

The lowest resonance of QCD

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$\pi\pi$ interaction

- Plays a crucial role whenever the strong interaction is involved at low energies

Example: Standard model prediction for muon magnetic moment

- Main experiments on $\pi\pi$ scattering were done in the seventies. What's new ?

- Significant theoretical progress, based on χ PT + dispersion theory

- New precision data:

$K \rightarrow \pi\pi\ell\nu$	E865	Brookhaven
pionic atoms	DIRAC	CERN
$K \rightarrow 3\pi$	NA48/2	CERN

- Lattice results on $M_\pi, F_\pi, a_0^2, \langle r^2 \rangle_s$

Analyticity and crossing

- $\pi\pi$ scattering is special: crossed channels are identical
- ⇒ $\text{Re } T(s, t)$ can be represented as a twice subtracted dispersion integral over $\text{Im } T(s, t)$ in physical region

S.M. Roy 1971

- The 2 subtraction constants can be identified with the S -wave scattering lengths:

$$a_0^0, a_0^2 \begin{array}{l} \leftarrow \text{isospin} \\ \leftarrow \text{angular momentum} \end{array}$$

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*

Roy equations

- Pioneering work on the physics of the Roy equations: Basdevant, Froggatt & Petersen 1974
- Dispersion integrals converge rapidly (2 subtractions)
- ⇒ Crude phenomenological information on $\text{Im } T(s, t)$ for energies above 800 MeV suffices
- ⇒ Given a_0^0, a_0^2 , the scattering amplitude can be calculated to within small uncertainties

Ananthanarayan, Colangelo, Gasser & L. 2001

Descotes, Fuchs, Girlanda & Stern 2002

⇒ a_0^0, a_0^2 are the essential parameters at low energy

- Main problem in early work: a_0^0, a_0^2 poorly known
Experimental information near threshold is meagre

Low energy theorems

- Chiral perturbation theory provides the missing piece: theoretical prediction for a_0^0, a_0^2

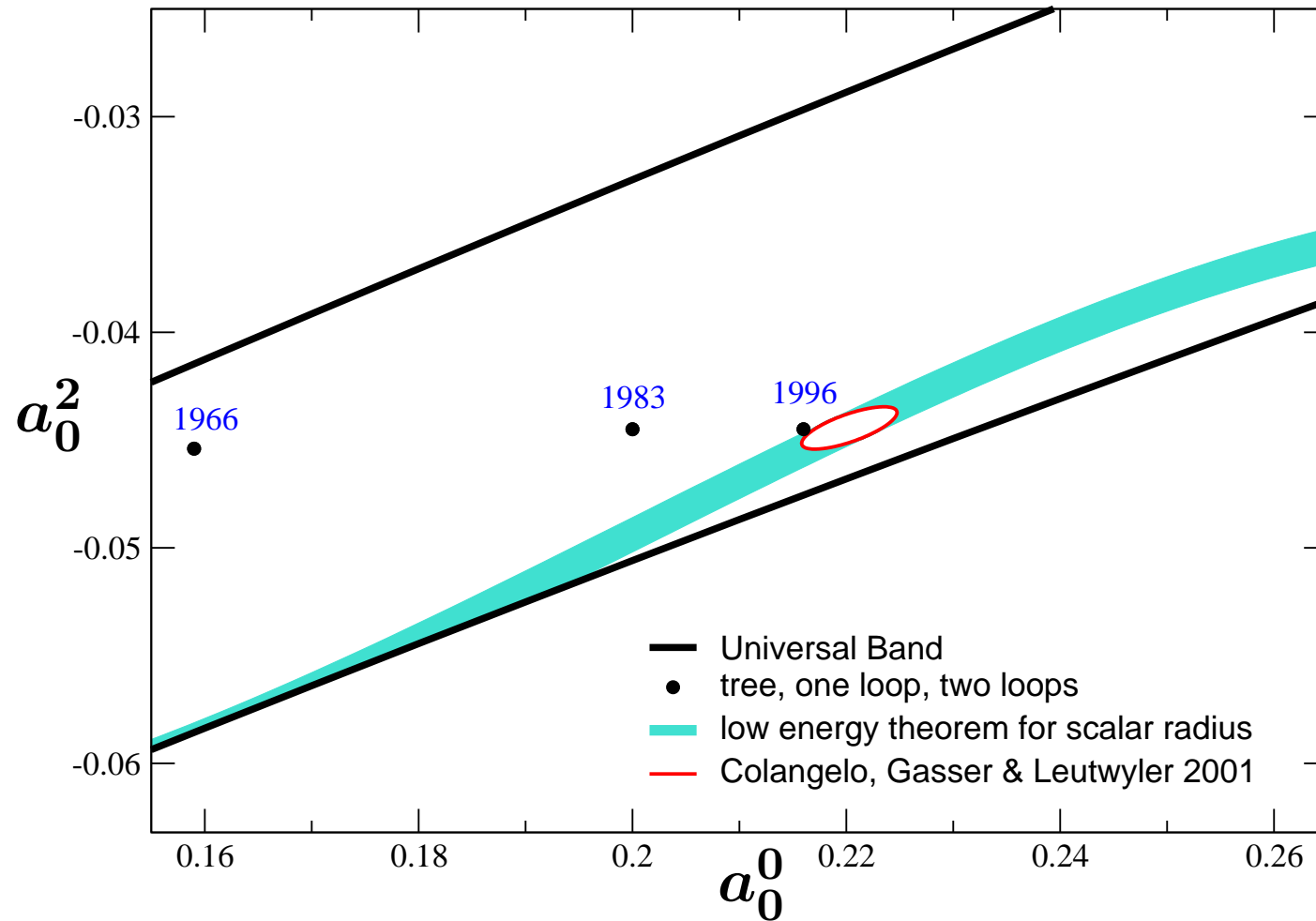
Weinberg 1966, Gasser & L. 1983, Bijmens, Colangelo, Ecker, Gasser & Sainio 1996

