On the history of the strong interaction

H. Leutwyler

University of Bern

Quantum Chromodynamics: History and Prospects

Oberwölz, September 2012

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- not a historian
- my own recollections, mainly based on memory

  memory does not improve with age . . .

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⇒ Tian Yu Cao, *From Current Algebra to Quantum Chromodynamics, Cambridge University Press, 2010*

I. From nucleons to quarks

II. History of the gauge field concept

III. QCD

Part I

From nucleons to quarks

Beginnings

- Discovery of the neutron — Chadwick 1932

$\Rightarrow$ Nuclei = p + n

- Isospin — Heisenberg 1932
- Investigation of the strong interaction started with the nuclear forces

  Yukawa $\sim$ 1935 — Stueckelberg

$$V_{e.m.} = -\frac{e^2}{4\pi r} \qquad\qquad V_s = -\frac{h^2}{4\pi r}\, e^{-\frac{r}{r_0}}$$

$$\frac{e^2}{4\pi} \simeq \frac{1}{137} \qquad\qquad \frac{h^2}{4\pi} \simeq 13$$

long range — short range

$$r_0 = \infty \qquad\qquad r_0 = \frac{\hbar}{M_\pi c} = 1.4 \cdot 10^{-15}\,\text{m}$$

$$M_\gamma = 0 \qquad\qquad M_\pi c^2 \simeq 135\,\text{MeV}$$

Strong interaction

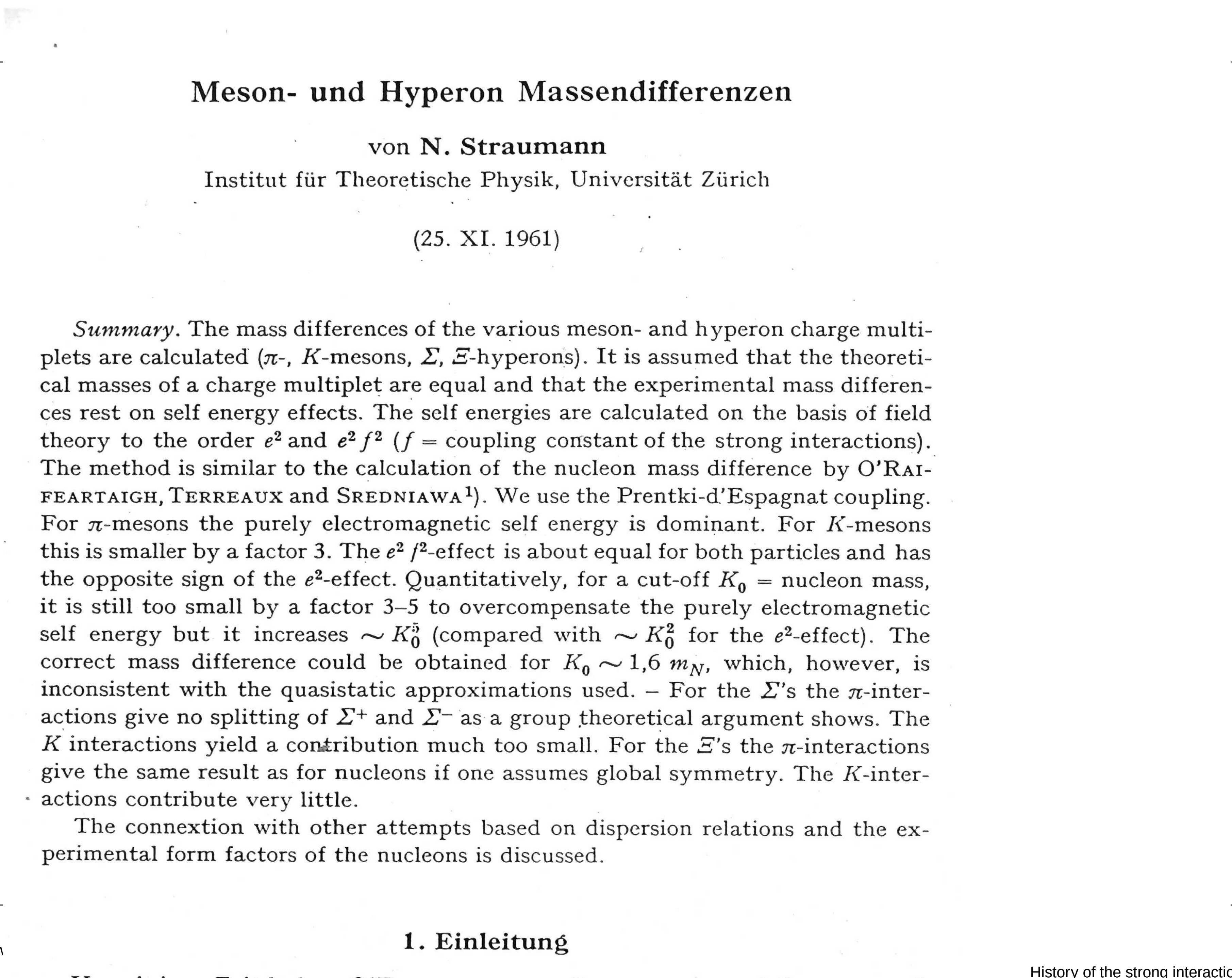
- Nonrelativistic potentials are much more flexible than quantum field theories. Suitable potentials between two nucleons, with attraction at large distances, repulsion at short distances do yield a decent understanding of nuclear structure.

  Paris potential, Bonn potential, shell model of the nucleus

$\Rightarrow$ Nuclear reactions, processes responsible for the luminosity of the sun, stellar structure, nuclear reactors, α-decay, . . . were well understood more than fifty years ago.

