Qualitative aspects of the Standard Model

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Standard Model

The Standard Model is a miracle:

- since a long time, we know that the microscopic world is governed by three types of interaction:

  strong, electromagnetic, weak
- these have qualitatively very different properties
- nevertheless, they are all generated by gauge fields

advertisement, 1973

IG Physik, Gesellschaft mit besonderer Haftung

Im Falle eines Falles

klebt ein EICHFELD

wirklich alles !

Bezugsquellennachweis

J.C.Maxwell, A Treatise on Electricity and Magnetism, Clarendon Press, Oxford (1873)
H.Weyl, Z.Phys.56 (1929) 330
C.N.Yang and R.Mills, Phys.Rev.96 (1954) 191

Gauge fields

- Prototype: electromagnetic field, Maxwell $\sim$ 1860 survived relativity and quantum theory, unharmed
- Quanta of the field: photons, Einstein 1905

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  In particle language: $e^- \longrightarrow e^- + \gamma$
- Field produces force between charged particles

  Interaction energy between two electrons:

$$V = \frac{e^2}{4\pi\epsilon_0 r} = \alpha_{em}\frac{\hbar c}{r}$$

$$\alpha_{em} = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

⇒ Strength of e.m. interaction characterized by a number

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occur in 3 “colours”: $\boldsymbol{u} = \begin{pmatrix} \boldsymbol{u_1} \\ \boldsymbol{u_2} \\ \boldsymbol{u_3} \end{pmatrix}$ $\boldsymbol{d} = \begin{pmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_2} \\ \boldsymbol{d_3} \end{pmatrix}$

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  Gluons can generate transitions: $u_1 \longrightarrow u_2 + G$
- The colour is the source of the gluon field

  Quarks carry colour $\Rightarrow$ surrounded by a gluon field

$\Rightarrow$ Quarks take part in the strong interaction

- Electrons or photons do not have colour

$\Rightarrow$ do not participate in the strong interaction

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Gauge field of flavour

- The weak interaction is also due to a gauge field
- Quanta of the field: W^+, W^-, Z
- Internal quantum number here: “flavour”

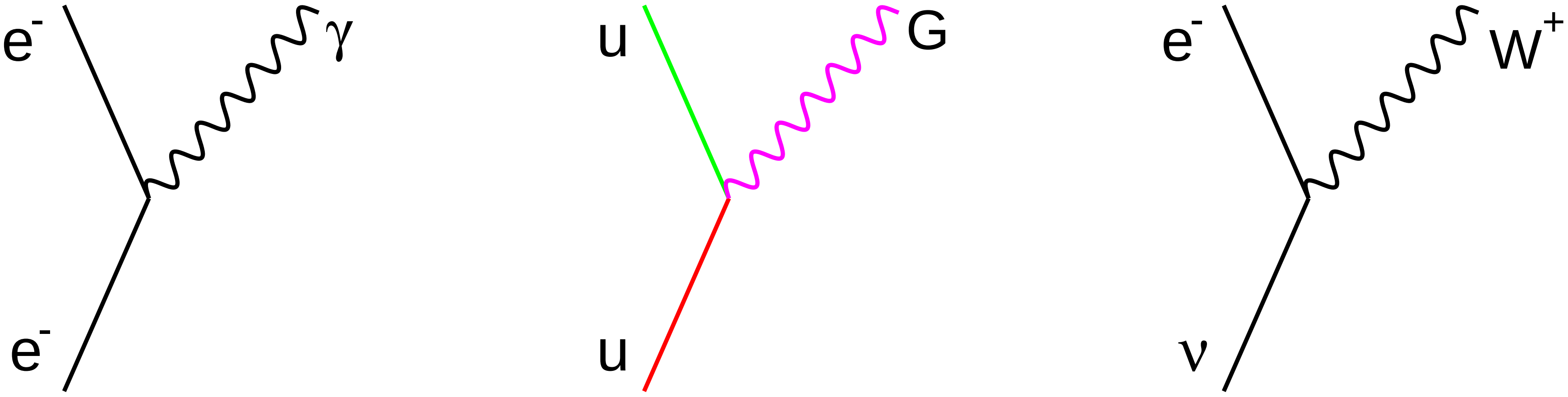
  e_L and ν_L form a flavour doublet: $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

  $W^\pm, Z$ can generate transitions: $\nu_L \longrightarrow e_L + W^+$
- The flavour is the source of the weak gauge field