- Most accurate results for a_0^0, a_0^2 are obtained by matching the chiral and dispersive representations near the center of the Mandelstam triangle

Colangelo, Gasser & L. 2001

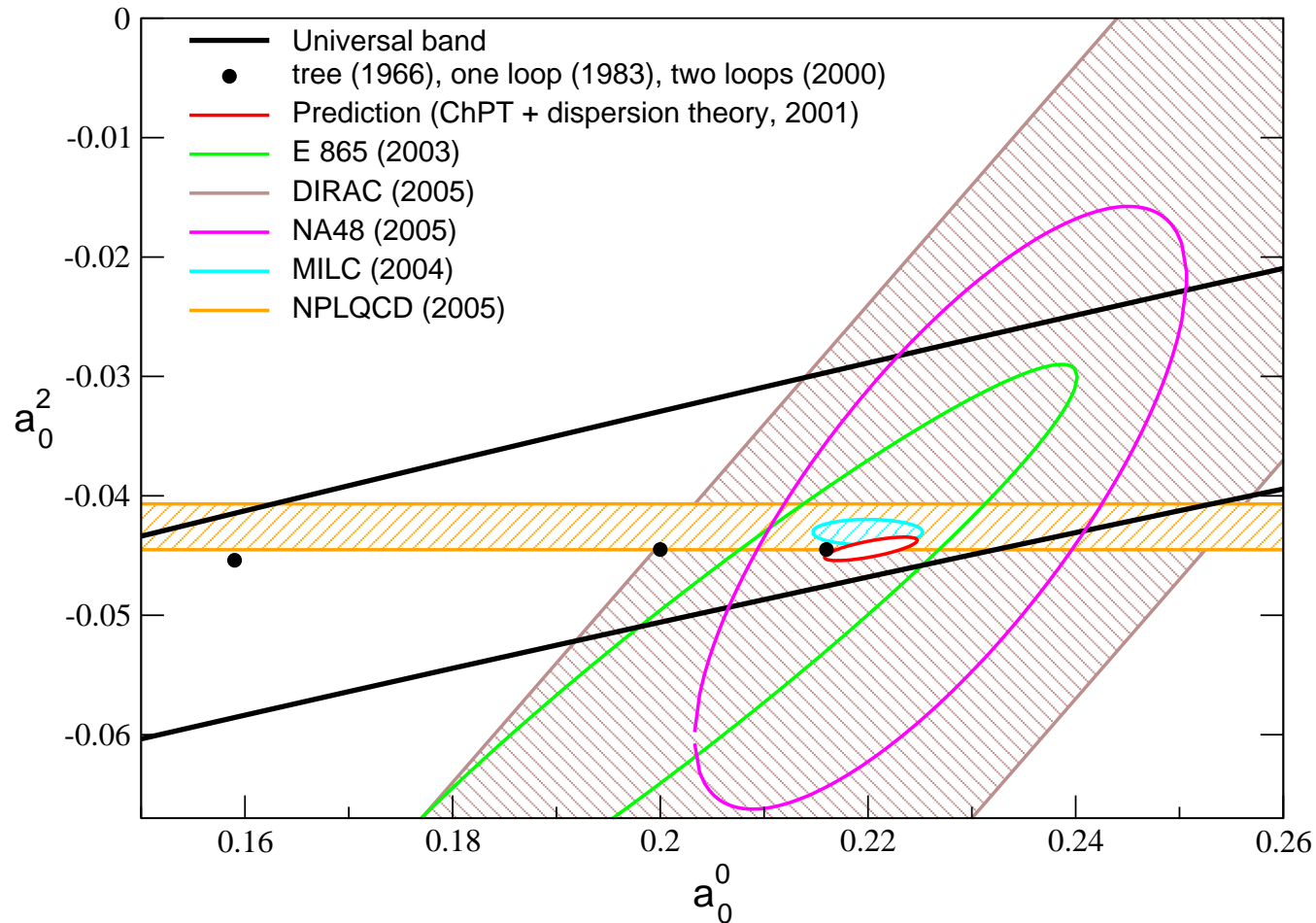
- In combination with the low energy theorems for a_0^0, a_0^2 , the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to an incredible degree of accuracy

Predictions for the S-wave $\pi\pi$ scattering lengths



Sizeable corrections in a_0^0 , while a_0^2 nearly stays put

Tests of the predictions for a_0^0 , a_0^2 : experiment & lattice



Theory is ahead of experiment ...



There is the broad object seen in $\pi\pi$ scattering, often called “background”, which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance² which we identify as the lightest glueball with quantum numbers $J^{PC} = 0^{++} \dots$

² we refer to it as **red dragon**

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

The red dragon

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001

- Does QCD have a resonance near threshold ?
- Why care ?
 - Concerns the nonperturbative domain of QCD
 - Quark and gluon degrees of freedom useless there
 - ⇒ Understanding very poor, pattern of energy levels ?
 - Lowest resonance: σ ? ρ ?
- Resonance \leftrightarrow pole on second sheet
 - Poles are universal
 - Pole position is unambiguous, even if width is large
 - Where is the pole closest to the origin ?