- These phenomena concern interactions among nucleons with small relative velocities. Experimentally, it had become possible to explore relativistic collisions, but a description in terms of nonrelativistic potentials cannot cover these.
- Many attempts at formulating a theory of the strong interaction based on elementary fields for baryons and mesons were undertaken, Yukawa interaction for the strong forces, perturbation theory with coupling constants of order 1, . . .

$\Rightarrow$ uncountable PhD theses 1945 -1965



Meson- und Hyperon Massendifferenzen

von **N. Straumann**

Institut für Theoretische Physik, Universität Zürich

(25. XI. 1961)

Summary. The mass differences of the various meson- and hyperon charge multiplets are calculated (π-, K-mesons, Σ, Ξ-hyperons). It is assumed that the theoretical masses of a charge multiplet are equal and that the experimental mass differences rest on self energy effects. The self energies are calculated on the basis of field theory to the order e^2 and $e^2 f^2$ (f = coupling constant of the strong interactions). The method is similar to the calculation of the nucleon mass difference by O'Raifeartaigh, Terreaux and Sredniawa[1]). We use the Prentki-d'Espagnat coupling. For π-mesons the purely electromagnetic self energy is dominant. For K-mesons this is smaller by a factor 3. The $e^2 f^2$-effect is about equal for both particles and has the opposite sign of the e^2-effect. Quantitatively, for a cut-off K_0 = nucleon mass, it is still too small by a factor 3–5 to overcompensate the purely electromagnetic self energy but it increases $\sim K_0^5$ (compared with $\sim K_0^2$ for the e^2-effect). The correct mass difference could be obtained for $K_0 \sim 1{,}6\, m_N$, which, however, is inconsistent with the quasistatic approximations used. – For the Σ's the π-interactions give no splitting of Σ^+ and Σ^- as a group theoretical argument shows. The K interactions yield a contribution much too small. For the Ξ's the π-interactions give the same result as for nucleons if one assumes global symmetry. The K-interactions contribute very little.

The connextion with other attempts based on dispersion relations and the experimental form factors of the nucleons is discussed.

1. Einleitung

Quantum field theory versus S-matrix theory

- Absolutely nothing worked even halfway, beyond general principles like Lorentz invariance, causality, unitarity, crossing symmetry.
  Analyticity, dispersion relations, CPT theorem, spin + statistics $\surd$
- There was considerable progress in renormalization theory, but faith in quantum field theory was in decline, even concerning QED (Landau-pole).
- Many people had doubts whether the strong interaction could at all be described by means of a local quantum field theory.
- Replace quantum field theory by S-matrix theory ?
  Heated debates $\Rightarrow$ Pietschmann, Eur. Phys. J. **H36** (2011) 75
- Regge poles Veneziano model 1968

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Fifty years ago, the quantum field theory of the strong interaction consisted of a collection of beliefs, prejudices and assumptions. Most of these turned out to be wrong.

Flavour symmetries

Isospin	Heisenberg 1932
Strangeness quantum number	Gell-Mann, Nakano & Nishijima 1953
Gell-Mann-Nishijima formula	
Eightfold way	Gell-Mann, Ne’eman 1961
Pattern of symmetry breaking, Ω^-	Gell-Mann, Okubo 1961/1962

Chiral symmetry

- Main characteristic of the strong interaction at low energies:
  energy gap is small, $M_\pi \simeq$ 135 MeV
- In 1960, Nambu found out why that is so: Nobel prize 2008
  - Has to do with a hidden, approximate, continuous symmetry.
  - Some of the generators of the symmetry carry negative parity: "chiral symmetry".
  - Origin of the symmetry was entirely unclear.
  - An analog of spontaneous magnetization occurs in particle physics:
    For dynamical reasons, the state of lowest energy is not symmetric.

  $\Rightarrow$ Chiral symmetry is hidden, “spontaneously broken”.
  - Nambu realized that the spontaneous breakdown of a continuous symmetry entails massless particles and concluded that the pions must play this role:

  $\Rightarrow$ No gap at all if the symmetry was exact - pions would be massless.
  - "Goldstone theorem" came later: Goldstone, Salam, Weinberg 1962
  - In reality, the symmetry is not perfect, but nearly so.

  $\Rightarrow$ There is an energy gap: the pions are not massless, only light.
- At large distances, the nuclear forces are indeed dominated by pion exchange.
  Yukawa formula $\surd$

Quark model

- Quark model — Gell-Mann, Zweig 1962
- Protons, neutrons are composed of quarks

  p $= uud$ n $= udd$
- Remarkably simple and successful picture, explains the pattern of energy levels – but

  why do the quarks not show up in experiment ?

⇒ The existence of quarks was considered doubtful.
"Such particles [quarks] presumably are not real but we may use them in our field theory anyway . . ." — Gell-Mann, Physics 1, 1964, 63

- Quarks were treated like the veal used to prepare a pheasant in the royal french cuisine. Conceptual basis of such a cuisine ?
- Puzzle: why is the symmetry not exact ?
  Exact consequences of approximate properties ?
  Charges & currents form an exact algebra
  even if they do not commute with the Hamiltonian. — Gell-Mann 1964
- Test of current algebra: size of $\langle N|A^\mu|N\rangle \sim g_A$ — Adler, Weisberger 1965
- Prediction derived from current algebra: $\pi\pi$ scattering lengths — Weinberg 1966

Behaviour at short distances

- Scaling of the ep cross section in the deep inelastic region. Bjorken 1968
- Deep inelastic scattering experiments by the MIT-SLAC collaboration in 1968/69: First experimental evidence for point-like constituents within the proton.
  Friedman, Kendall, Taylor, Nobel prize 1990
- Feynman called these "partons", leaving it open whether they were the quarks or something else.
- Operator product expansion Wilson 1969
  "Non-Lagrangian models of current algebra"

Color

- Quarks carry an internal quantum number
  - Greenberg had introduced an internal degree of freedom of this type in 1964, referring to this as "parastatistics".
  - In 1965, Bogolubov, Struminsky & Tavkhelidze, Han & Nambu and Miyamoto independently pointed out that some of the problems encountered in the quark model disappear if the u, d and s quarks occur in 3 states, "three-triplet model".
  - Gell-Mann coined the term "color" for the new quantum number.
- In his lectures at the Schladming Winter School in 1972, Gell-Mann thoroughly discussed the role of the quarks and gluons: theorists had to navigate between Scylla and Charybdis, trying to abstract neither too much nor too little from models built with these objects. Basic tool at that time: ”Current algebra on the light cone".