  Quarks, electrons, neutrini, . . . all carry flavour

⇒ are all participating in the weak interaction

Gauge field interactions in particle language



electromagnetic	strong	weak
QED	QCD	QFD
charge	colour	flavour
photon	gluons	$W^{\pm}, Z$

Behaviour at short distance

- At short distances (1 TeV $\leftrightarrow$ $\mathbf{2 \cdot 10^{-19}}$ m)
  all of the forces obey the inverse square law

  $$\boldsymbol{V} = \text{constant} \times \frac{\boldsymbol{\hbar c}}{\boldsymbol{r}} \qquad \text{interaction energy}$$

- the constant is a pure number

$\Rightarrow$ strength of interaction is fixed by 3 pure numbers

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	electromagnetic	strong	weak
●	$\dfrac{e^2}{4\pi\epsilon_0\hbar c}$	$\dfrac{g_s^2}{4\pi}$	$\dfrac{g_w^2}{4\pi}$

⇒ Nature uses a generalization of QED !

- Possibly, the strength of the three interactions even becomes the same at $r \sim 10^{-30}$ m (GUT)

Why are the three interactions so different ?

- strong $\simeq$ weak ??
- $\frac{1}{r}$ – law describes an interaction of long range
  Strong and weak interactions are short range !
- photons can be seen by eye, gluons not

etc. etc. etc.

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Two features are responsible for the difference :

- properties of the vacuum
- photons do not carry charge, but gluons carry colour

Properties of the vacuum

- At the time of Torricelli (1608–1647), the vacuum was a terrifying state: horror vacui
- Das Nichts nichtet. M. Heidegger
- The vacuum is the state of lowest energy
  Energy is conserved → vacuum is stable

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- The vacuum is the state of lowest energy
  Energy is conserved → vacuum is stable
- Properties of the ground state are determined by the dynamics $\mathbf{H}|0\rangle = 0$

⇒ The vacuum is a complicated state

- The state of lowest energy happens to be transparent for photons, opaque for W,Z
- Why ?

Standard Model

$$\boxed{\text{vacuum = condensate of Higgs particles}}$$

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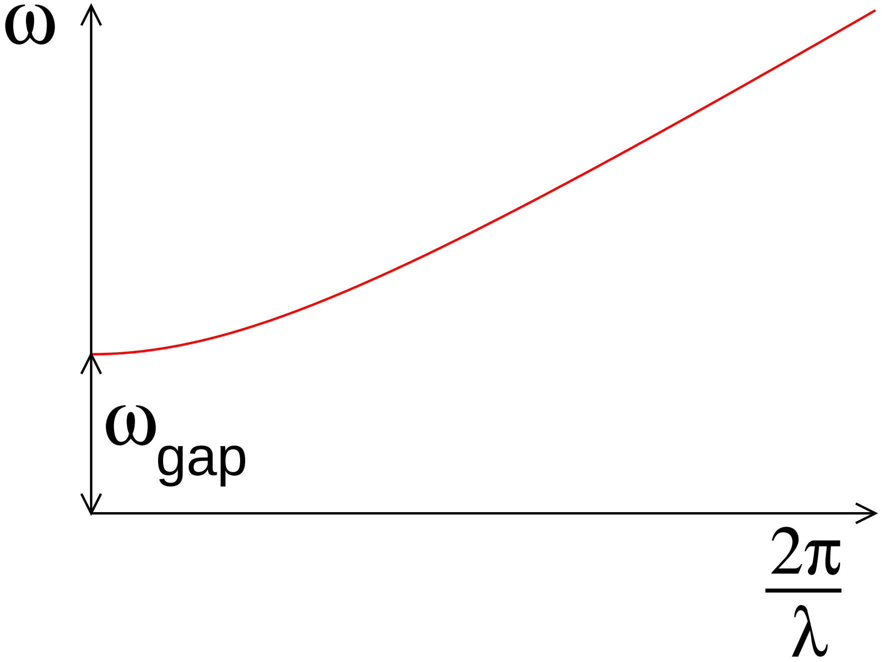
- The Higgs particles do carry flavour

