Recent developments in light flavor hadron physics

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Light flavour hadrons

- Main characteristic: small energy gap, $M_\pi \simeq 140$ MeV
- Hidden, approximate symmetry
- Symmetry becomes exact for $m_u, m_d \to 0$

$\Rightarrow$ Energy gap disappears: pions become massless

- In reality $m_u, m_d \neq 0$, but small

$\Rightarrow$ Symmetry is nearly perfect

- The state of lowest energy is not symmetric   Nambu 1961

$\Rightarrow$ Chiral symmetry is hidden, “spontaneously broken”

- Very strong experimental evidence $\surd$
  Very strong evidence from lattice calculations $\surd$
  Analytic understanding of the ground state still poor

Quarks masses as perturbations

- Masses of u, d enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d\}$$

  H_0 describes u, d as massless, s, c, b, t as massive

  H_0 is invariant under $\text{SU(2)}_{\text{L}} \times \text{SU(2)}_{\text{R}}$

- H_0 treats the pions as massless particles

  H_1 gives them a mass

Expansion in powers of m_u, m_d	$\Longleftrightarrow$	Perturbation series in powers of H_1

Gell-Mann-Oakes-Renner formula

- First order perturbation theory yields:

$$M_\pi^2 = (m_u + m_d) \times |\langle 0|\,\bar{u}u\,|0\rangle| \times \frac{1}{F_\pi^2}$$

$$\Uparrow \text{ explicit} \qquad \Uparrow \text{ spontaneous}$$

Gell-Mann, Oakes & Renner 1968

Coefficient: decay constant F_π

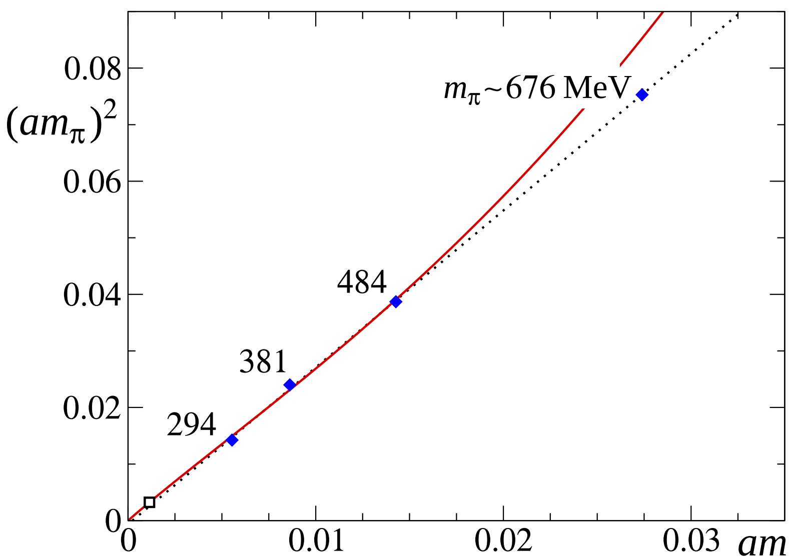
$$\langle 0|\,\bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\,p^\mu\sqrt{2}\,F_\pi$$

Value of F_π is known from $\pi^+ \rightarrow \mu^+\nu$

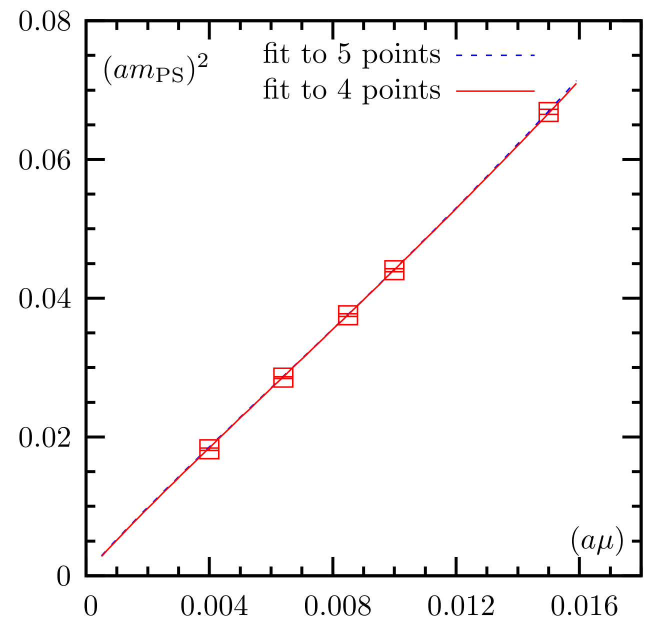
$\Rightarrow$ The pattern of the first few energy levels of QCD can be understood on the basis of this formula

Lattice results for M_π

- GMOR formula can now be checked on the lattice: determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- No quenching, quark masses are sufficiently light

$\Rightarrow$ legitimate to use χPT for the extrapolation to the physical values of m_u, m_d

Lattice

- Quality of data is impressive
- Proportionality of M_π^2 to the quark mass appears to hold out to values of m_u, m_d that are an order of magnitude larger than in nature
- Main limitation: systematic uncertainties
in particular: $N_f = 2 \rightarrow N_f = 3$

Expansion of M_π^2 in powers of the quark mass

- Consequences of hidden, approximate symmetry can be worked out by means of an effective field theory

Weinberg 1979

- GMOR formula represents leading term of χPT
- At NLO, the expansion contains a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

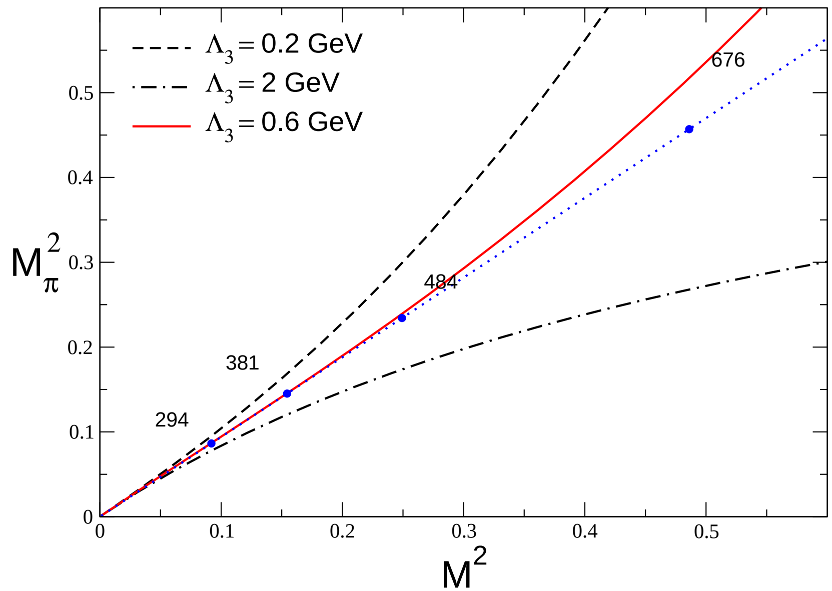
$$M^2 \equiv B(m_u + m_d)$$

Gasser & L. 1983

- Coefficient is determined by pion decay constant
Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3)×SU(3):
0.2 GeV $\lesssim \Lambda_3 \lesssim 2$ GeV