  He invited me to visit Caltech. I did that during three months in the spring break of 1973 and spent an extremely interesting period there.
- Color symmetry exact, strict confinement of color ?
- One of the possibilities considered for the interaction that binds the quarks together was an abelian gauge field analogous to the e.m. field.
- Fritzsch and Gell-Mann pointed out that if the gluons carry color, then the empirical observation that quarks appear to be confined might also apply to them: the spectrum of the theory might exclusively contain color neutral states.

QCD

- In his talk at the High Energy Physics Conference in 1972 (Fermilab), Gell-Mann discussed the proposal to describe the gluons in terms of a nonabelian gauge field coupled to color, relying on work done with Fritzsch.
- As it was known already that the electromagnetic and weak interactions are mediated by gauge fields, the idea that color might be a local symmetry as well does not appear as far fetched. In the proceedings, unpublished work by Wess is mentioned.
- Main problem at the time: All quantum field theories encountered in nature so far (including the electroweak theory) had the spectrum indicated by the degrees of freedom occurring in the Lagrangian.

⇒ Proposal does not look plausible. The observed spectrum of physical states differs qualitatively from what is expected from the fields needed to formulate the theory.

- Gell-Mann gave the theory a decent name:

$$\boxed{\text{Quantum chromodynamics}}$$

⇒ Is QCD qualitatively different from the quantum field theories encountered earlier ? Or is this wishful thinking ?

Part II

On the history of the gauge field concept

Electromagnetic interaction

- Final form of the laws obeyed by the electromagnetic field: Maxwell ∼ 1860 survived relativity and quantum theory, unharmed.
- Schrödinger equation for electrons in an electromagnetic field:

$$\frac{1}{i}\frac{\partial\psi}{\partial t} - \frac{1}{2m_e^2}(\vec{\nabla} + i\,e\vec{A})^2\psi - e\,\varphi\,\psi = 0$$

- Fock pointed out that this equation is invariant under a group of local transformations:

$$\vec{A}'(x) = \vec{A}(x) + \vec{\nabla}\alpha(x), \quad \varphi'(x) = \varphi(x) - \frac{\partial\alpha(x)}{\partial t}$$

$$\psi(x)' = e^{-ie\alpha(x)}\,\psi(x)$$

$\vec{A}', \varphi', \psi'$ describe the same physical situation as $\vec{A}, \varphi, \psi$.

Fock 1926

see Okun, "V. A. Fock and gauge symmetry", Phys.Usp.53:835-837,2010

- Weyl termed these gauge transformations, gauge group: U(1).

QED is fully characterized by gauge invariance

- Gauge invariance is the crucial property of QED.
- Illustrate the statement with the core of QED: photons + electrons.
  - Gauge invariance allows only 2 free parameters in the Lagrangian of this system: e, m_e.
  - Moreover, only one of these is dimensionless: $e^2/4\pi = 1/137.035\,999\,679\,(94)$.

  $\Rightarrow$ U(1) symmetry + renormalizability fully determine the properties of the e.m. interaction, except for this number, which so far still remains unexplained.
- Side remark: In QED, there is an additional operator of dimension ≤ 4: $F_{\mu\nu}\tilde{F}^{\mu\nu}$.
  The term represents a total derivative, $F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu f^\mu$.
  Since e.m. fields with a nontrivial behaviour at large distances do not appear to play a significant role (no instantons or the like), this term does not affect the physics.

Gauge fields from geometry

- Kaluza (1921) & Klein (1926): 5-dimensional Riemann space with a metric that is independent of the fifth coordinate is equivalent to a 4-dimensional world with

  $\boxed{\text{metric space with } d = 5 \supset \text{ gravity + U(1) gauge field + scalar field}}$

  $x^{5\prime} = x^5 + \alpha(\vec{x}, t) \iff$ gauge transformation

- Coordinate independent characterization: Riemann space that is invariant under translations in one direction, isometry, Killing vectors, isometry group: U(1).

  Fifth dimension can be compactified to a circle, U(1) generates motions on this circle.

- Klein (1938) and Pauli (1953) investigated generalizations of the Kaluza-Klein scenario. Pauli noted that Riemann spaces of dimension > 5 can admit *nonabelian* isometry groups that reduce the system to a 4-dimensional one:

  $\boxed{\text{metric space with } d > 5 \supset \text{ gravity + } \textit{nonabelian} \text{ gauge fields + several scalar fields}}$

- Pauli was motivated by the isospin symmetry of the meson-nucleon interaction, considered a Riemann space of dimension 6, with isometry group SU(2).

  $\Rightarrow$ Straumann, arXiv:0810.2213

Nonabelian gauge fields

- Pauli did not publish the idea, because he was convinced that the quanta of a gauge field were necessarily massless: gauge invariance does not allow one to put a mass term into the Lagrangian.

$\Rightarrow$ Concluded that the forces mediated by gauge fields would necessarily be of long range and hence in conflict with observation: the strong interaction has short range.