$f_0(600)$
or σ

$$I^G(J^{PC}) = 0^+(0^{++})$$

A REVIEW GOES HERE – Check our WWW List of Reviews

$f_0(600)$ T-MATRIX POLE \sqrt{s}

Note that $\Gamma \approx 2 \text{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(400–1200)–i(300–500) OUR ESTIMATE			
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●			
(541 ± 39)–i(252 ± 42)	1 ABLIKIM	04A BES2	$J/\psi \rightarrow \omega\pi^+\pi^-$
(528 ± 32)–i(207 ± 23)	2 GALLEGOS	04 RVUE	Compilation
(440 ± 8)–i(212 ± 15)	3 PELAEZ	04A RVUE	$\pi\pi \rightarrow \pi\pi$
(533 ± 25)–i(247 ± 25)	4 BUGG	03 RVUE	
532 – i272	BLACK	01 RVUE	$\pi^0\pi^0 \rightarrow \pi^0\pi^0$
(470 ± 30)–i(295 ± 20)	5 COLANGELO	01 RVUE	$\pi\pi \rightarrow \pi\pi$
(535 ⁺⁴⁸ _{–36})–i(155 ⁺⁷⁶ _{–53})	6 ISHIDA	01	$\Upsilon(3S) \rightarrow \Upsilon\pi\pi$
610 ± 14 – i620 ± 26	7 SUROVTSEV	01 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
(558 ⁺³⁴ _{–27})–i(196 ⁺³² _{–41})	ISHIDA	00B	$p\bar{p} \rightarrow \pi^0\pi^0\pi^0$
445 – i235	HANNAH	99 RVUE	π scalar form factor
(523 ± 12)–i(259 ± 7)	KAMINSKI	99 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}, \sigma\sigma$
442 – i 227	OLLER	99 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
469 – i203	OLLER	99B RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
445 – i221	OLLER	99C RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$
(1530 ⁺⁹⁰ _{–250})–i(560 ± 40)	ANISOVICH	98B RVUE	Compilation
420 – i 212	LOCHER	98 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
(602 ± 26)–i(196 ± 27)	8 ISHIDA	97	$\pi\pi \rightarrow \pi\pi$
(537 ± 20)–i(250 ± 17)	9 KAMINSKI	97B RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}, 4\pi$
470 – i250	10,11 TORNQVIST	96 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}, K\pi, \eta\pi$
~ (1100 – i300)	AMSLER	95B CBAR	$\bar{p}p \rightarrow 3\pi^0$
400 – i500	11,12 AMSLER	95D CBAR	$\bar{p}p \rightarrow 3\pi^0$
1100 – i137	11,13 AMSLER	95D CBAR	$\bar{p}p \rightarrow 3\pi^0$
387 – i305	11,14 JANSSEN	95 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
525 – i269	15 ACHASOV	94 RVUE	$\pi\pi \rightarrow \pi\pi$
(506 ± 10)–i(247 ± 3)	KAMINSKI	94 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
370 – i356	16 ZOU	94B RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
408 – i342	11,16 ZOU	93 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
870 – i370	11,17 AU	87 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
470 – i208	18 BEVEREN	86 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta, \dots$
(750 ± 50)–i(450 ± 50)	19 ESTABROOKS	79 RVUE	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
(660 ± 100)–i(320 ± 70)	PROTOPOP...	73 HBC	$\pi\pi \rightarrow \pi\pi, K\bar{K}$
650 – i370	20 BASDEVANT	72 RVUE	$\pi\pi \rightarrow \pi\pi$

Model independent determination of the pole

- All of the results quoted by the PDG are obtained by
 - (a) parametrizing the data for real values of s
 - (b) continuing this parametrization analytically in s

⇒ Result is sensitive to the parametrization used
- We found a model independent method:
 1. Poles on second sheet are zeros on first sheet
 2. The Roy equations are valid for complex values of s , in a limited region of the first sheet

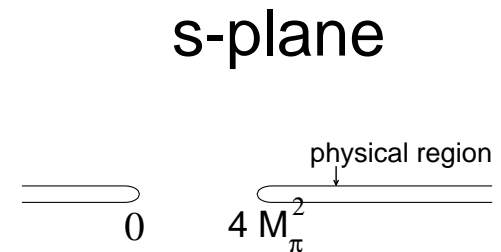
⇒ Exact representation of the partial waves in terms of observable quantities, valid for complex values of s

 3. Can evaluate this representation to good precision and determine the zeros numerically

Pole on second sheet \leftrightarrow zero on first sheet

- $S_0^0(s) = \eta_0^0(s) \exp 2i\delta_0^0(s)$

$S_0^0(s)$ is analytic in the cut plane



- For $0 < s < 4M_\pi^2$, $S_0^0(s)$ is real

$\Rightarrow S_0^0(s^*) = S_0^0(s)^*$

x in elastic interval: $S_0^0(x \pm i\epsilon) = \exp \pm 2i\delta_0^0(x)$

- Second sheet is reached by continuation across the elastic interval of the right hand cut

$$S_0^0(x - i\epsilon)^{II} = S_0^0(x + i\epsilon)^I = 1/S_0^0(x - i\epsilon)^I$$