Gasser & L. 1984

Lattice allows more accurate determination of Λ_3

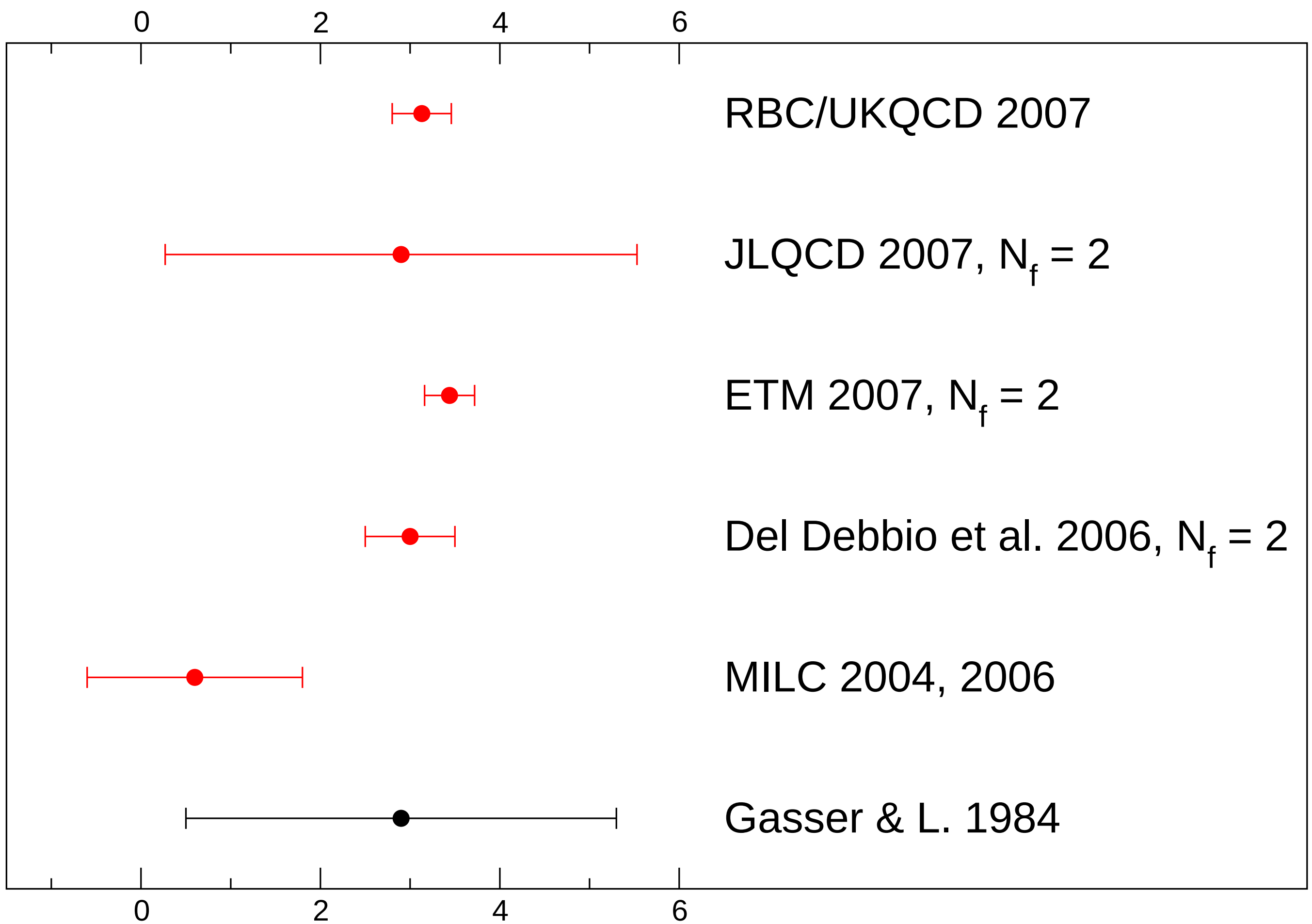


Express the result for Λ_3 in terms of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3{}^2}{M_\pi^2}$

$\bar{\ell}_3 = 2.9 \pm 2.4 \;\leftrightarrow\; 0.2\,\text{GeV} \lesssim \Lambda_3 \lesssim 2\,\text{GeV}$ Gasser & L. 1984

$\bar{\ell}_3 = 3.0 \pm 0.5 \;\leftrightarrow\; 0.5\,\text{GeV} \lesssim \Lambda_3 \lesssim 0.8\,\text{GeV}$ Del Debbio et al. 2006

Lattice results for Λ_3



$$\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

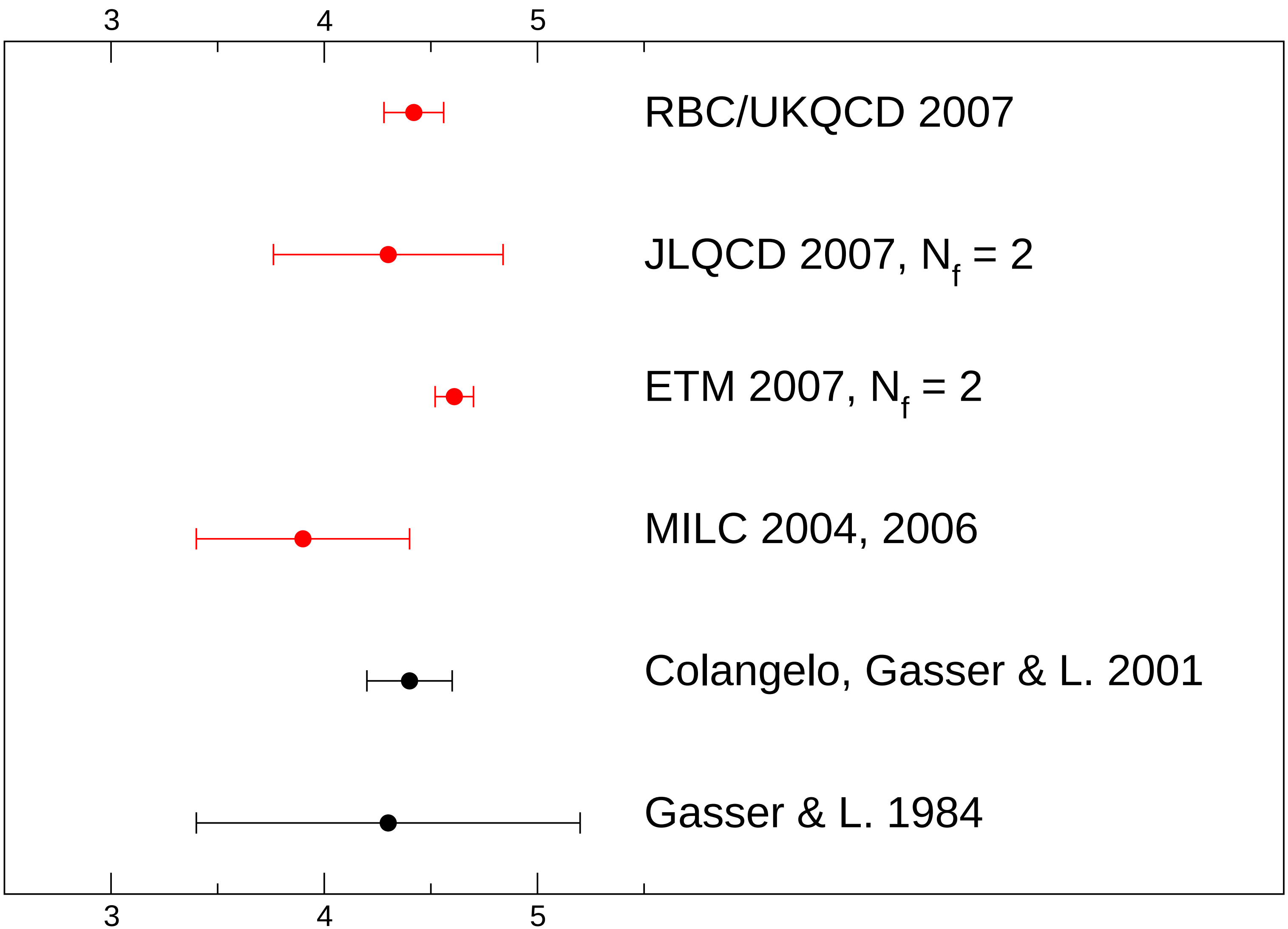
$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

F is value of pion decay constant in limit $m_u, m_d \to 0$

- Structure is the same, coefficients and scale of logarithm are different
- Quark mass dependence of F_π can also be measured on the lattice $\Rightarrow$ measurement of Λ_4
- Alternative method: determine the scalar form factor of the pion, radius $\langle r^2 \rangle_s \Leftrightarrow \bar{\ell}_4$ Hashimoto, Simula

Lattice results for Λ_4



$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

NNLO

- The next order contains the square of a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{x}{2} \ln \frac{M^2}{\Lambda_3^2} + \frac{17x^2}{8} \left(\ln \frac{M^2}{\Lambda_M^2} \right)^2 + x^2 k_M + O(M^6) \right\}$$

$$F_\pi = F \left\{ 1 - x \ln \frac{M^2}{\Lambda_4^2} - \frac{5x^2}{4} \left(\ln \frac{M^2}{\Lambda_F^2} \right)^2 + x^2 k_F + O(M^6) \right\}$$

$$x \equiv \left(\frac{M}{4\pi F} \right)^2$$

Colangelo 1995, Bijnens et al. 1996, Bürgi 1996

- For physical value of m_u, m_d, the NNLO terms are tiny

$\Rightarrow$ Size of $\Lambda_M, k_M, \Lambda_F, k_F$ barely known

- Must become clearly visible if m_u, m_d are made larger

$\pi\pi$ interaction

- Plays a crucial role whenever the strong interaction is involved at low energies