- Yang and Mills (1954).
- Ronald Shaw (student of Salam) independently formulated nonabelian gauge fields in his PhD thesis, Cambridge 1955.
- Higgs (1964), Brout & Englert (1964) and Guralnik, Hagen & Kibble (1964) showed that Pauli's objection can be overcome: in the presence of scalar fields, gauge fields can pick up mass $\Rightarrow$ forces mediated by gauge fields can be of short range.
- The work of Glashow (1961), Weinberg (1967) and Salam (1968) demonstrated that nonabelian gauge fields are relevant for physics: the framework discovered by Higgs et al. does accommodate a realistic description of the e.m. and weak interactions.

Nobel prize 1979

Asymptotic freedom

- Already in 1965, Terentyev and Vanyashin found that the renormalization of the electric charge of a vector field is of opposite sign to the one of the electron (the numerical value of the coefficient was not correct). In the language of SU(2) gauge field theory, their result implies that the β-function is negative at one loop. Silagadze, Vysotsky
- The first correct calculation of the β-function of a nonabelian gauge field theory was carried out by Khriplovich, for the case of SU(2), relevant for the electroweak interaction. He found that β is negative and concluded that the interaction becomes weak at short distance.

  Khriplovich, Yad. Phys. 10 (1969) 409 [Sov. J. Nucl. Phys. 10 (1970) 235]
- t'Hooft was not aware of the work of Khriplovich. In his PhD thesis, he performed the calculation of the β-function for an arbitrary gauge group, including the interaction with fermions and Higgs scalars. He demonstrated that the theory is renormalizable and confirmed that, unless there are too many fermions or scalars, the β-function is negative. Nobel prize 1999, together with Veltman

Implications for QCD

- Politzer and Gross & Wilczek (1973) discussed the consequences of a negative β-function and suggested that this might explain Bjorken scaling, which had been observed at SLAC in 1969.
- They pointed out that QCD predicts specific modifications of the scaling laws. In the meantime, there is strong experimental evidence for these. Nobel prize 2004
- A detailed account of the history of the quantum theory of gauge fields can be found in the 1998 Erice lectures: ’t Hooft, The glorious days of physics: Renormalization of gauge theories, arXiv:hep-th/9812203

Part III

QCD

QCD

- Reasons for proposing QCD as a theory of the strong interaction were given in Fritzsch, Gell-Mann & L. , Phys. Lett. **B47** (1973) 365.

  Abstract: ”It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang-Mills gauge model based on colored quarks and color octet gluons.”

  - Before the paper was completed, preprints by Politzer and Gross & Wilczek were circulated - they are quoted and asymptotic freedom is given as argument #4 in favour of QCD.
  - Also, important open questions were pointed out, in particular, the U(1) problem.

- Many considered QCD a wild speculation. On the other hand, several papers concerning gauge field theories that include the strong interaction appeared around the same time, e.g.
  Pati and Salam, Phys. Rev. **D8** (1973) 1240, **D10** (1974) 275
  Weinberg, Phys. Rev. Lett. 31 (1973) 494

November revolution

- Discovery of the J/ψ, announced at Brookhaven and SLAC on 11.11.1974.
- Confirmed three days later at ADONE, Frascati.
- ψ' found ten days later at SLAC, subsequently many further related states.
- Now know that these are bound states formed with the c-quark and its antiparticle and that there are two further heavy quarks: b, t.

Standard Model

- Only gradually, particle physicists abandoned their outposts in no man’s and no woman’s land, returned to the quantum fields and resumed discussion in the good old *Gasthaus zu Lagrange.* Jost

⇒ Standard Model, clarified the picture enormously

- At sufficiently high energies, quarks and gluons do manifest themselves: jets.
- Like the neutrini, they have left their theoretical place of birth and can be seen flying around like ordinary, observable particles.

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- Like the neutrini, they have left their theoretical place of birth and can be seen flying around like ordinary, observable particles.
- The success of the Standard Model is amazing:

Gauge fields are renormalizable in $d = 4$, but it looks unlikely that the Standard Model is valid much beyond the explored energy range. Presumably it represents an effective theory. No reason for an effective theory to be renormalizable.

⇒ Why is the Standard Model renormalizable ?

Theoretical paradise

- Turn the electroweak interaction off and ignore the masses of the quarks.
- $m_u = m_d = m_s = 0$

  $m_c = m_b = m_t = \infty$

  $\boxed{\text{QCD with 3 massless quarks}}$
- Lagrangian contains a single parameter: g, net color of a quark.
  g depends on the radius of the region considered.
- Color contained within radius r

$$\alpha_s \equiv \frac{g^2}{4\pi} = \frac{2\pi}{9\,|\ln(r\,\Lambda_{QCD})|}$$

- Intrinsic scale Λ_{QCD} is meaningful, but not dimensionless.

⇒ No dimensionless free parameter.
All dimensionless physical quantities are pure numbers, determined by the theory.
Cross sections can be expressed in terms of Λ_{QCD} or in the mass of the proton.

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$\boxed{\text{Massless QCD is how theories should be}}$

Symmetries of massless QCD

- Interactions of u, d, s are identical. If the masses are set equal to zero, there is no difference at all: Lagrangian symmetric under $u \leftrightarrow d \leftrightarrow s$.
- More symmetry: For massless fermions, right and left do not communicate.
  $\Rightarrow$ Lagrangian of QCD with 3 massless flavours is invariant under SU(3)$_R$ × SU(3)$_L$.
  $\Rightarrow$ QCD explains the presence of the mysterious symmetry discovered by Nambu.
- Nambu had conjectured that chiral symmetry breaks down spontaneously.
  Can it be demonstrated that this happens in QCD ?

