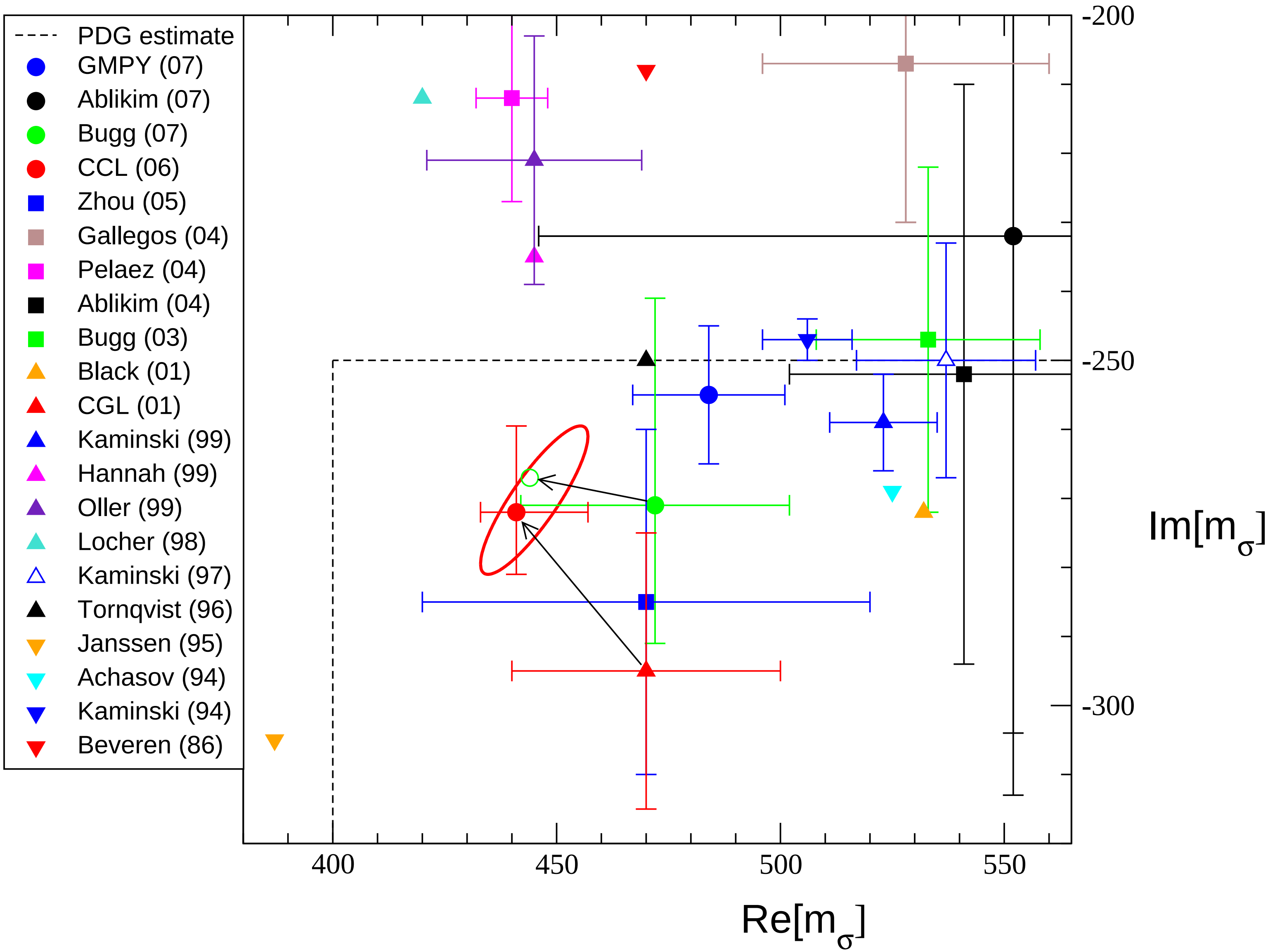


Vicinity of the pole



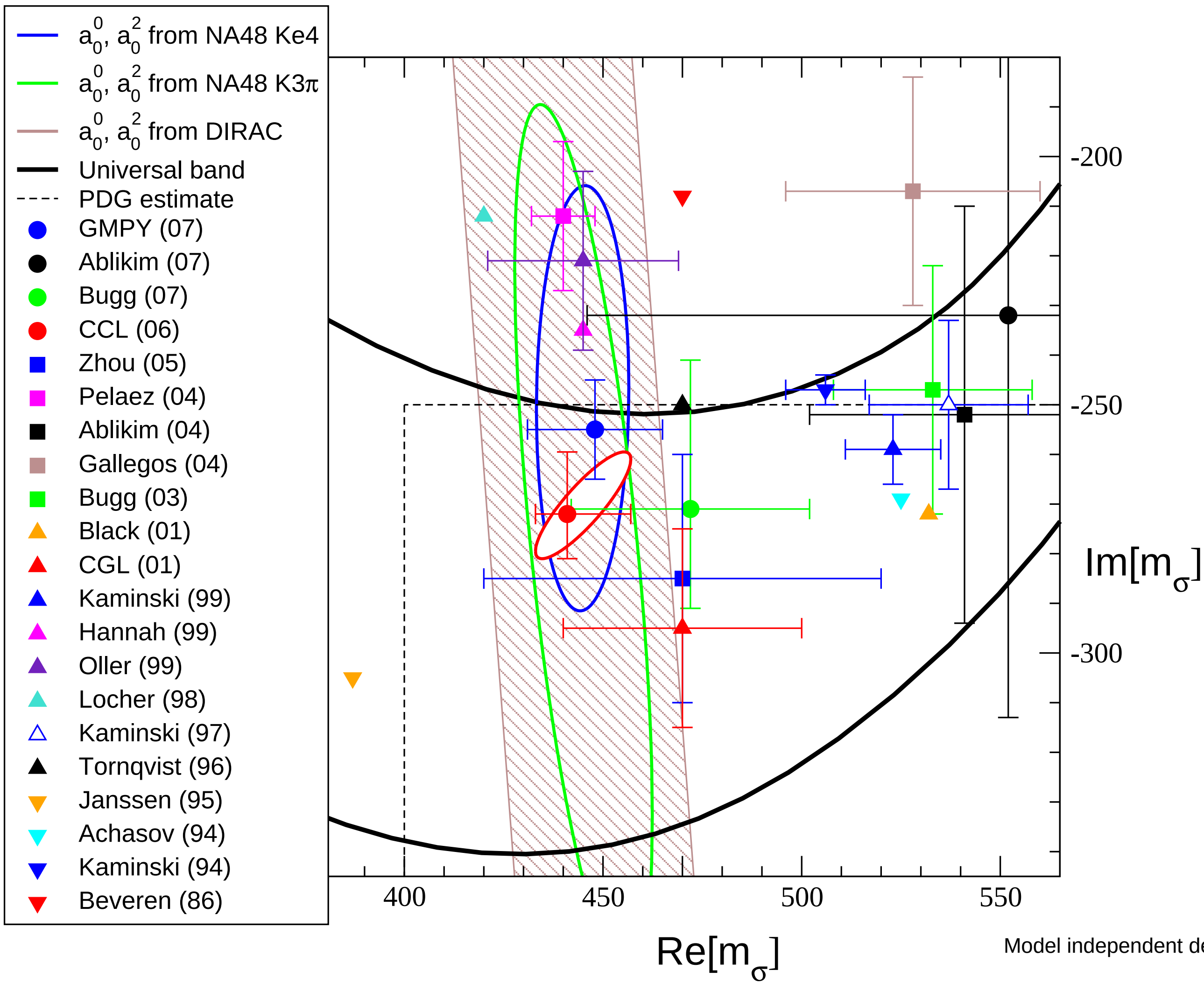


Analytic continuation versus dispersion theory



Ignore the theoretical predictions for a_0^0, a_0^2

- Replace the low energy theorems for a_0^0, a_0^2 by the experimental results from DIRAC and NA48
- $a_0^0, a_0^2 \in$ universal band



Physical interpretation of the σ

- Glueball ? $\bar{q}q$? $\bar{q}q\bar{q}q$?
- Hadrons in terms of quarks and gluons ?
- Fock space is basis of perturbation theory – not clear how to use it quantitatively in nonperturbative domain
- My qualitative picture of the σ:
  - Can understand position of σ pole in terms of F_π

  $\Rightarrow$ Low energy properties of the pions are relevant

  $\Rightarrow$ Physics of the $\sigma \in$ Goldstone boson dynamics

  $\Rightarrow$ Head of the dragon contains only little glue

  $\Rightarrow$ Wave function has large tetra-quark component
- This picture is by no means commonly accepted

  Törnqvist, Ishida, Jaffe, Minkowski, Ochs, Bugg, Pennington, Peláez, Oller, Hannah, Guo, Su, Xiao, Zheng, Zhou, Chen, Hosaka, Zhu, Liu, Maiani, Polosa, Piccinini, Riquer, Isidori, Nicolaci, Pacetti, Menessier, Narison, Fariborz, Jora, Schechter, van Beveren, Kleefeld, Rupp, Scadron, Ynduráin, García-Martín, ...