  Example: Standard model prediction for muon magnetic moment

- The interaction among the pions is largely determined by the hidden symmetry

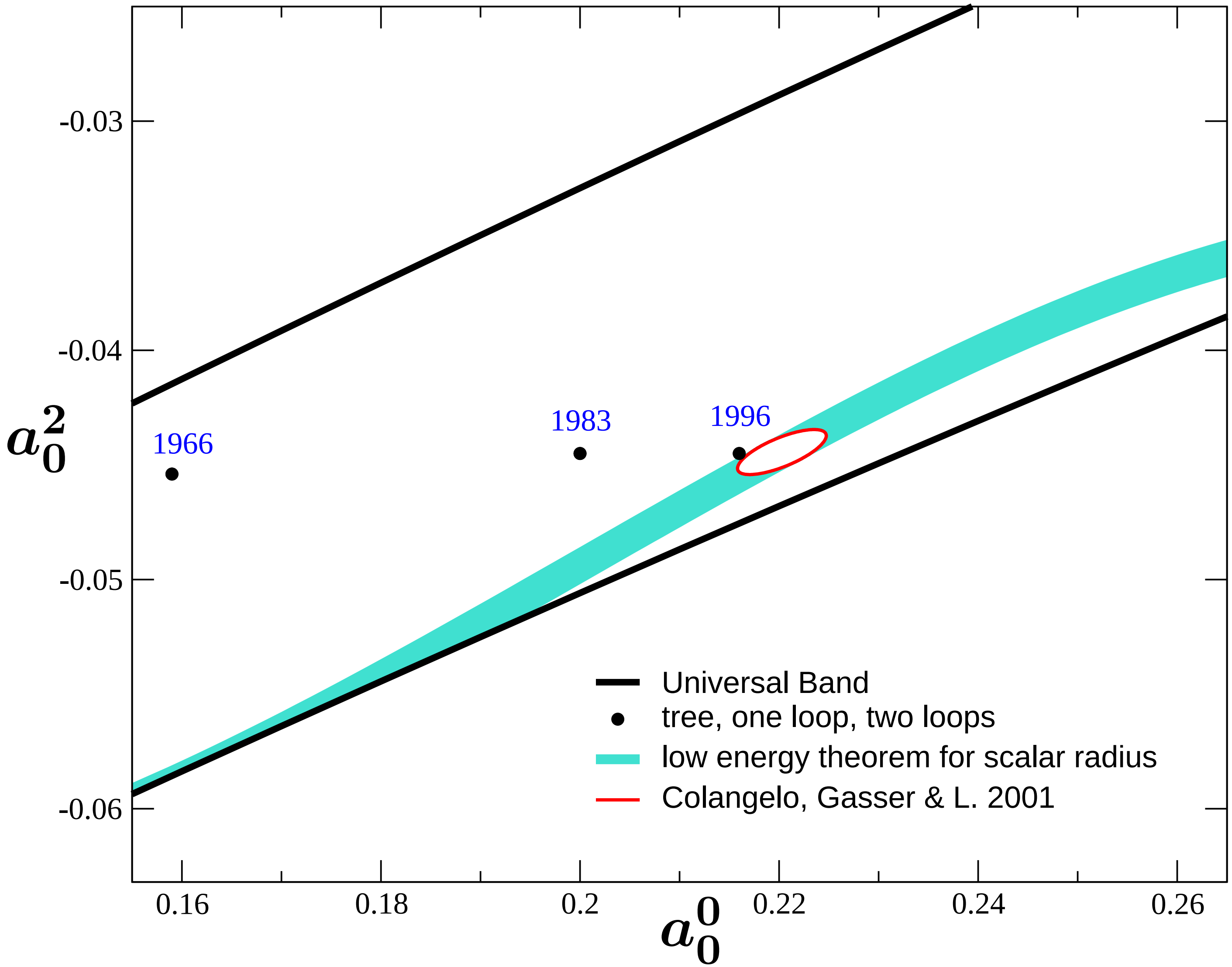
- Chiral perturbation theory makes very sharp predictions for the S-wave scattering lengths a_0^0, a_0^2

  Weinberg 1966, Gasser & L. 1983, Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

$\Rightarrow$ Subtraction constants in dispersion relations

$\Rightarrow$ Below 1 GeV, the dispersion relations for the partial waves (Roy equations) fix the scattering amplitude to an incredible degree of accuracy Colangelo, Gasser & L. 2001

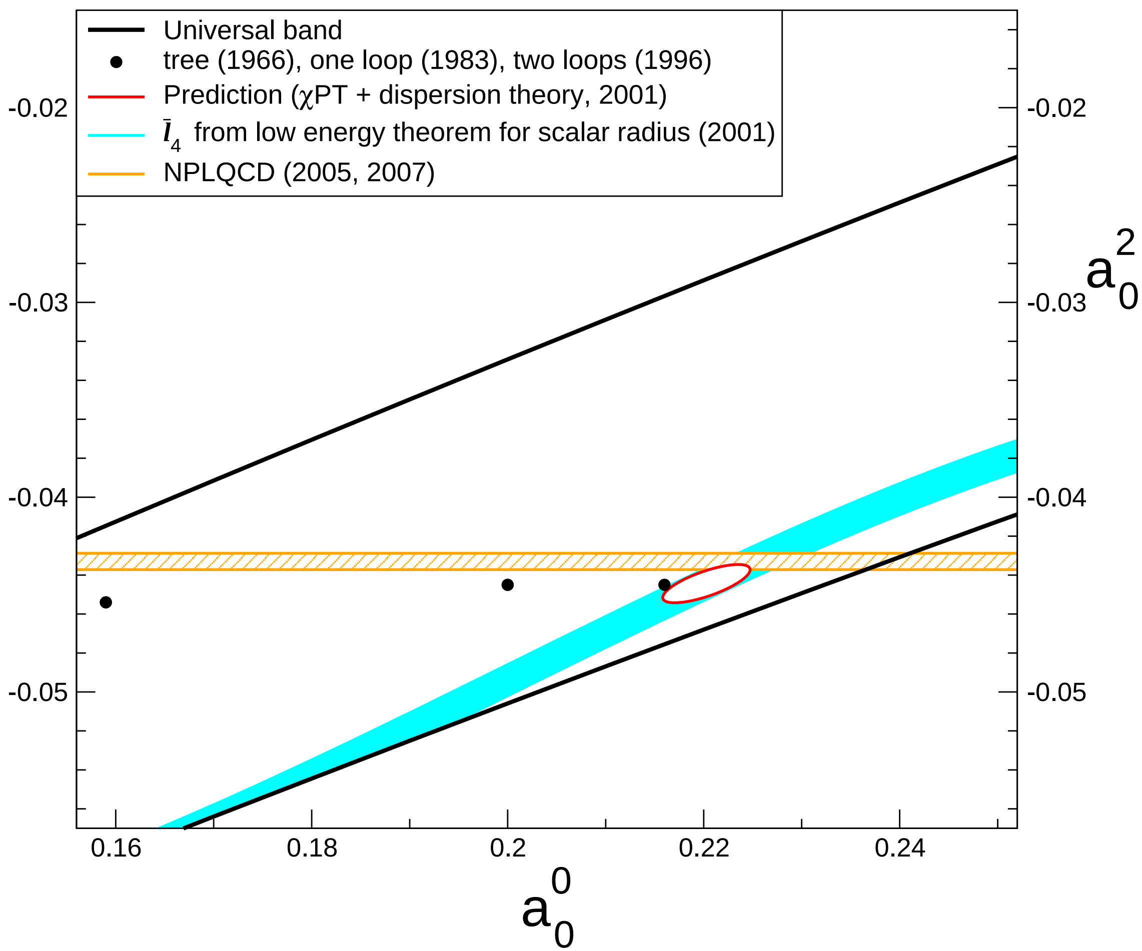
Predictions for the S-wave $\pi\pi$ scattering lengths



Sizeable corrections in a_0^0, while a_0^2 nearly stays put

Lattice result for a_0^2

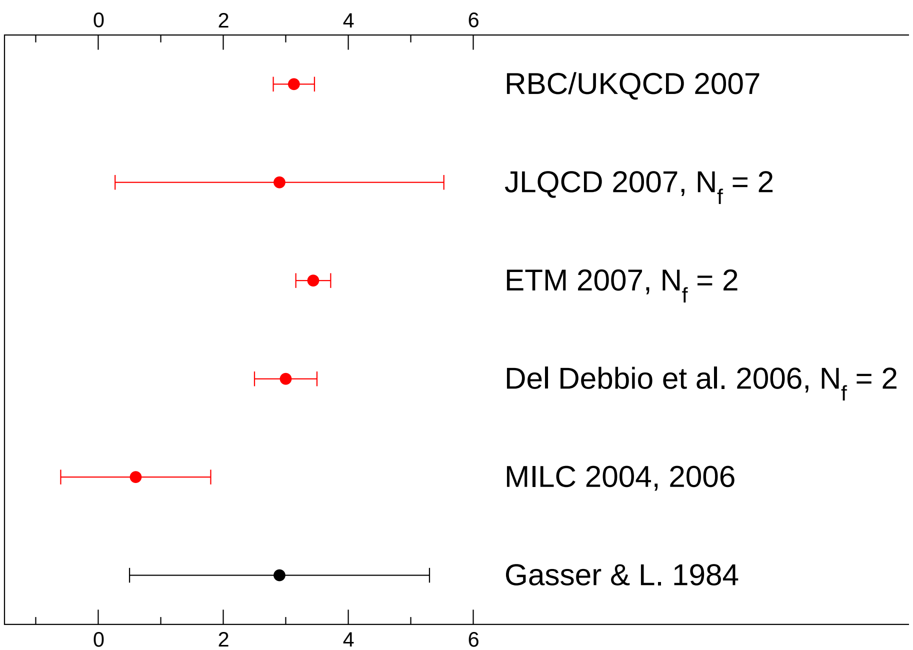
- Lattice allows direct measurement of a_0^2 via volume dependence of energy levels