Analyticity \Rightarrow $S_0^0(s)^{II} = 1/S_0^0(s)^I$ valid $\forall s$

Pole in $S_0^0(s)^{II} \iff$ zero in $S_0^0(s)^I$

Roy equation for the isoscalar S -wave

$$S_0^0(s) = 1 + 2i\rho t_0^0(s) \quad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2)b + \int_{4M_\pi^2}^{\infty} ds' \{ K_0(s, s') \text{Im} t_0^0(s') \\ + K_1(s, s') \text{Im} t_1^1(s') + K_2(s, s') \text{Im} t_2^2(s') \} \\ + \text{higher partial waves}$$

- The subtraction constants are determined by a_0^0, a_0^2 :

$$a = a_0^0, \quad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$$

- The kernels are elementary functions, e.g.

$$K_0(s, s') = \underbrace{\frac{1}{\pi(s' - s)}}_{r.h.cut} + \underbrace{\frac{2 \ln\{(s + s' - 4M_\pi^2)/s'\}}{3\pi(s - 4M_\pi^2)} - \frac{5s' + 2s - 16M_\pi^2}{3\pi s'(s' - 4M_\pi^2)}}_{l.h.cut}$$

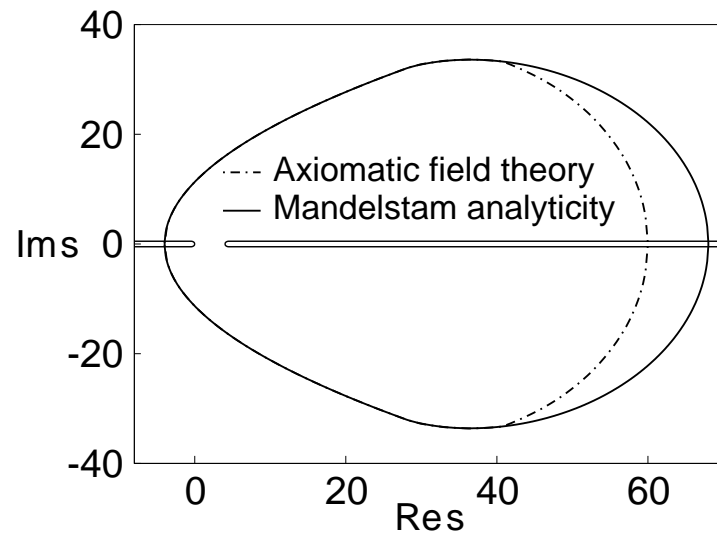
- Left hand cut is essential for convergence:

$$K_0(s, s') \sim 1/s'^3 \text{ for large } s'$$

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$
- Equations are valid for complex s in a limited region of the first sheet

I. Caprini, G. Colangelo and H. Leutwyler,
Phys. Rev. Lett. 96 (2006) 132001



- Proof is based on first principles, general quantum field theory

A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing*, Lecture Notes in Physics, vol. 3, 1969.

G. Mahoux, S. M. Roy and G. Wanders,
Nucl. Phys. B 70 (1974) 297.

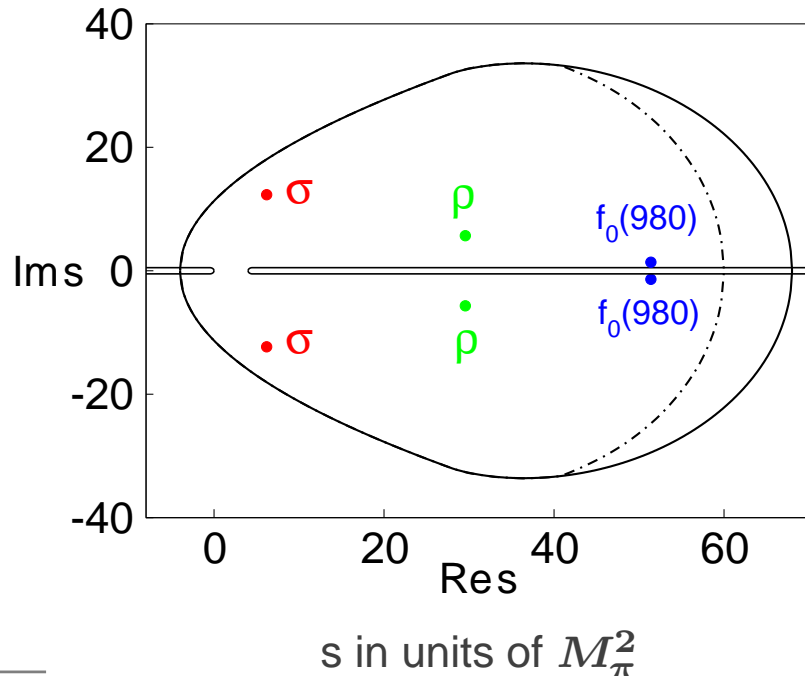
⇒ Exact representation for $S_0^0(s)$ in this region
Do not need to parametrize the amplitude