$\Rightarrow$ Comprehensive review : Klempt & Zaitsev, arXiv:0708.4016

Conclusion

- Low energy pion physics: theory ahead of experiment
  - Precision experiments carried out and under way
  - Lattice makes slow, but steady progress
  - So far, all tests confirm the theory

Conclusion

- Low energy pion physics: theory ahead of experiment
  - Precision experiments carried out and under way
  - Lattice makes slow, but steady progress
  - So far, all tests confirm the theory
- Limitations of our approach:
  - Calculations cannot be done on back of an envelope
  - Analysis only covers low energies
    Extension to higher energies is under way
  - Only a few applications have been worked out:
    $\pi\pi$ scattering, pion form factors, hadronic vacuum polarization in muon $g-2$
  - Much is yet to be done: $J/\psi \rightarrow \omega\pi\pi$, $D \rightarrow 3\pi$, $\gamma\gamma \rightarrow \pi\pi$, πK, πN, ...

Conclusion

- Model independent method for analytic continuation
  - The lowest resonance of QCD occurs at

    $M_\sigma = 441\,{}^{+16}_{-8}$ MeV $\qquad \Gamma_\sigma = 544\,{}^{+18}_{-25}$ MeV

    and carries vacuum quantum numbers
  - Crossing symmetry plays an essential role:
    Fixes contributions from left hand cut
    Ensures fast convergence, low energy dominance
  - Pole occurs at low value of s, closer to left hand cut than to singularities from $K\bar{K}$, $f_0(980)$
  - Result for Γ_σ relies on theory for a_0^2
    Experiments concerning a_0^2 would be most welcome

SPARES

NNLO

- The next order contains the square of a logarithm:

$$M_\pi^2 = M^2\left\{1+\frac{x}{2}\ln\frac{M^2}{\Lambda_3^2}+\frac{17x^2}{8}\left(\ln\frac{M^2}{\Lambda_M^2}\right)^2+x^2k_M+O(M^6)\right\}$$

$$F_\pi = F\left\{1-x\ln\frac{M^2}{\Lambda_4^2}-\frac{5x^2}{4}\left(\ln\frac{M^2}{\Lambda_F^2}\right)^2+x^2k_F+O(M^6)\right\}$$