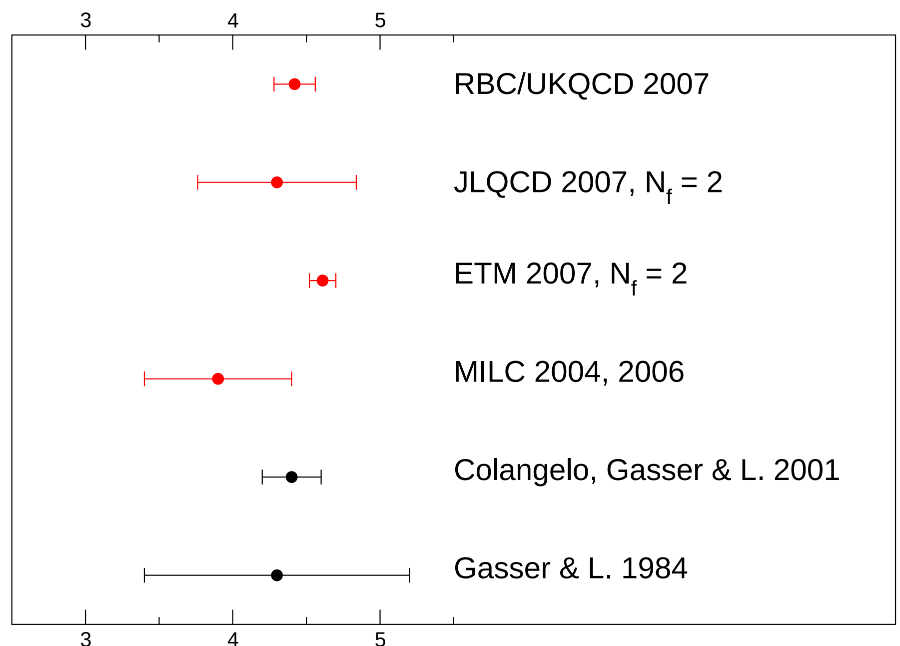


Can use lattice results for ℓ_3, ℓ_4

- Uncertainty in prediction for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4

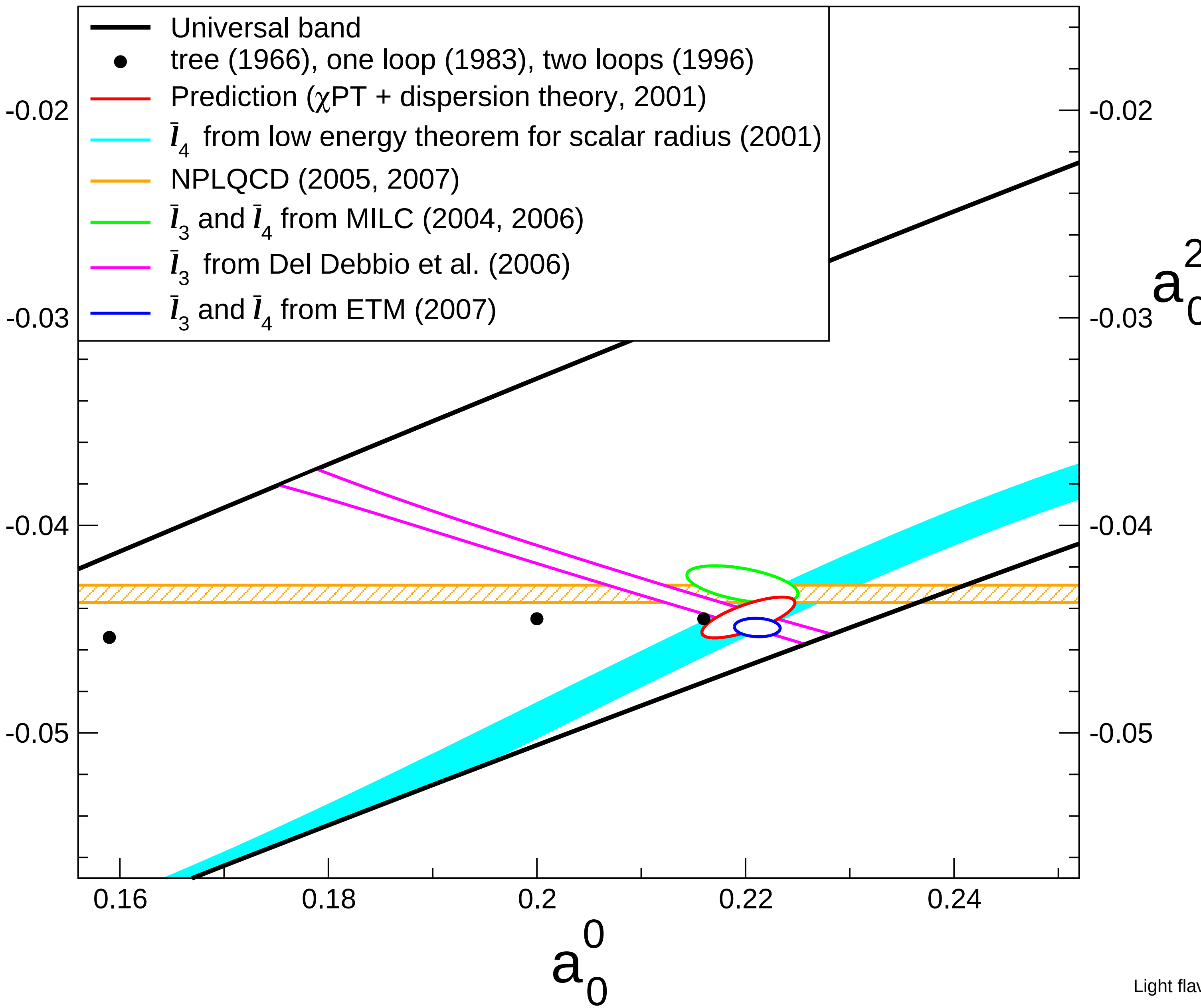
$\Rightarrow$ Can make use of the lattice results for these





$$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M_\pi^2} \qquad \bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

Calculate a_0^0, a_0^2 from lattice results for ℓ_3, ℓ_4



Experiments on light flavours at low energy

- Production experiments $\pi N \to \pi\pi N$, $\psi \to \pi\pi\omega$ ...

  Problem: pions are not produced in vacuo

$\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple

  Accuracy rather limited

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- $\pi^+\pi^-$ atoms, DIRAC
- $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$ precision data from E865, NA48/2

Pionic atoms

- $\pi^+\pi^-$ atoms provide an ideal laboratory
- Decay through the strong interaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$
  Decay rate $\propto (a_0^0 - a_0^2)^2$
- Interference of e.m. and strong interactions in bound state and decay is well understood

$\Rightarrow$ Can reliably measure low energy properties of the $\pi\pi$ scattering amplitude in this way

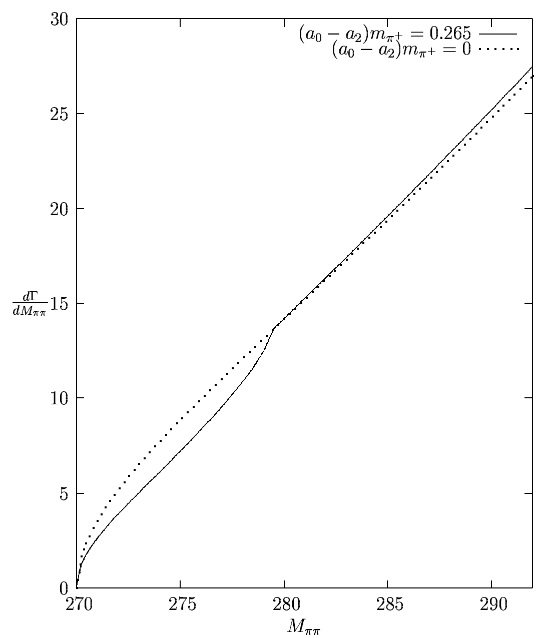
- Prediction for the lifetime: $\tau = 2.9 \pm 0.1$ fs

Gasser, Lyubovitskij, Rusetsky & Gall 2001

- Experimental result: $\tau = 2.91\,^{+0.49}_{-0.62}$ fs   DIRAC 2005
- Experiment on πK-atoms is under way $\Rightarrow$ fabulous tool to explore strange quarks at low energy