Evaluation of the pole position

- Insert our solutions of the Roy equations
For the central solution, $S_0^0(s)$ has two pairs of zeros in the region of validity of the representation:

$$s = (6.2 \pm i 12.3) M_\pi^2 \quad \sigma$$

$$s = (51.4 \pm i 1.4) M_\pi^2 \quad f_0(980)$$



- ⇒ 1. Lowest resonance of QCD has vacuum quantum numbers
2. Pole on lower half of sheet II occurs in vicinity of

$$m_\sigma = 441 - i 272 \text{ MeV}$$

$$= M_\sigma - \frac{i}{2} \Gamma_\sigma$$

Error analysis

- Results depend on phenomenological input used when solving the Roy equations, subject to uncertainties
Can follow error propagation explicitly
- Pole position of $f_0(980)$ sensitive to input used for $\eta_0^0(s)$
- Pole position of σ mainly depends on 3 input variables:
$$a_0^0, a_0^2, \delta_A \equiv \delta_0^0(800 \text{ MeV})$$
 - Information about a_0^0, a_0^2 is in good shape
 - Substantial uncertainties in phenomenology of δ_A
 - Use conservative range: $\delta_A = 82.3^\circ \begin{smallmatrix} +10^\circ \\ -4^\circ \end{smallmatrix}$

Error analysis

- Noise from remaining input variables is very small:

$$m_\sigma = (441 \pm 4) - i(272 \pm 6) \text{ MeV}$$

but the values of a_0^0 , a_0^2 , δ_A are crucial:

$$\begin{aligned} m_\sigma &= (441 \pm 4) - i(272 \pm 6) \\ &+ (-2.4 + i 3.8) \frac{a_0^0 - 0.22}{0.005} \\ &+ (0.8 - i 4.0) \frac{a_0^2 + 0.0444}{0.001} \\ &+ (5.3 + i 3.3) \frac{\delta_A - 82.3}{3.4} \end{aligned} \quad \text{numbers in MeV}$$

- Final result: insert the predictions for a_0^0 , a_0^2 , use the phenomenological range for δ_A and add errors up:

$$m_\sigma = 441 \begin{matrix} +16 \\ -8 \end{matrix} - i 272 \begin{matrix} +9 \\ -13 \end{matrix} \text{ MeV}$$

Curvature due to the left hand cut

- Left hand cut generates curvature
Main contribution on the left stems from the ρ
- Most pole determinations neglect the left hand cut
Pole from σ is too close for this to be justified
- Can estimate contributions from left hand cut with χ PT

Z.Y. Zhou, G.Y. Qin, P. Zhang, Z.G. Xiao, H.Q. Zheng, N. Wu, JHEP 0502 (2005) 043

Estimate is crude \Rightarrow sizeable uncertainties

Outcome for pole position agrees with our result

Calculate pole position from phenomenology

- Ignore the representation of the scattering amplitude obtained from the Roy equations

- Instead use a phenomenological one

J. R. Peláez and F. J. Ynduráin Phys. Rev. D71 (2005) 074016 ← PY
(improved representation for energies above 1 GeV:

R. Kaminski, J. R. Peláez and F. J. Ynduráin, hep-ph/0603170)

- Insert it in formula for $S_0^0(s)$ and calculate the zeros
With the central values of PY, this gives