$$x \equiv \left(\frac{M}{4\pi F}\right)^2$$

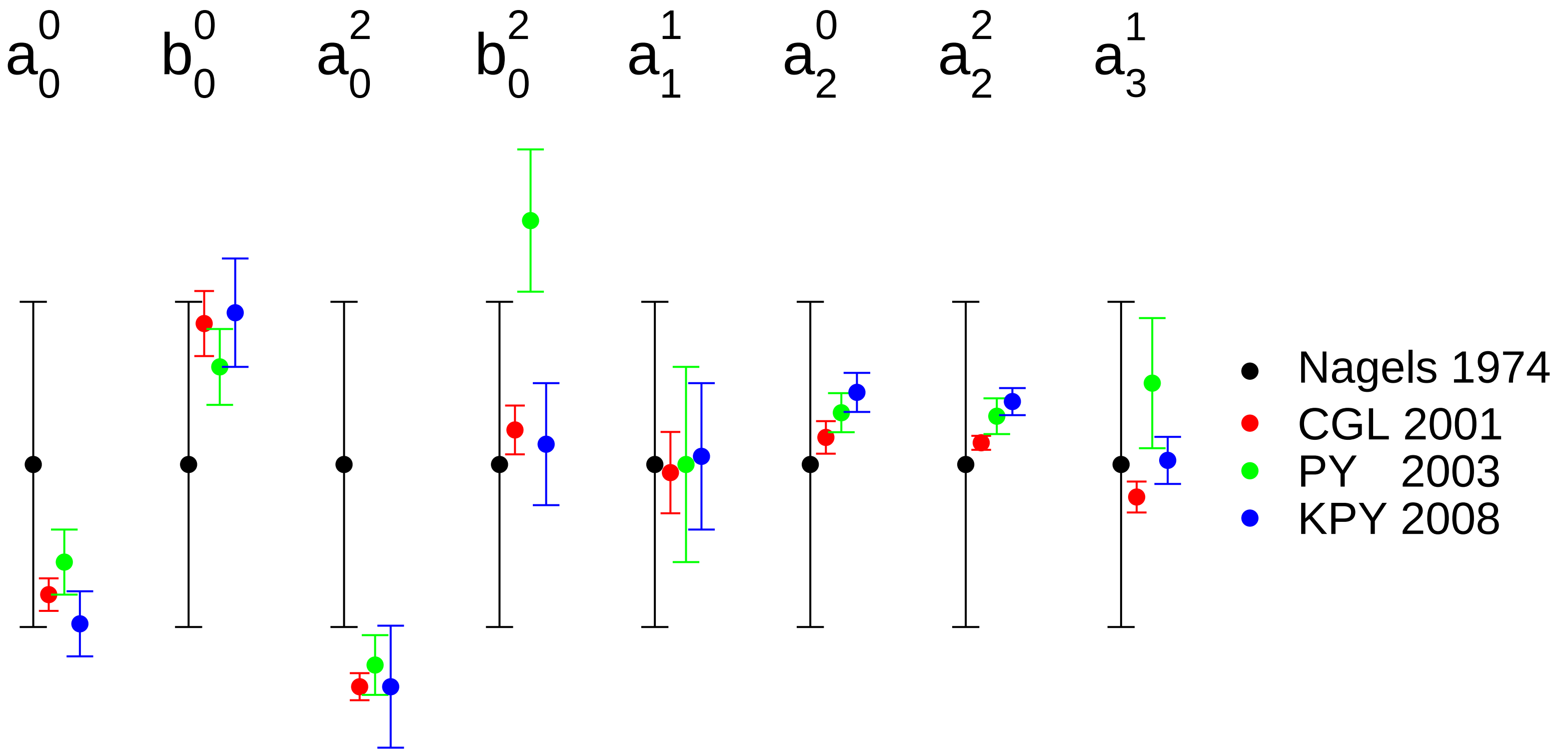
Colangelo 1995, Bijnens et al. 1996, Bürgi 1996

- For physical value of m_u, m_d, the NNLO terms are tiny

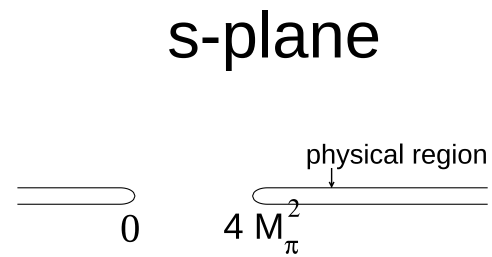
$\Rightarrow$ Size of $\Lambda_M, k_M, \Lambda_F, k_F$ barely known

- Must become clearly visible if m_u, m_d are made larger

Comparison with recent phenomenological results



Pole on second sheet ↔ zero on first sheet



- $S_0^0(s) = \eta_0^0(s)\exp 2i\delta_0^0(s)$

  $S_0^0(s)$ is analytic in the cut plane

- For $0 < s < 4M_\pi^2$, $S_0^0(s)$ is real

$\Rightarrow$ $S_0^0(s^\star) = S_0^0(s)^\star$

  x in elastic interval: $S_0^0(x \pm i\epsilon) = \exp \pm 2i\delta_0^0(x)$

- Second sheet is reached by continuation across the elastic interval of the right hand cut

  $S_0^0(x - i\epsilon)^{II} = S_0^0(x + i\epsilon)^{I} = 1/S_0^0(x - i\epsilon)^{I}$

  Analyticity $\Rightarrow$ $\boxed{S_0^0(s)^{II} = 1/S_0^0(s)^{I}}$ valid $\forall s$