  Massless QCD with 3 flavours: $\underset{\text{Lagrangian}}{\text{SU(3)}_R\times\text{SU(3)}_L} \overset{?}{\Rightarrow} \underset{\text{ground state}}{\text{SU(3)}_{R+L}}$
- $\nexists$ an analytic proof, but the work done on the lattice demonstrates beyond any doubt that this does happen in QCD:
  - The calculated mass spectrum agrees with experiment.
  - For $m_u = m_d = m_s$, the states form degenerate multiplets of SU(3)$_{R+L}$.
  - In the limit $m_u, m_d, m_s \rightarrow 0$, the pseudoscalar octet becomes massless, as required by the Goldstone theorem.
- The 8 lightest mesons do have the quantum numbers of the Nambu-Goldstone bosons:
  $\pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, \eta$
  but massless they are not . . .

Quark masses

- Real world $\neq$ paradise

  $m_u\ ,\ m_d\ ,\ m_s \neq 0$

  Quark masses break chiral symmetry, allow the left to talk to the right.

⇒ The multiplets are split, the Nambu-Goldstone boson multiplet is not massless.

- Chiral symmetry broken in two ways:

  spontaneously $\langle 0|\,\bar{q}_R q_L\,|0\rangle \neq 0$

  explicitly $m_u\ ,\ m_d\ ,\ m_s \neq 0$

Pattern of light quark masses

- Even before the discovery of QCD, attempts at estimating the masses of the quarks were made.
- Bound state models for mesons and baryons:

  $m_u + m_u + m_d \simeq M_p \qquad m_u \simeq m_d \simeq 300$ MeV

  “constituent masses”
- With the discovery of QCD, the mass of the quarks became an unambiguous concept: quark masses occur in the Lagrangian of the theory.
- First crude estimate within QCD relied on a model for the wave functions of π, K, ρ, based on SU(6) (spin-flavour-symmetry)

  $$\frac{(m_u + m_d)}{2} = \frac{F_\pi M_\pi^2}{3F_\rho M_\rho} \simeq 5\,\text{MeV}, \qquad m_s \simeq 135\,\text{MeV}$$

  L. 1974, "Is the quark mass as small as 5 MeV ?"
- Not very different from the pattern found within the Nambu-Jona-Lasinio model (1961) or the one obtained from sum rules by Okubo (1969).

Crude picture for m_u, m_d, m_s

- Difference between m_u and m_d ? In 1975, Jürg Gasser and I analyzed the Cottingham formula with the data available by then.

$\Rightarrow$ e.m. self-energy of proton $>$ neutron

$\Rightarrow$ $M_p < M_n$ cannot be due to the e.m. interaction

must be due to $m_u < m_d$.

$\Rightarrow$ Isospin not a symmetry of the strong interaction ! In fact, an apparently very strong breaking was needed:

- $m_u \simeq 4$ MeV, $m_d \simeq 7$ MeV, $m_s \simeq 135$ MeV   Gasser + L. 1975
  - m_u and m_d are very different
  - m_u and m_d are small compared to m_s
  - "constituent masses" $\notin$ Lagrangian of QCD
- Took quite a while before this bizarre pattern was generally accepted.

Weinberg 1977

Approximate symmetries are natural in QCD

- Why is isospin such a good quantum number ?

  (a) Divergences of perturbation theory do not represent a disease, but are intimately connected with the structure of the theory: dimensional transmutation.

  $\Rightarrow$ QCD has an intrinsic scale, Λ_{QCD}.

  (b) $m_d - m_u \ll$ scale of QCD, not $\ll m_u + m_d$

- Why is the eightfold way a decent approximate symmetry ?

  $m_s - m_u \ll$ scale of QCD

- Isospin is an even better symmetry because

  $m_d - m_u \ll m_s - m_u$

- $m_u \ll m_s \Rightarrow m_u, m_d, m_s \ll$ scale of QCD

  $\Rightarrow$ Masses of the light quarks represent perturbations.

  Can neglect these in a first approximation.

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  ⇒ Masses of the light quarks represent perturbations.

  Can neglect these in a first approximation.

⇒ $\boxed{\text{In first approximation, the world is a paradise.}}$

Quark masses as perturbations

- Masses of the light quarks enter the Hamiltonian via

$$H_{QCD} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d + m_s \, \bar{s}s\}$$

H_0 describes u, d, s as massless, c, b, t as massive.

H_0 is invariant under SU(3)$_L \times$SU(3)$_R$.

H_0 treats π, K, η as massless particles, H_1 gives them a mass.

Expansion in powers of $m_u\,, m_d\,, m_s$	$\Longleftrightarrow$	Perturbation series in powers of H_1

Chiral perturbation theory

Gell-Mann-Oakes-Renner formula

- First order χPT yields (formula does not appear like this in the paper)

$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \text{explicit}}}{+} m_d) \times |\langle 0|\, \underset{\substack{\Uparrow \\ \text{spontaneous}}}{\bar{u}u}\, |0\rangle| \times \frac{1}{F_\pi^2}$$

Gell-Mann, Oakes & Renner 1968

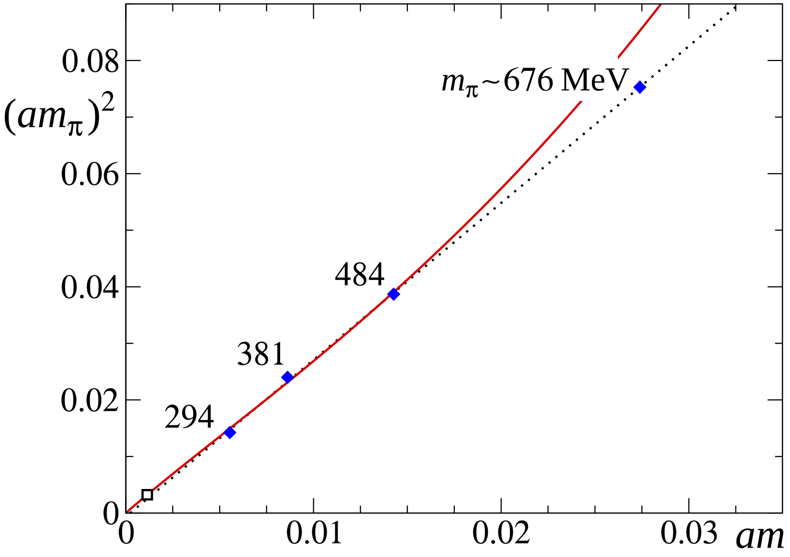
Coefficient: pion decay constant $\langle 0|\, \bar{d}\gamma^\mu \gamma_5\, u|\pi^+\rangle = i\, p^\mu \sqrt{2}\, F_\pi$

Value of F_π is known from $\pi^+ \rightarrow \mu^+ \nu$.