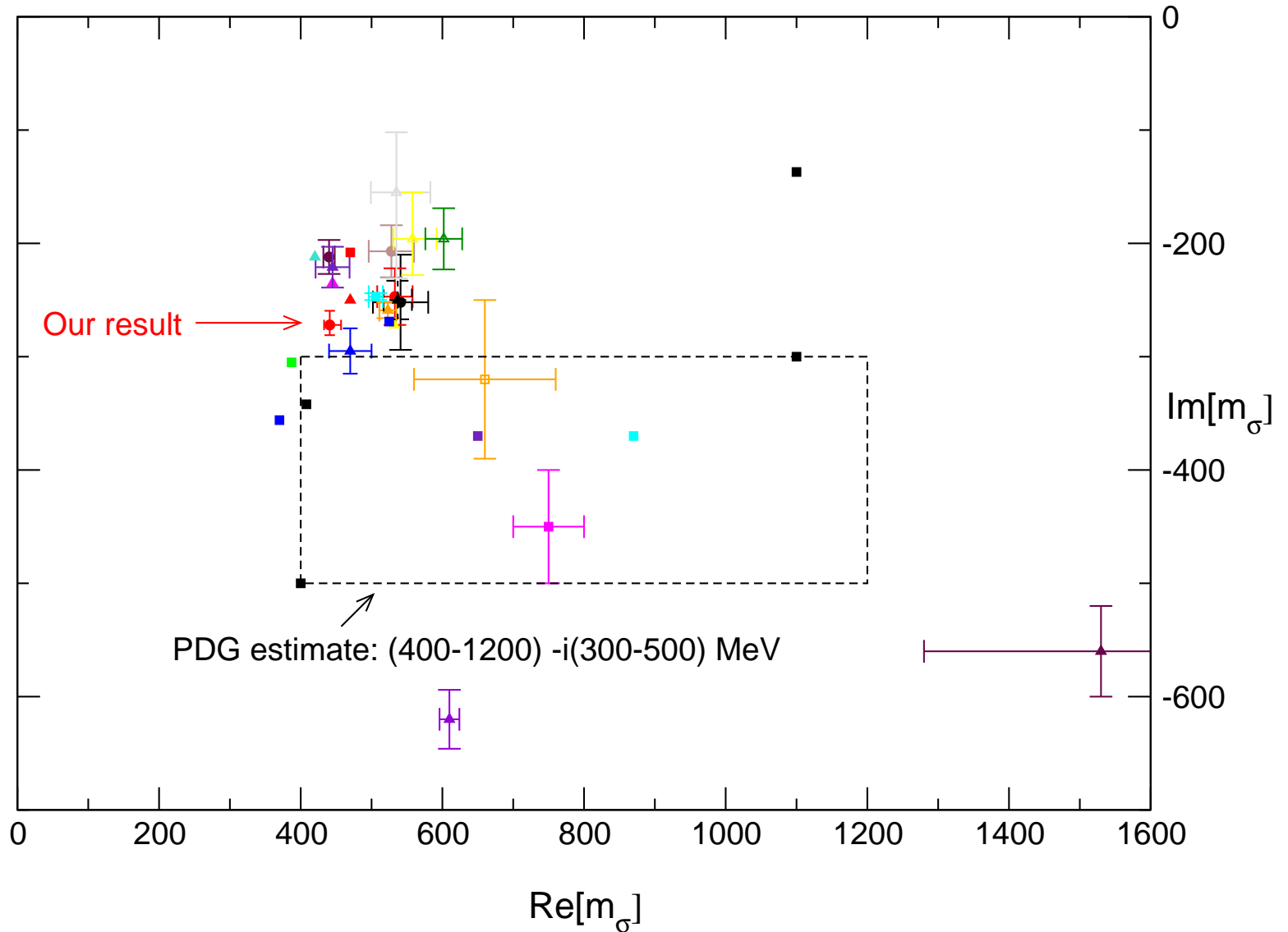
$$m_\sigma = 445 - i 241 \text{ MeV}$$

- Uncertainties in phenomenology are large
Those in a_0^0 , a_0^2 alone give

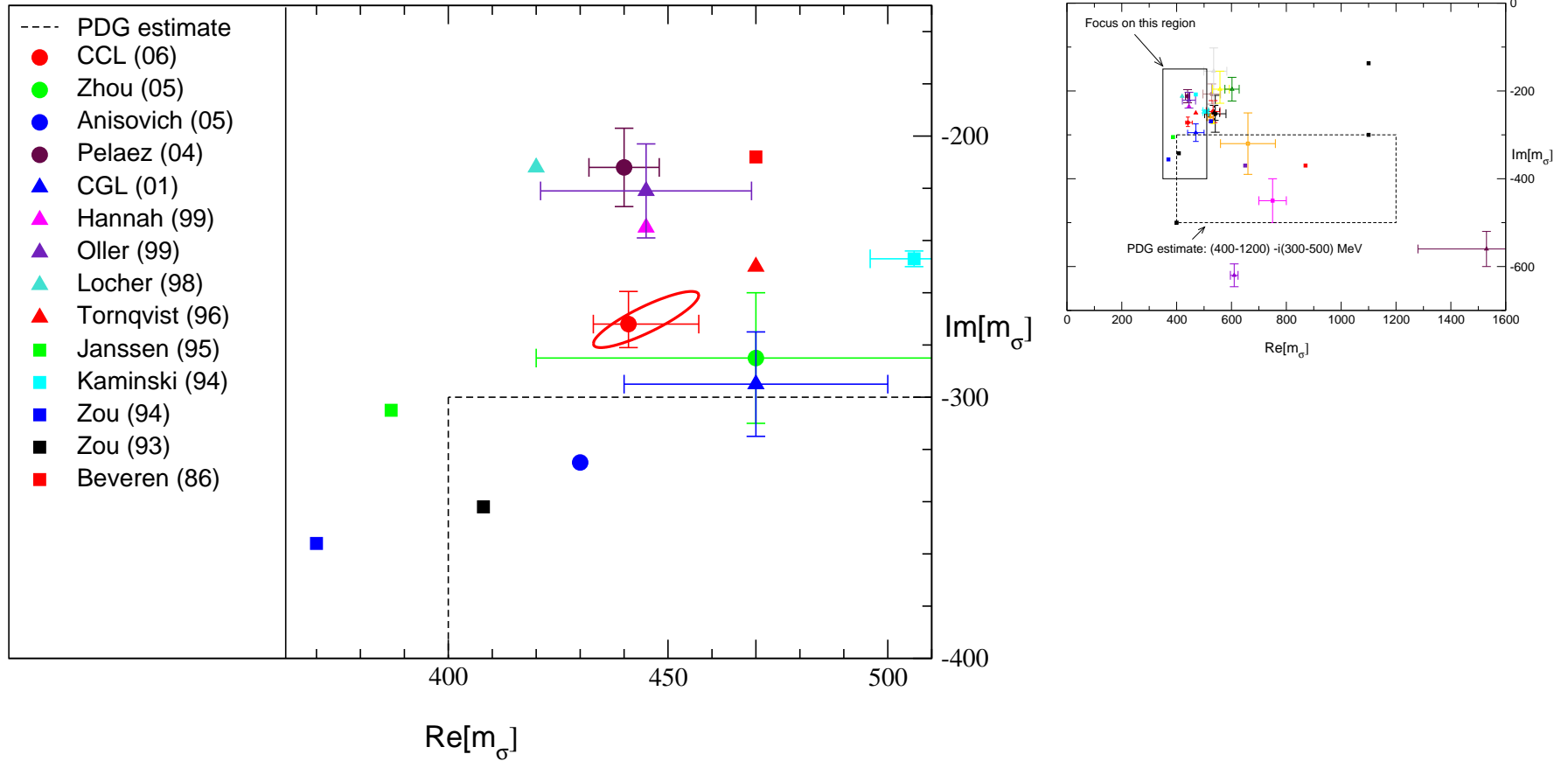
$$m_\sigma = (445 \pm 8) - i(241 \pm 22) \text{ MeV}$$