  Pole in $S_0^0(s)^{II}$ $\Longleftrightarrow$ zero in $S_0^0(s)^{I}$

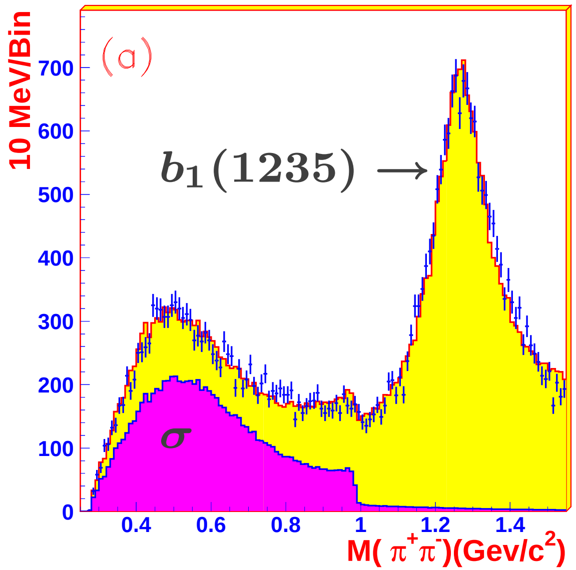
Curvature due to the left hand cut

- Left hand cut generates curvature
  Main contribution on the left stems from the ρ
- Most pole determinations neglect the left hand cut
  Pole from σ is too close for this to be justified
- Can estimate contributions from left hand cut with χPT

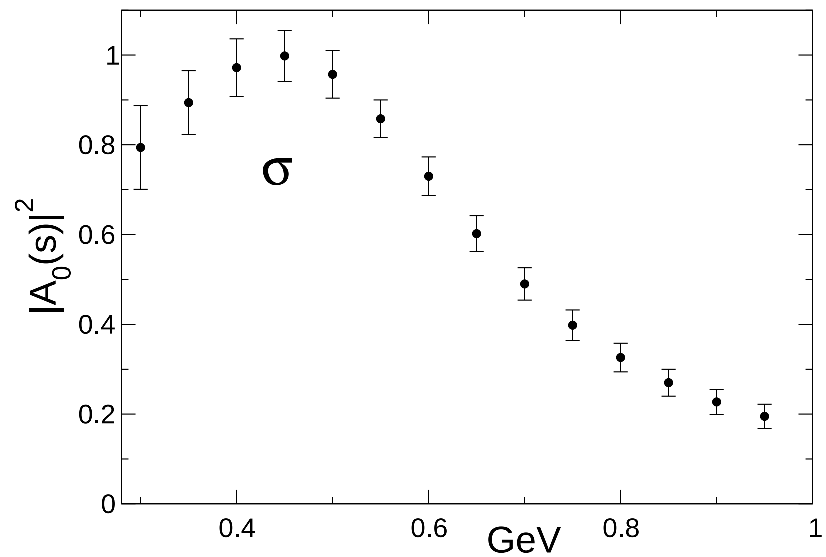
  Zhou, Qin, Zhang, Xiao, Zheng, Wu, JHEP 0502 (2005) 043

$\Rightarrow$ Outcome for pole position agrees with our result within the quoted errors

BES data on $J/\psi \to \omega\pi\pi$



BES, Phys. Lett. B598 (2004) 149



S-wave projection (D. Bugg, priv. comm.)

Outcome for pole position:

$m_\sigma = (541 \pm 39) - i\,(252 \pm 42)$ MeV BES 2004

(simple parametrization à la Breit-Wigner, $K\bar{K}$ and $\eta\eta$ final states neglected)

$m_\sigma = (472 \pm 30) - i\,(271 \pm 30)$ MeV Bugg hep-ph/0608081

(reanalysis based on a more complicated model)

Result is model dependent $\Rightarrow$ systematic uncertainty

Model independent discussion of $J/\psi \to \omega\pi\pi$

- Neglect rescattering on the ω and 4π final states

$\Rightarrow$ Watson theorem fixes phase of decay amplitude:

$$A_0(s) = |A_0(s)|\, e^{i\delta_0^0(s)} \quad \text{for } 4M_\pi^2 < s < 4M_K^2$$

$\uparrow$

S-wave projection of decay amplitude

I. Caprini, Phys. Lett. B638 (2006) 468

- Situation is the same as for the scalar form factor

$$F_0(s) = \langle \pi\pi \,\text{out}\, |\, \bar{u}u\, | 0\rangle = |F_0(s)|\, e^{i\delta_0^0(s)}$$