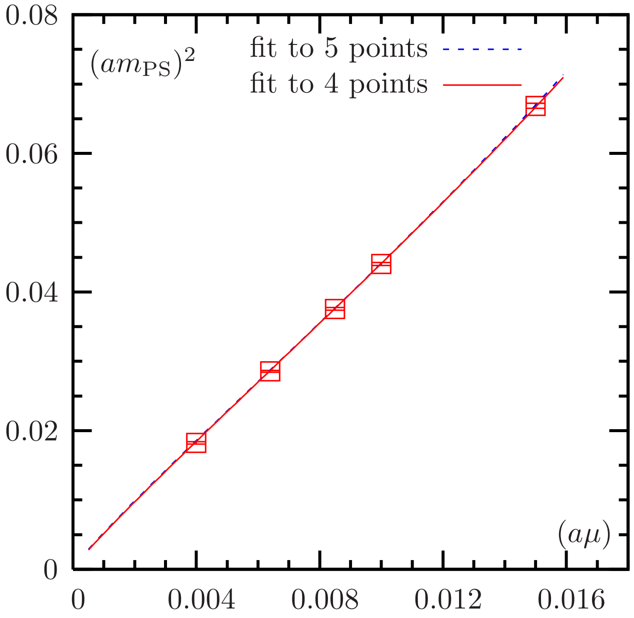
- Correlates $m_u + m_d$ with the quark condensate $|\langle 0|\bar{u}u\,|0\rangle|$.
- Formula receives corrections from higher orders.

Lattice

- Simulations of QCD on a lattice now reach small lattice spacings, small quark masses. Quality of the available lattice data is impressive ⇒ can make contact with physics.
- Gell-Mann-Oakes-Renner formula can be checked on the lattice: can calculate M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

⇒ Proportionality of M_π^2 to $m_u + m_d$ holds out to quark mass values that are an order of magnitude larger than in nature.

- Many other observables can now reliably and accurately be calculated on the lattice.

Mass pattern of the Nambu-Golstone bosons

- $M_\pi^2 = (m_u + m_d)\,B + O(m^2)$

⇒ The energy gap of QCD is small because $m_u\,, m_d$ happen to be small.

- $M_{K^+}^2 = (m_u + m_s)\,B + O(m^2)$

  $M_{K^0}^2 = (m_d + m_s)\,B + O(m^2)$

⇒ $M_K^2 \gg M_\pi^2$, because m_s happens to be large compared to $m_u\,, m_d$.

- Nambu-Goldstone boson masses measure the strength of symmetry breaking.

⇒ strongly violate SU(3).