Cusp in $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$

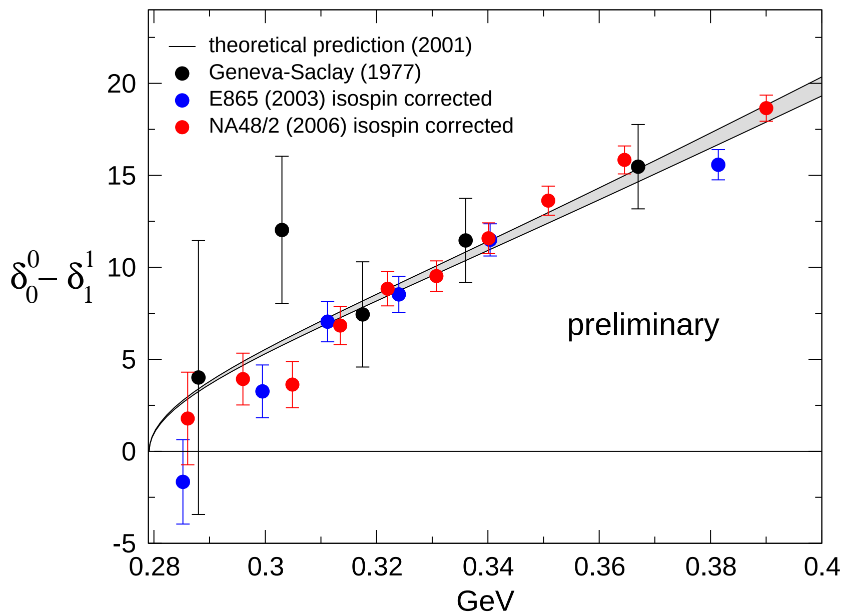
- Accurate data in the threshold region of the decay $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ allow a determination of $a_0^0 - a_0^2$
- NA48/2 has collected $\sim 10^8$ decays in this channel !



taken from N. Cabibbo, hep-ph/0405001

K_{e4} decay

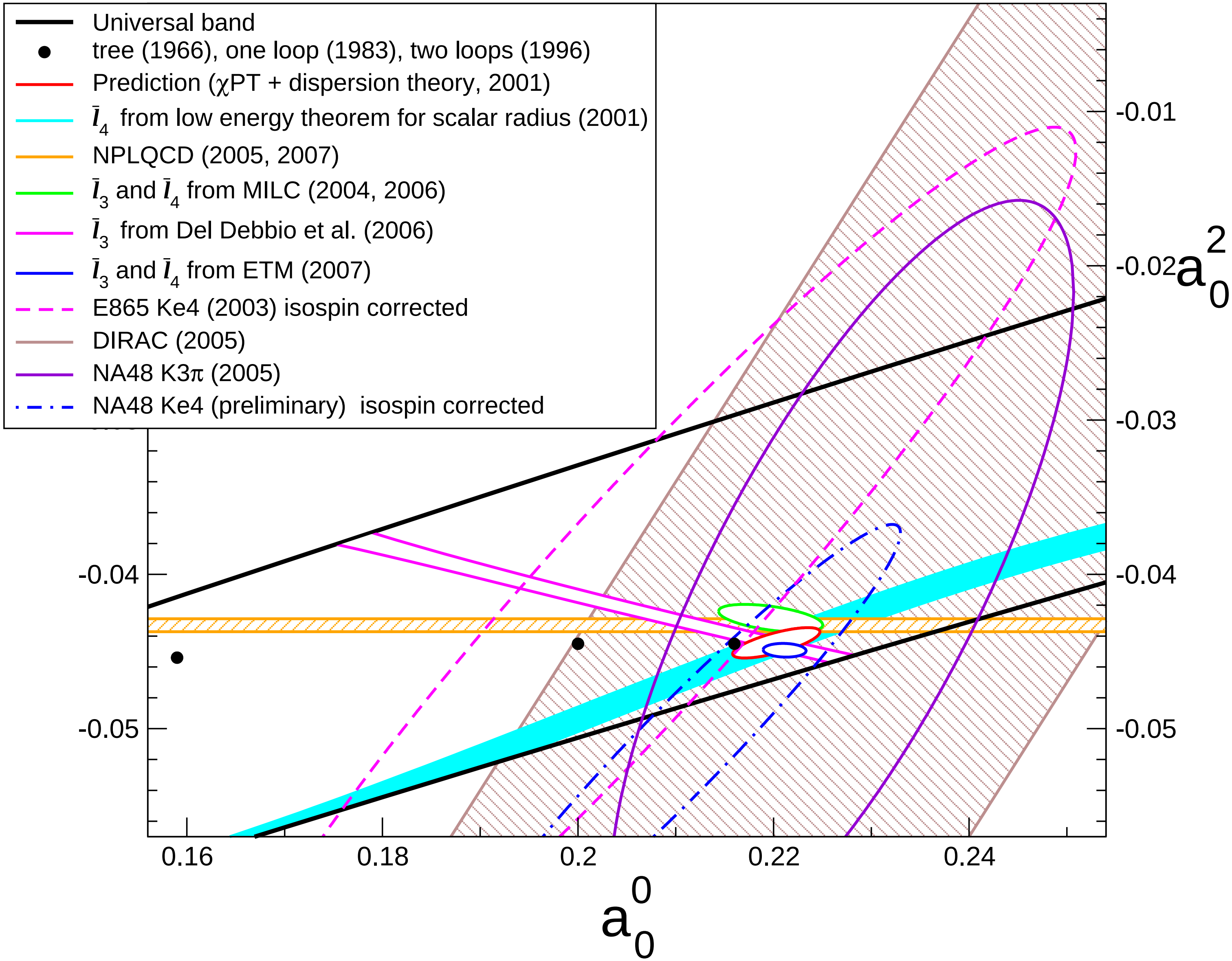
- $K \to \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Brigitte Bloch-Devaux 2007

Tests of the predictions for a_0^0, a_0^2: experiment and lattice



Puzzling results for $K_L \to \pi\mu\nu$

SEARCHING FOR THE ‘TOTALLY UNEXPECTED’
IN THE LHC ERA

- Hadronic matrix element of weak current:

$$\langle K^0|\bar{u}\gamma^\mu s|\pi^-\rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

- Scalar form factor $\sim \langle K^0|\partial_\mu(\bar{u}\gamma^\mu s)|\pi^-\rangle$

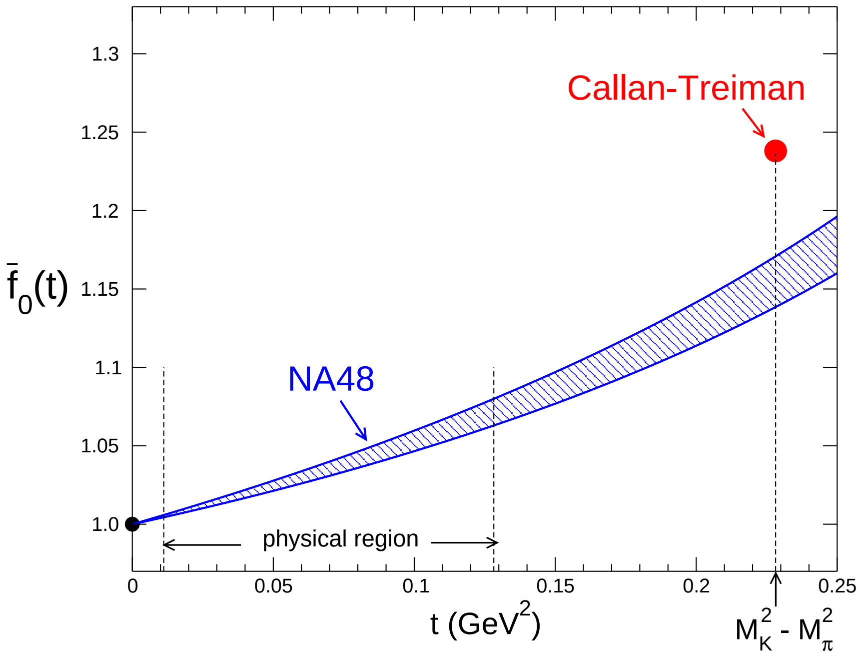
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

- Low energy theorem of Callan and Treiman (1966):

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi}\left\{1 + O(m_u, m_d)\right\} \simeq 1.19$$

$f_0(0) = f_+(0) \simeq 0.96$ relevant for determination of V_{us}

Comparison with experiment



NA48, hep-ex/0703002

141 authors, 2.3×10^6 events

Plot shows normalized scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- Callan-Treiman relation in this normalization:

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)}$$

- Experimental value: $\dfrac{F_K}{F_\pi f_+(0)} = 1.2438 \pm 0.0040$

Passemar, Proc. KAON 2007, arXiv:0708.1235

Corrections, extrapolation

- Callan-Treiman-relation is exact only for $m_u, m_d \to 0$

  Corrections of NLO were worked out long ago, are tiny

  Gasser & L. 1985

  Form factor now known to NNLO Post & Schilcher 2002,

  Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005

  Including the uncertainties from $m_u, m_d \neq 0$:

  $$\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$$

  Bernard, Oertel, Passemar & Stern, preliminary

$\Rightarrow$ Cannot blame the discrepancy on the prediction

- CT-point not in physical region, extrapolation needed

  Curvature can be calculated with dispersion theory

  Jamin, Oller & Pich 2004, Bernard, Oertel, Passemar & Stern 2006

$\Rightarrow$ Cannot blame the discrepancy on the extrapolation

Slope of the scalar form factor

- Definition of the slope $$\bar{f}_0(t) = 1 + \frac{\lambda_0\, t}{M_{\pi^+}^2} + O(t^2)$$
- Callan-Treiman-relation implies sharp prediction:

$$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$

Jamin, Oller & Pich 2004

- Update with current experimental information

$$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

Bernard, Oertel, Passemar & Stern, preliminary

- To be compared with the result of NA48:

$$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$

Fit with dispersive representation of BOPS

$$\lambda_0 = (11.7 \pm 0.7_{\text{stat}} \pm 1.0_{\text{syst}}) \times 10^{-3}$$

Linear fi t

Implications

- NA48 data on $K_L \to \pi\mu\nu$ disagree with SM

  If confirmed, the implications are dramatic:

$\Rightarrow$ Righthanded currents ? Bernard, Oertel, Passemar & Stern 2006

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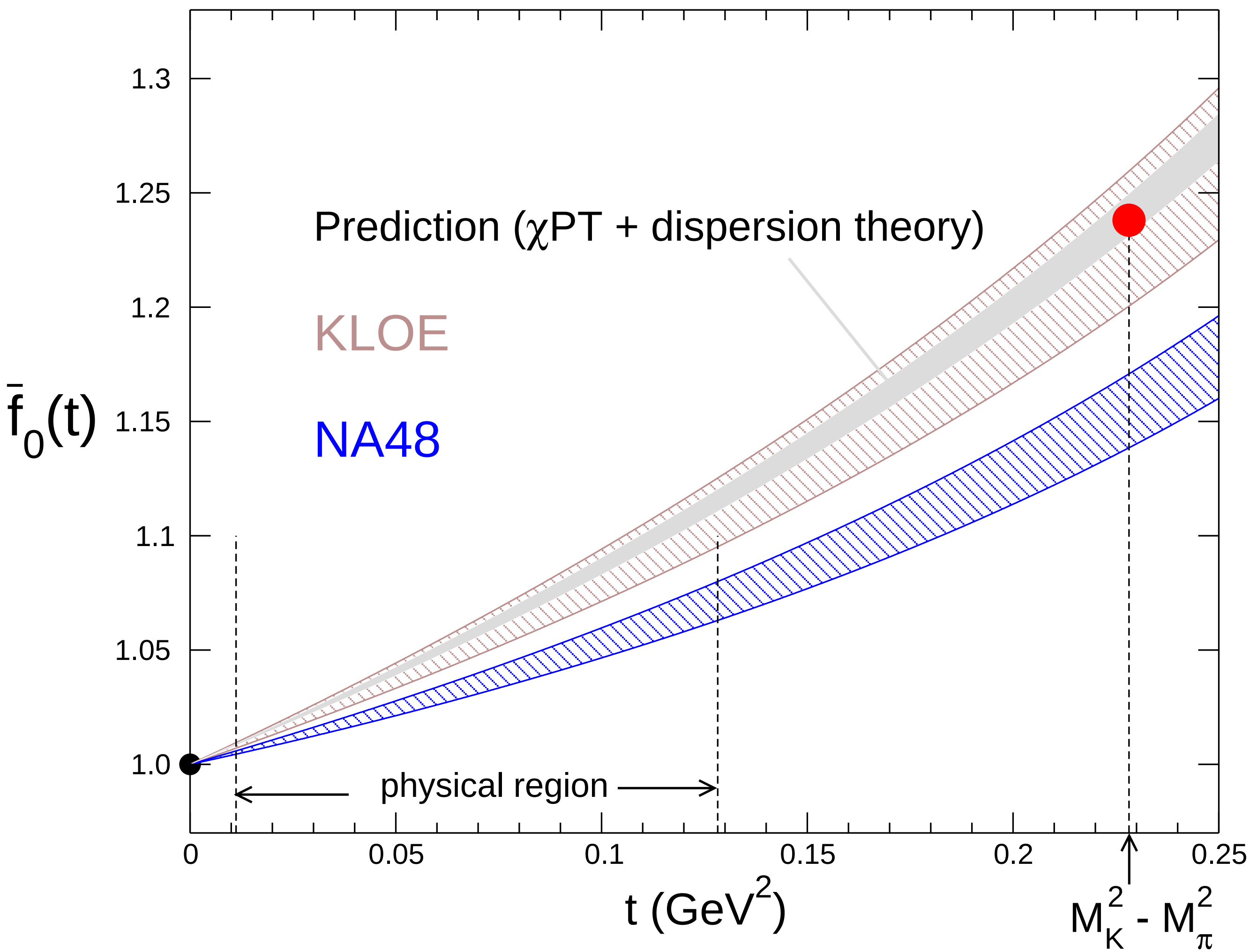
- In the meantime, new data from KLOE

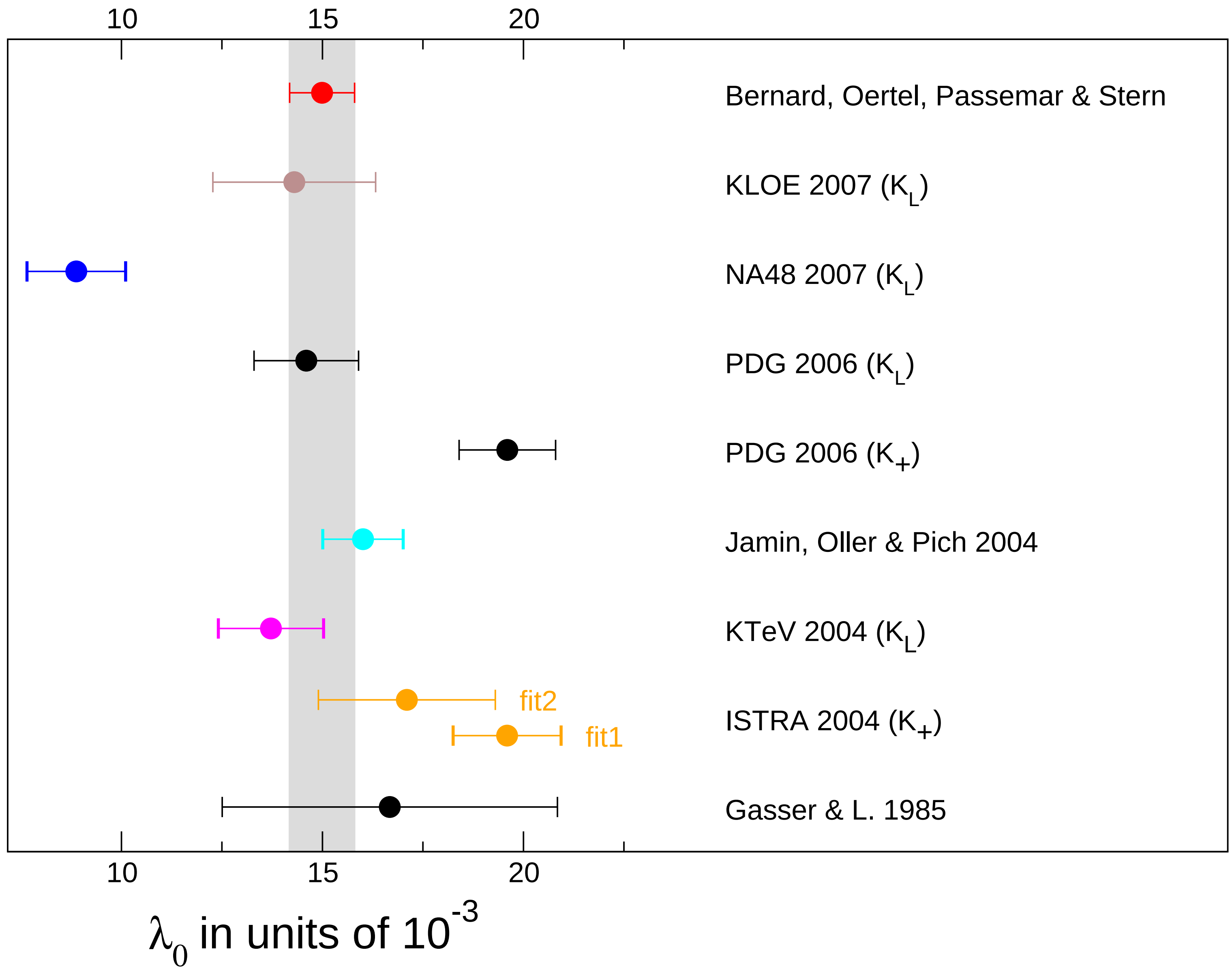
  Results are consistent with Callan-Treiman-relation