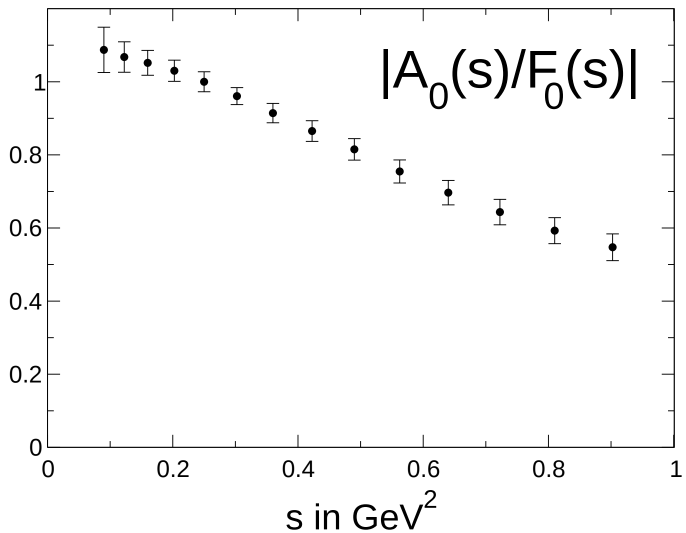
$\Rightarrow$ $A_0(s)/F_0(s)$ is real for $0 < s < 4M_K^2$

- Both $A_0(s)$ and $F_0(s)$ have a pole from the σ on the second sheet, drops out in $A_0(s)/F_0(s)$

- r.h. cut in $A_0(s)/F_0(s)$ only starts at $4M_K^2$

$\Rightarrow$ $A_0(s)/F_0(s)$ can vary only slowly with s

Comparison with scalar form factor



- $F_0(s)$ taken from Ananthanarayan et al. (2004), based on central solution of the Roy equations
- Model of Lähde and Meißner, hep-ph/0606133 describes J/ψ decays into $\omega\pi\pi, \omega K\bar{K}, \phi\pi\pi$, $\phi K\bar{K}$ in terms of scalar form factors, uses crude approximation: $A_0(s)/F_0(s) \simeq$ constant

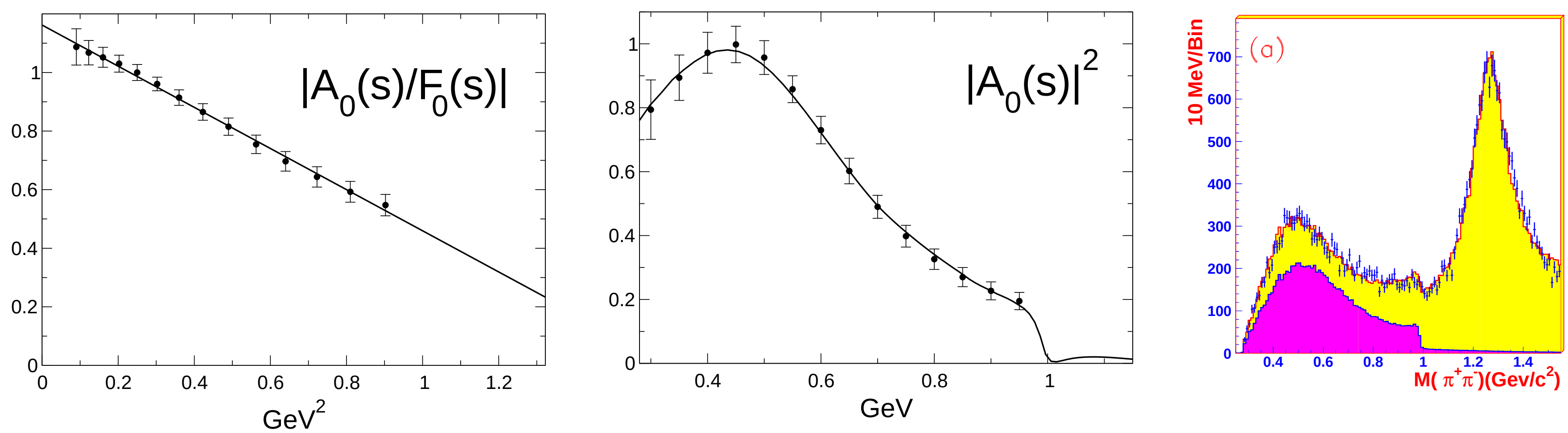
- Dispersion relation for $R(s) \equiv A_0(s)/F_0(s)$:

$$R(s) = R_0 + R_1\, s + \frac{(s - 2M_K^2)^2}{\pi} \int \frac{dx\, \mathrm{Im} R(x)}{(x - 2M_K^2)^2\,(x - s)}$$