- ⇒ Calculation confirms our result, but errors are larger

Comparison with compilation of PDG



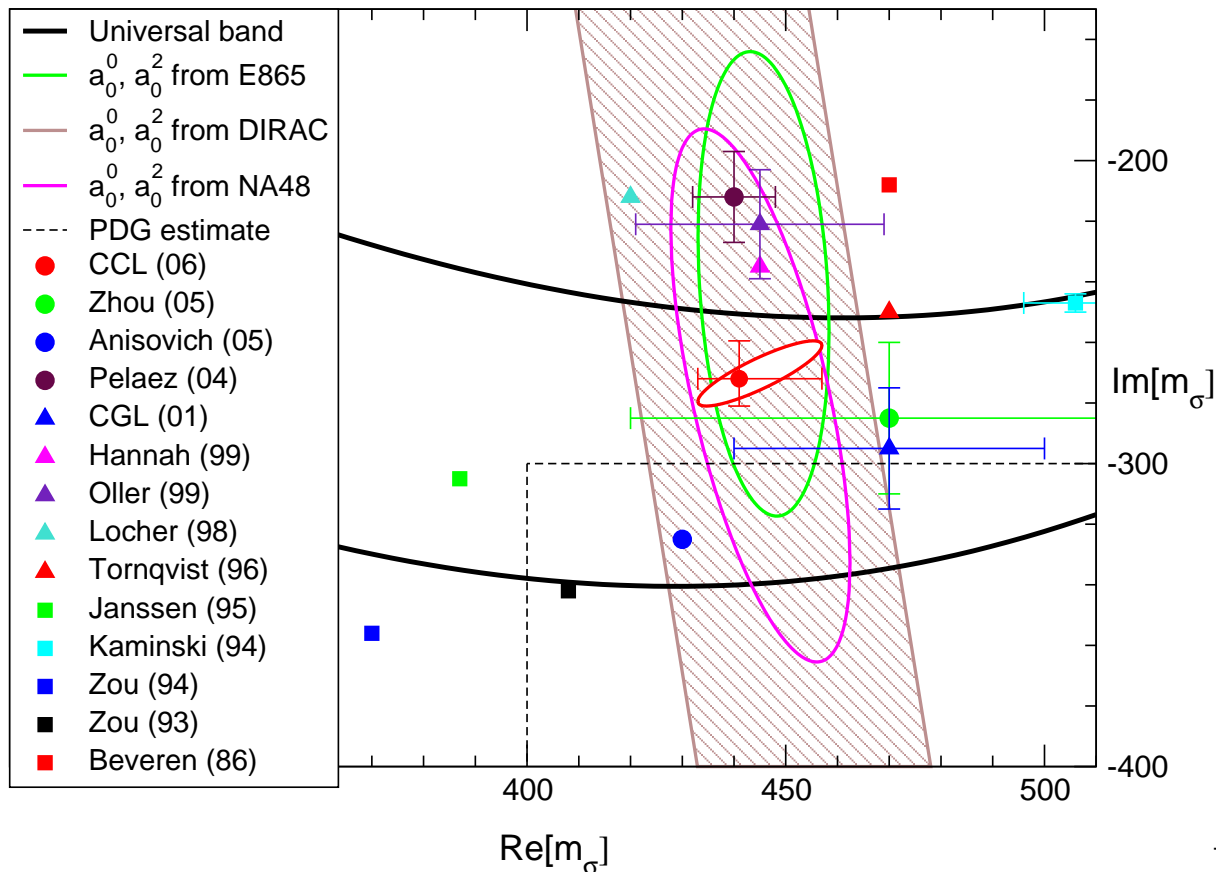
Vicinity of the pole



Results for $Re[m_\sigma]$ and $Im[m_\sigma]$ are strongly correlated

Ignore the theoretical predictions for a_0^0, a_0^2

- Replace the low energy theorems for a_0^0, a_0^2 by the experimental results from E865, DIRAC and NA48
- $a_0^0, a_0^2 \in$ universal band



Why are our errors so incredibly small ?