Comparison of theory and experiment



I thank Emilie Passemar for generous help with the numerics, in particular for providing the curves shown in this fi gure

Comparison of results for the slope



Extension to SU(3)×SU(3)

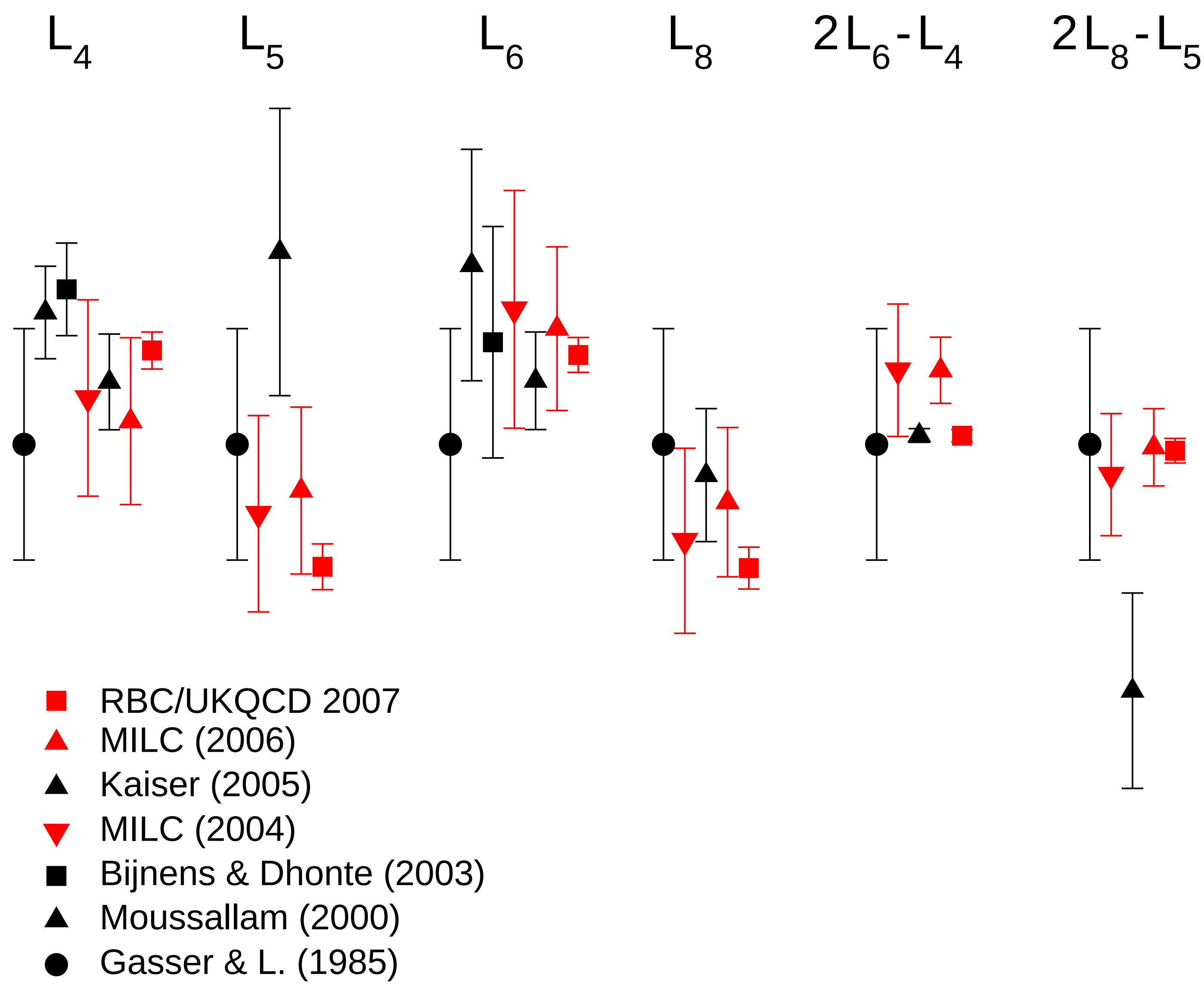
- The eightfold way is an approximate symmetry
- Only coherent way to understand this within QCD:
  $m_s - m_d, m_d - m_u$ can be treated as perturbations
- $m_u, m_d \ll m_s$

$\Rightarrow$ m_s can be treated as a perturbation

$\Rightarrow$ Expansion in powers of m_s should work

$\Rightarrow$ Meaningful to determine the effective coupling constants relevant for SU(3)×SU(3)

Effective coupling constants of SU(3)×SU(3)



Expansion in powers of m_s

- m_s is much larger than m_u, m_d

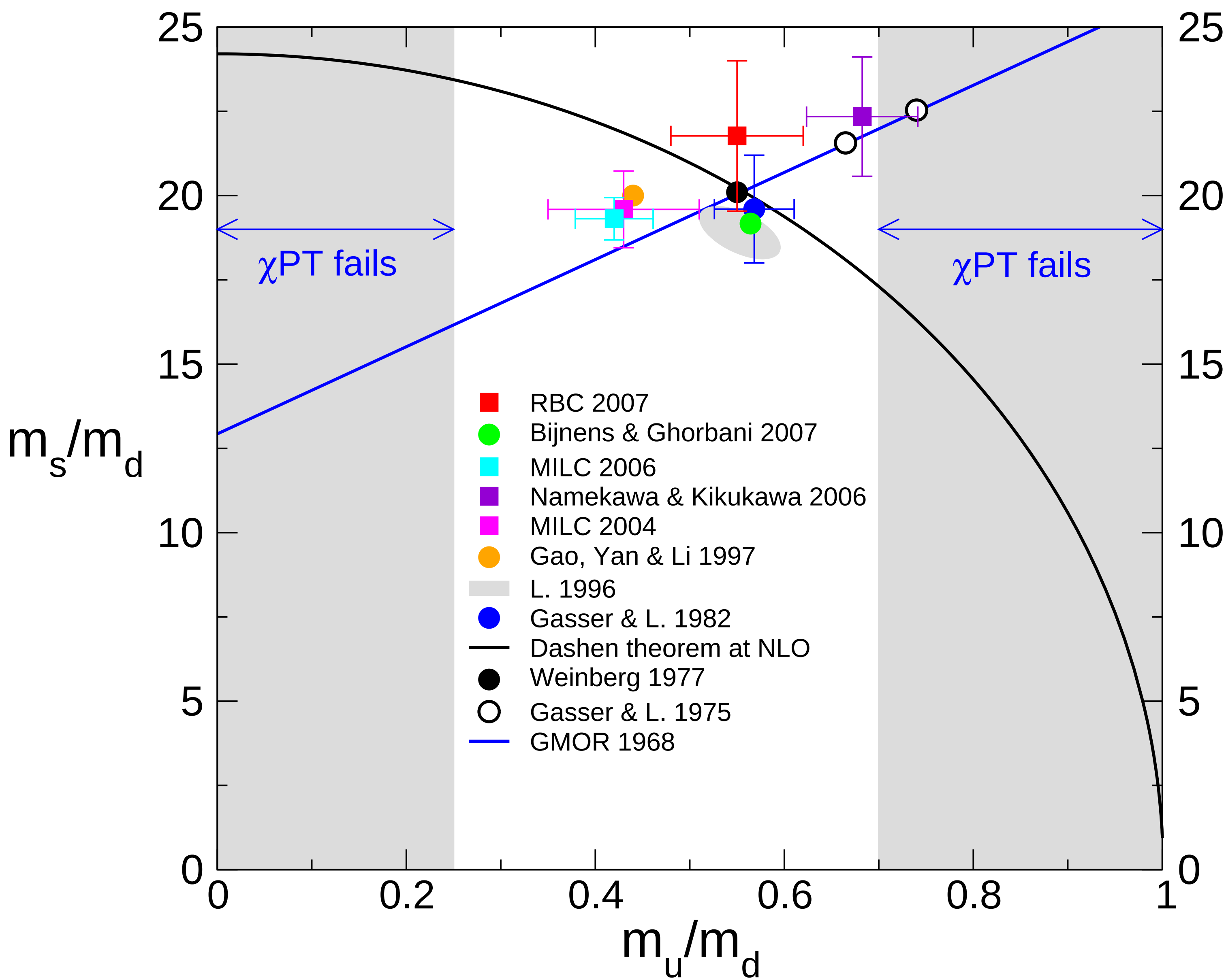
$\Rightarrow$ Expansion converges more slowly

$\Rightarrow$ Higher order terms are more important

- Estimates for these are rather crude

$\Rightarrow$ χPT based on SU(3)$\times$SU(3) does not have the same precision as in the case of SU(2)$\times$SU(2)