- Check: first order perturbation theory also yields

  $M_\eta^2 = \frac{1}{3}\,(m_u + m_d + 4m_s)B + O(m^2)$

⇒ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 ✓

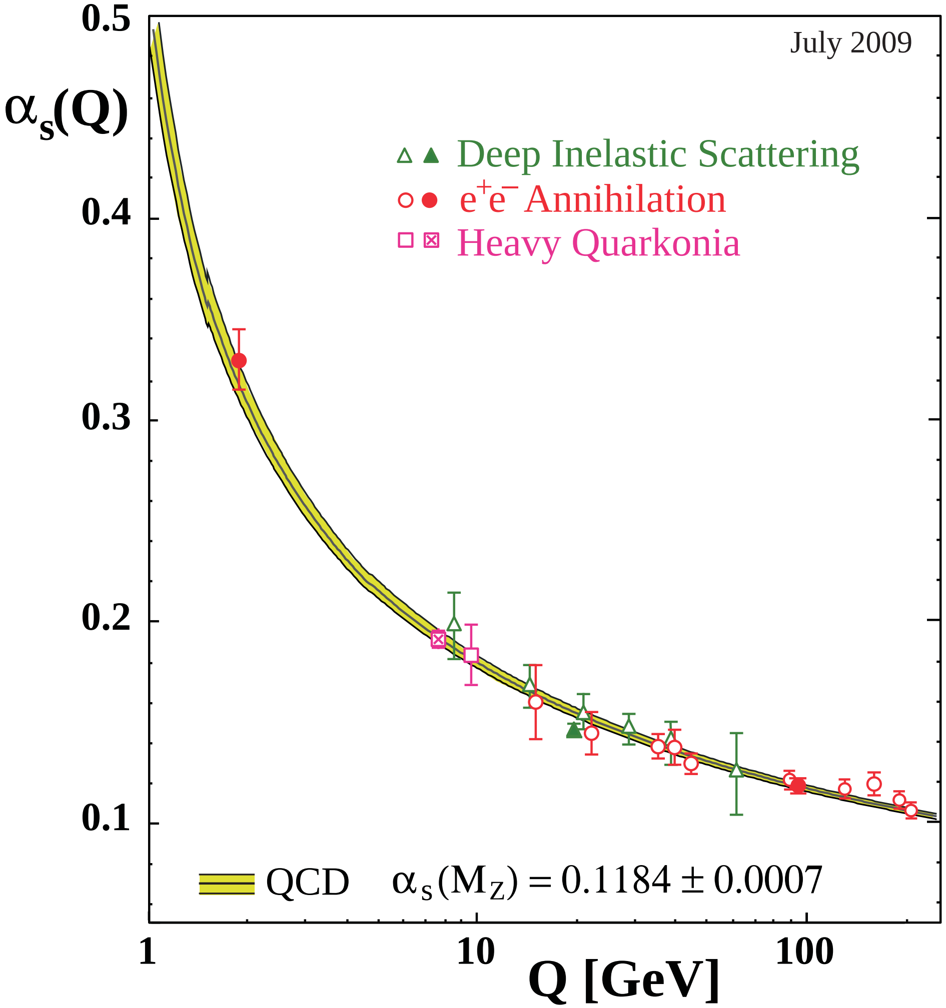
High energies: perturbative QCD

- At short distances, QCD can be analyzed by means of perturbation theory
- At high energies, inclusive processes, such as

  $$e + p \longrightarrow e + \text{anything}$$

  can be calculated in terms of the running coupling constant $\alpha_s(\mu)$ and the running quark masses $m_u(\mu), \ldots, m_t(\mu)$.
- Main characteristics of the jets generated by quarks or gluons with large transverse momenta can be analyzed within perturbation theory
- Effective theories have been developed to cope with the infrared singularities encountered if some of the loop momenta become soft or collinear
- Very successful and very active field of research, useful in particular also for processes where QCD merely generates unwanted background

Running coupling constant of QCD



taken from minireview on QCD
by Dissertori and Salam (PDG online)
Plot is due to Bethke 2009

Sources test very different scales

Clear evidence for asymptotic freedom

Low energies: χPT, lattice

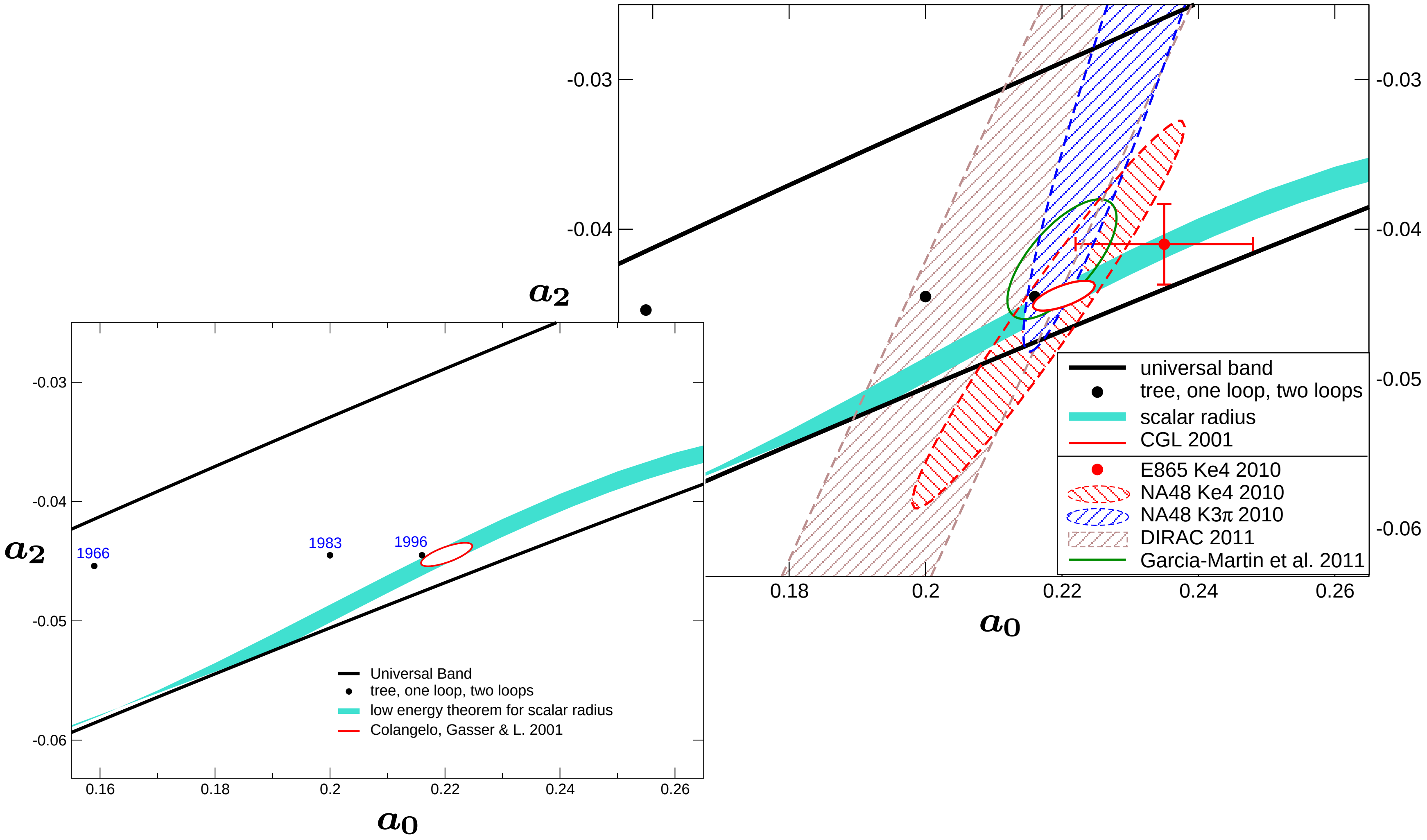
- For many physical quantities a quantitative understanding of the low energy properties of the strong interaction is needed. Discuss only one example: $\pi\pi$ interaction.
- Quantitative description relies on dispersion theory. The S-wave scattering lengths a_0 and a_2 play a crucial role: subtraction constants in the dispersive representation of the scattering amplitude. Roy 1971
- Prediction at leading order of χPT:

$$a_0 = \frac{7M_\pi^2}{32\pi F_\pi^2} = 0.16, \qquad a_2 = -\frac{M_\pi^2}{16\pi F_\pi^2} = -0.045$$