- The σ occurs at low energies
- At low energies, the subtraction term dominates

$$t_0^0(s) \simeq a_0^0 + (2a_0^0 - 5a_0^2) \frac{(s - 4M_\pi^2)}{12M_\pi^2}$$

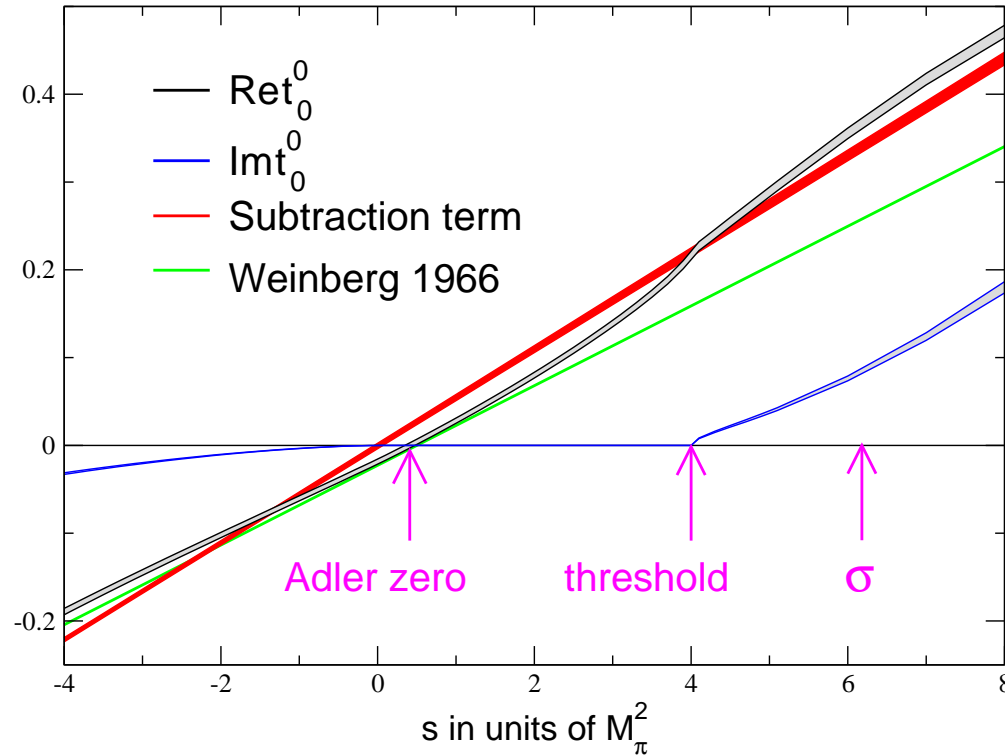
Insert low energy theorem for a_0^0, a_0^2

⇒ Roy equation reduces to Weinberg formula

$$t_0^0(s) \simeq \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Dispersion integrals only represent a correction

At low energies, the subtraction term dominates



$$s = (0.41 \pm 0.06) M_\pi^2 \quad \text{Adler zero}$$

$$s = (6.2 - i 12.3) M_\pi^2 \quad \text{pole from } \sigma$$

At low energies, Goldstone bosons interact only weakly

Estimate pole position on back of an envelope

- Approximate $t_0^0(s)$ with the Weinberg formula

$$t_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Where are the zeros of $S_0^0(s)$ in this approximation ?

$$1 + 2i \sqrt{1 - 4M_\pi^2/s} t_0^0(s) = 0$$

⇒ Cubic equation for s

- Pair of complex zeros, $m_\sigma = 365 - i 291$ MeV
- Correction from higher orders amounts to

$$\Delta m_\sigma = 76 \begin{matrix} +16 \\ -8 \end{matrix} + i 19 \begin{matrix} +9 \\ -13 \end{matrix} \text{ MeV}$$

For the quantity that counts, the accuracy is modest

- Real zero on sheet II, near $s = 0$ (full amplitude has kinematic singularity: vanishes on sheet II at $s = 0$)

Physical interpretation of the σ

- The head of the dragon is not made of glue
- The dragon likes flavoured food, pions in particular
Markushin and Locher 1999
 - ⇒ Physics of the $\sigma \in$ Goldstone boson dynamics
 - ⇒ Wave function has large tetraquark component
Jaffe 1977
- Physics of the $f_0(980) \in$ Goldstone boson dynamics
The interaction among two kaons is relevant
- Physics of the $\kappa \in$ Goldstone boson dynamics ?
 - ⇒ Roy-Steiner equations for $K\pi$ scattering
Büttiker, Descotes, Moussallam 2004

Conclusion

- Low energy pion physics: theory ahead of experiment
 - Precision experiments carried out and under way
 - Lattice makes slow, but steady progress
 - So far, all tests confirm the theory
- Limitations of our approach:
 - Calculations cannot be done on back of an envelope
 - Method only covers low energies
 - Only a few applications have been worked out:
 $\pi\pi$ scattering, pion form factors, hadronic vacuum polarization in SM prediction for muon $g - 2$
 $\gamma\gamma \rightarrow \pi^0\pi^0$ M. Pennington, hep-ph/0604212
- Much is yet to be done: $J/\psi \rightarrow \omega\pi\pi$, $D \rightarrow 3\pi$, ...
 πK , κ , ...

Conclusion

- Model independent method for analytic continuation
 - The lowest resonance of QCD occurs at
$$M_\sigma = 441 \begin{matrix} +16 \\ -8 \end{matrix} \text{ MeV} \quad \Gamma_\sigma = 544 \begin{matrix} +18 \\ -25 \end{matrix} \text{ MeV}$$
and carries vacuum quantum numbers
 - Crossing symmetry plays an essential role:
Fixes contributions from left hand cut
Ensures fast convergence, low energy dominance
 - Pole occurs at low value of s , closer to left hand cut than to singularities from $K\bar{K}$, $f_0(980)$
 - Result for Γ_σ relies on theory for a_0^2
Experiments concerning a_0^2 would be most welcome



VISIT THE RED DRAGON

GENTLE ANIMAL
LOOK IN HIS EYES FROM CLOSE
SMELL HIS GOOD BREATH
BRING YOUR PIONS ALONG AND
FEED HIM YOURSELF

The management denies responsibility for incidents involving the dragon's tail