- Plot does not show any curvature $\Rightarrow$ integral is small

$$R(s) \simeq R_0 + R_1\, s$$

Comparison of $J/\psi \to \omega\pi\pi$ and scalar form factor

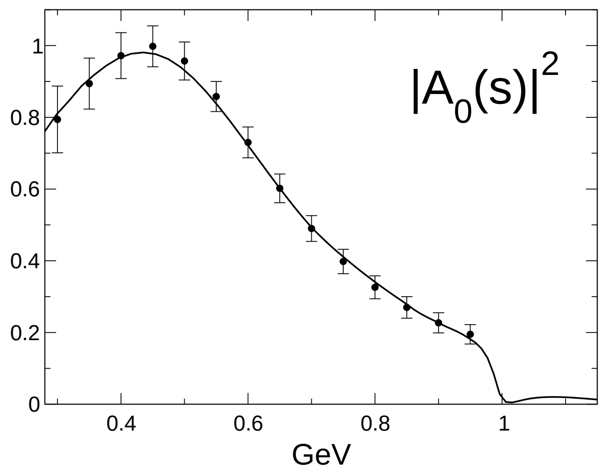


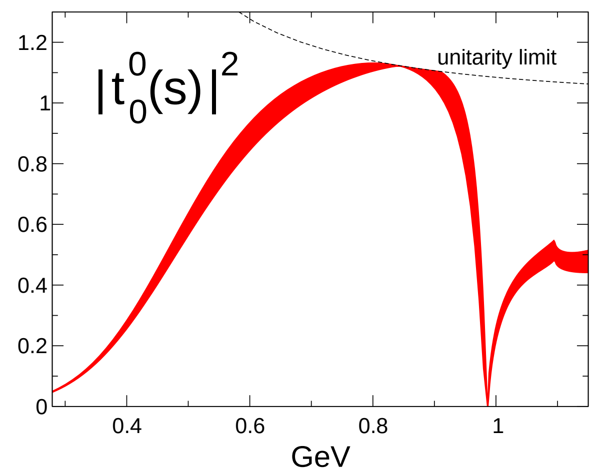
- Full line corresponds to the approximation

  $A_0(s) \simeq R_0\,(1 - s/s_0)\,F_0(s)$, with $s_0 = 1.65\,\text{GeV}^2$
- Observed energy dependence is consistent with the assumption that rescattering on the ω can be neglected
- In contrast to the case of $\pi\pi$ scattering, chiral symmetry does not determine the two subtraction constants occurring here

Comparison of $J/\psi \to \omega\pi\pi$ and $\pi\pi$ scattering

- $A_0(s)$ and $t_0^0(s)$ have approximately the same phase but profile is not the same: Adler zero in $t_0^0(s)$





- Need two subtractions – these make the difference
- Data on $J/\psi \to \omega\pi\pi$ are better
  Theory is weaker (unitarity, subtractions, rescattering)

$\Rightarrow$ Uncertainty in pole position from $J/\psi \to \omega\pi\pi$ larger

L., in Proc. MESON 2006, hep-ph/0608218

The κ

- $K\pi$ scattering amplitude obeys an analog of the Roy equations. Pole from κ can be calculated on this basis

$$m_\kappa = (658 \pm 13) - i\,(278.5 \pm 12)\,\text{MeV}$$

Descotes-Genon and Moussallam 2006

- Confirms an earlier calculation, where the l.h. cut was estimated with χPT Zhou and Zheng 2006
- Back-of-the-envelope calculation for $K\pi$ gives

$$m_\kappa = 671 - i\,262\,\text{MeV}$$

$\Rightarrow$ Physics of σ and κ is very similar

Qualitative picture for κ, $f_0(980)$, ...

- Can also understand the κ pole in terms of F_π
- $f_0(980)$ and $a_0(980)$
  - Suspiciously close to $K\bar{K}$ threshold - an accident ?
  - Interaction among two kaons plays important role
- Multiplet pattern ?
  - The $\pi\pi$, πK, $K\bar{K}$ thresholds strongly break SU(3)

  $\Rightarrow$ Expect strong symmetry breaking in the masses and widths of the lowest 0^+ resonances
  - Do these form complete SU(3) multiplets at all ?