Weinberg 1966

- χPT allows to analyze the contributions of higher order. Chiral expansion has been worked out to NNLO. Using dispersion theory (Roy equations), this leads to remarkably sharp predictions for a_0, a_2.
- These predictions triggered new low energy precision experiments:
  - $\pi^+\pi^-$ atoms, DIRAC.
  - $K^\pm \to \pi^0\pi^0\pi^\pm$, $K^0 \to \pi^0\pi^0\pi^0$: cusp near threshold, NA48/2.
  - $K^\pm \to \pi^+\pi^- e^\pm \nu$ data: E865, NA48/2.

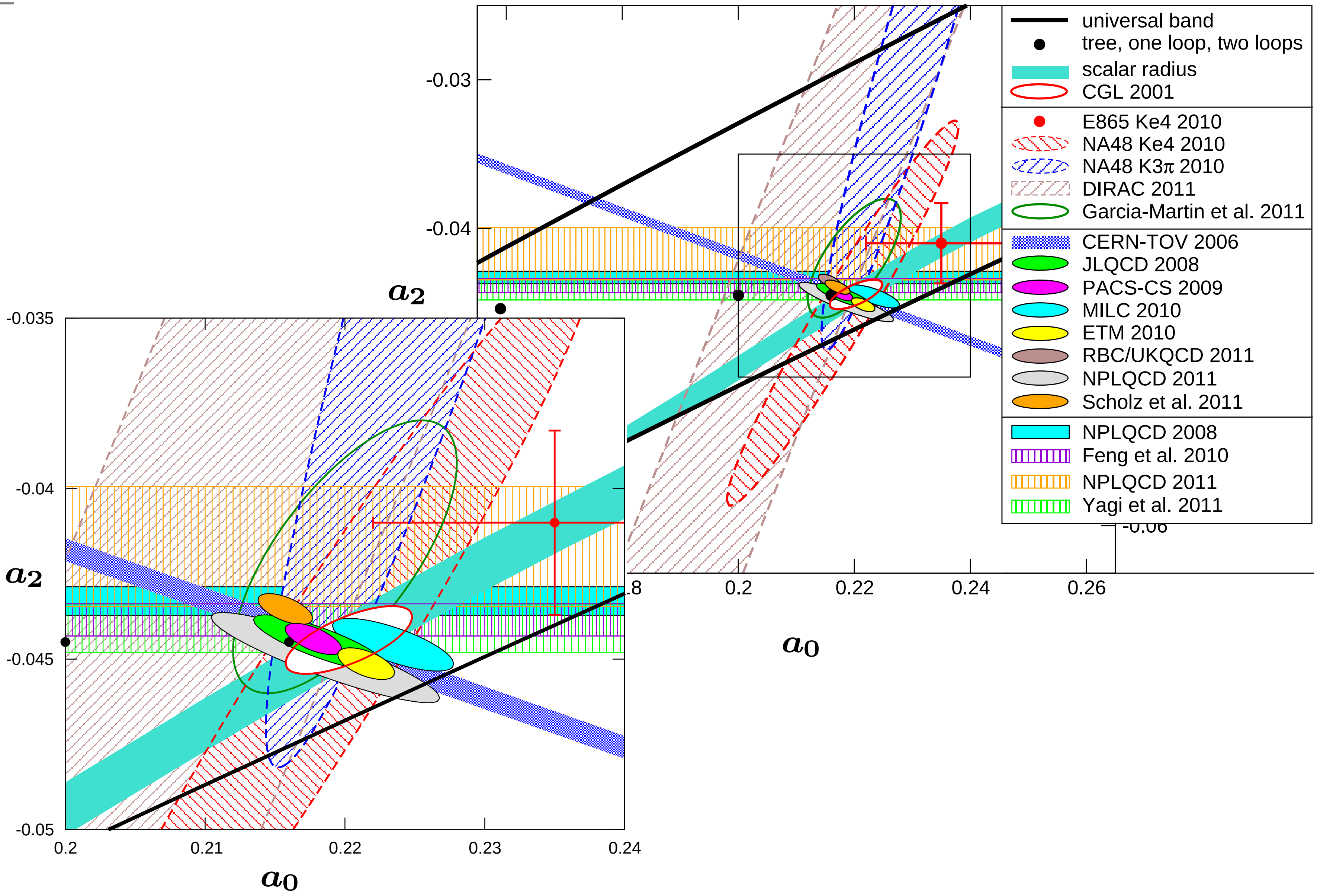
Experimental tests of the prediction



Determination of the scattering lengths on the lattice

- Uncertainty in χPT prediction for a_0, a_2 is dominated by the uncertainty in the relevant coupling constants of the effective Lagrangian at NLO. These can now reliably be determined on the lattice, from the quark mass dependence of M_π and F_π.
- Direct determination of a_2 via dependence of the energy levels on the size of the box.

Compare the lattice results with prediction and experiment



Conclusion

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- To work out the quantitative consequences of this insight is a fascinating challenge.
- Models (AdS, CFT, superconductors, NJL . . .) may help developing an intuitive understanding of QCD.

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- To work out the quantitative consequences of this insight is a fascinating challenge.
- Models (AdS, CFT, superconductors, NJL . . .) may help developing an intuitive understanding of QCD
  . . . but they are a meagre replacement for the real thing.
- Much work remains to be done to confront low energy precision experiments with our present understanding of the laws of nature: lattice simulations, effective field theory methods, dispersion theory, . . .

Merci vielmal