Results for quark mass ratios



Lattice results for quark masses

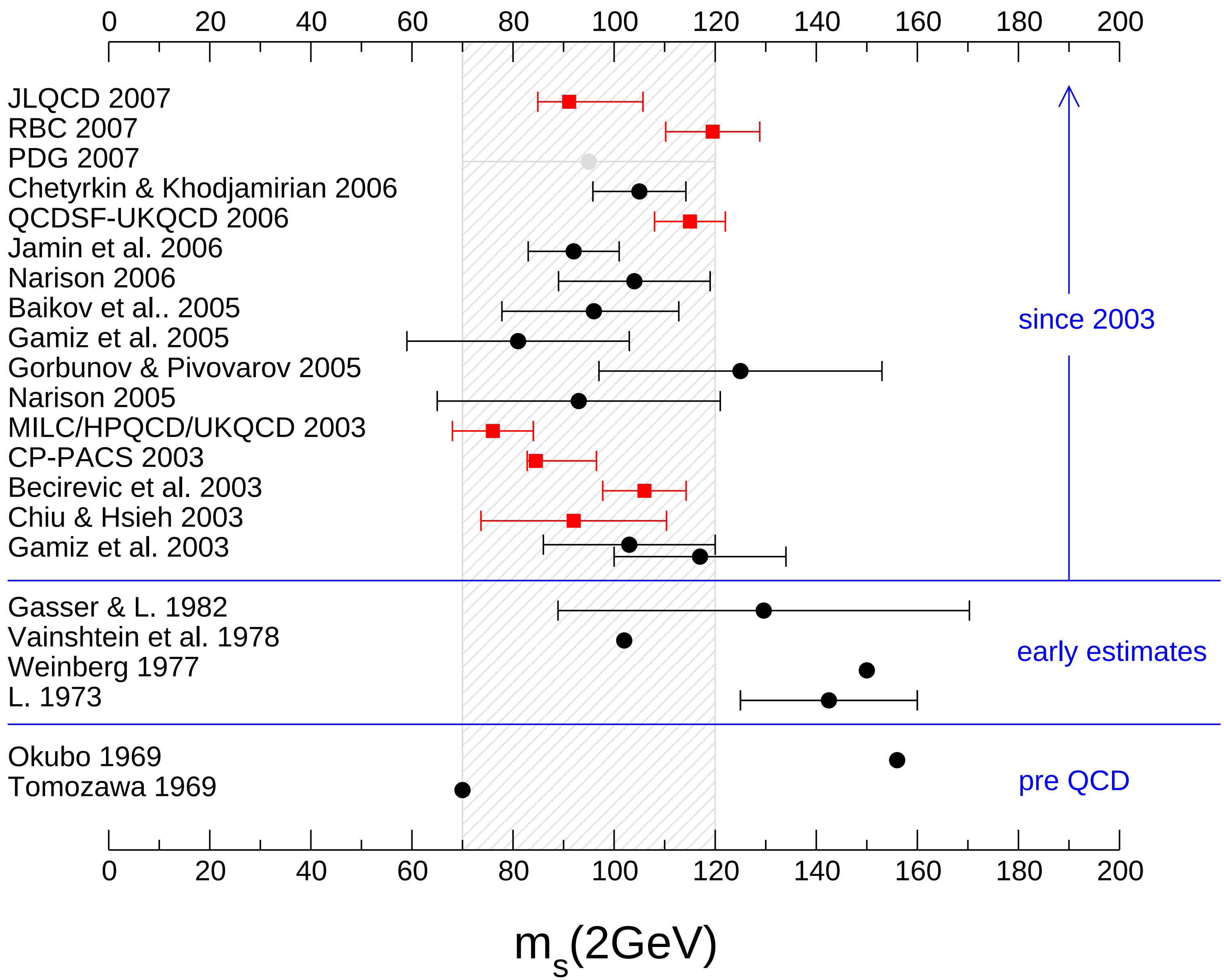
- Considerable progress with light dynamical fermions

$\Rightarrow$ Systematic uncertainties in extrapolations to the quark mass values of physical interest are being reduced

$\Rightarrow$ Should soon become possible to determine the SU(3)$\times$SU(3) coupling constants relevant at NNLO
In particular: logarithmic scales at NNLO ?

- Value of m_s is now known quite accurately, both from sum rules and from lattice results

Mass of the strange quark



Summary and conclusions

- Expansion in powers of m_u, m_d yields a very accurate low energy representation of QCD
- Lattice results clearly confirm the GMOR relation: M_π is proportional to $\sqrt{m_u + m_d}$

$\Rightarrow$ Energy gap of QCD is understood very well

- Lattice approach allows an accurate measurement of the effective coupling constant ℓ_3 already now
- Even for ℓ_4, the lattice starts becoming competitive with analytic methods
- Expect significant results for NNLO terms very soon

Summary and conclusions – ctd.

- Experimental discrepancy between NA48 and the other $K_{\mu 3}$ data needs to be resolved
- Important to analyze existing data on charged K-decays (isospin breaking)
- Dispersion theory fixes the shape of the form factors

  The most recent analyses properly account for the curvature
  Publishing linear fits is useless

- KTeV, ISTRA should be reanalyzed

  ISTRA: 0.54×10^6 events
  KTeV: 1.9×10^6 events
  NA48: 2.3×10^6 events

Summary and conclusions – ctd.

- Lattice work on m_s has reached an accuracy comparable to sum rule evaluations of τ decay
- Numerical value depends on the scale

$$\boxed{\begin{array}{l} m_s(2\,\text{GeV}) \simeq 100\ \text{MeV} \\ m_s(1\,\text{GeV}) \simeq 135\ \text{MeV} \end{array}}$$

- m_u, m_d more difficult to reach, uncertainties larger
- Pattern $m_u \simeq \frac{1}{2}\,m_d \simeq \frac{1}{40}\,m_s$ is confirmed

Summary and conclusions – ctd.

Many topics in low energy light flavour physics were not covered in this talk:

- V_{us} $\Rightarrow$ talk by Maltman
- rare kaon decays
- $\gamma + \gamma$ collisions
- σ, κ, $f_0(980)$, $a_0(980)$, ...

$\Rightarrow$ Excellent discussion of current situation in kaon physics:

Theory: G. Isidori, conference summary at KAON 07, arXiv:0709.2438 [hep-ph]

Experiment: M. Antonelli, talk at Lepton-Photon 07, http://chep.knu.ac.kr/lp07

SPARES

Compilation

$\bar{\ell}_3$	$\bar{\ell}_4$	
3.13 ± 0.33	4.42 ± 0.14	RBC/UKQCD 2007
2.9 ± 2.6	4.3 ± 0.5	JLQCD 2007
3.62 ± 0.12	4.61 ± 0.09	ETM Collaboration 2007
3.0 ± 0.5	$-$	Del Debbio et al. 2006
0.6 ± 1.2	3.9 ± 0.5	MILC 2004, 2006
$-$	4.4 ± 0.2	Colangelo, Gasser & L. 2001
2.9 ± 2.4	4.3 ± 0.9	Gasser & L. 1984

Entries for 2007: Talks by Boyle, Matsufuru/Noaki, Urbach at Lattice 2007, www.physik.uni-regensburg.de/lat07

Pattern of lowest levels

- $M_\pi^2 = (m_u + m_d)\, B + O(m^2)$

$\Rightarrow$ The energy gap of QCD is small because m_u, m_d happen to be small

- $M_{K^+}^2 = (m_u + m_s)\, B + O(m^2)$
  $M_{K^0}^2 = (m_d + m_s)\, B + O(m^2)$

$\Rightarrow$ M_K^2 is much larger than M_π^2, because m_s happens to be large compared to m_u, m_d

- Goldstone boson masses measure the strength of symmetry breaking $\Rightarrow$ strongly violate SU(3)

- Check: first order perturbation theory also yields
  $M_\eta^2 = \frac{1}{3}\,(m_u + m_d + 4m_s)B + O(m^2)$

$\Rightarrow$ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 $\surd$

Magnitude of the perturbations due to m_u, m_d, m_s

- $\langle 0|\,\bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\,p^\mu\sqrt{2}\,F_\pi$
  $\langle 0|\,\bar{s}\gamma^\mu\gamma_5 u|K^+\rangle = i\,p^\mu\sqrt{2}\,F_K$

  Difference between F_K and F_π comes from $m_s \neq m_d$

- Observed ratio: $\dfrac{F_K}{F_\pi} = 1.19 \pm 0.01$

  Branching fraction of $K \rightarrow \pi e \nu$ changed by $> 3\,\sigma$ in 2004 ! $1.22 \rightarrow 1.19$

$\Rightarrow$ $m_s - m_d$ generates correction of order 20%

- $m_u, m_d \ll m_s \Rightarrow$ correction mainly comes from m_s
- effects from m_u, m_d are tiny