⇒ W,Z do take notice

⇒ W,Z-waves of low frequency cannot propagate

⇒ For such waves, the vacuum is opaque

Frequency versus wave number



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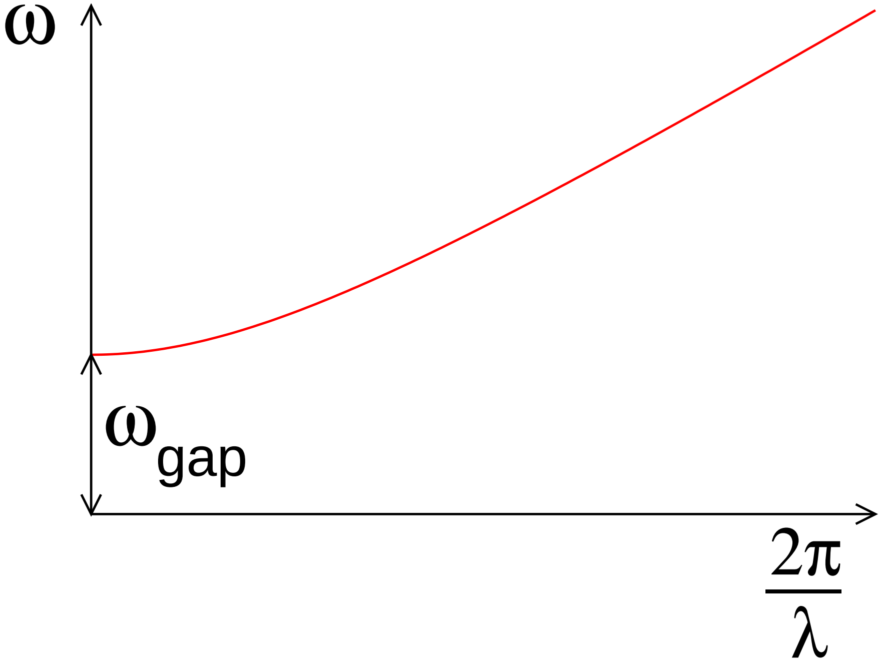

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- $M_\gamma, M_{\text{G}} = 0 \;\Rightarrow\;$ γ, G have $v = c$
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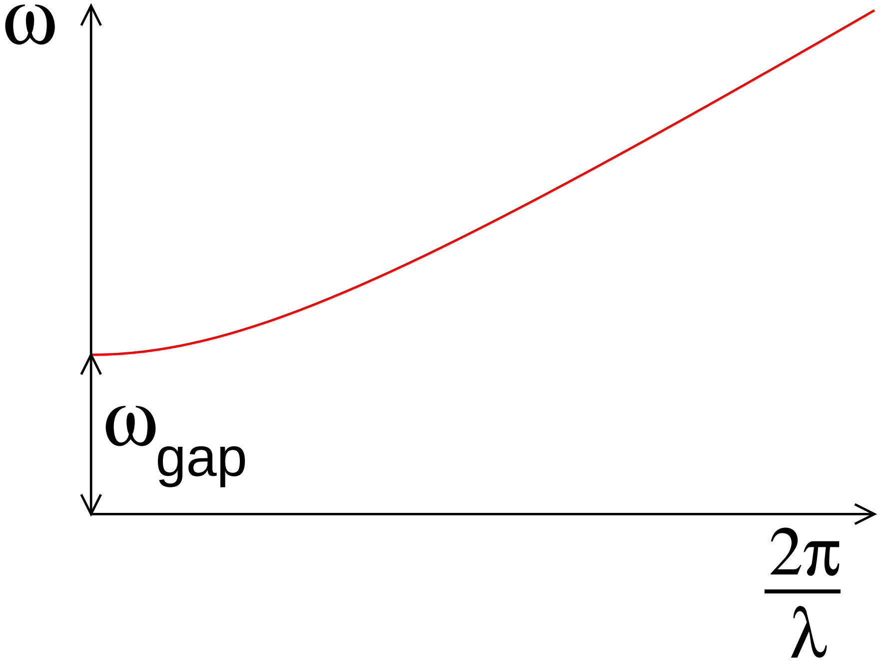

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- Penetration depth for small frequencies:

$$d = \frac{\hbar}{Mc} \qquad d_{\text{W}}, d_{\text{Z}} \sim 2 \cdot 10^{-18}\,\text{m}$$

Consequence for strength of weak interaction

- Interaction energy is reduced for $r \gtrsim d$:

$$\frac{g_w^2}{4\pi r} \Rightarrow \frac{g_w^2}{4\pi r} \cdot e^{-\frac{r}{d}}$$

Penetration depth of the weak interaction is small:

$$d = \frac{\hbar}{M_W c} = 2.4542(9) \times 10^{-18}\ \text{m}$$

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- Effective strength at low energies:

$$\int d^3r\, \frac{g_w^2}{4\pi r} \cdot e^{-\frac{r}{d}} = g_w^2 d^2$$

⇒ At low energies, the weak interaction is weak

Fermi constant

- The weak interaction is known since the 1930’s
- The Fermi constant is the effective strength

  Expressed in terms of Standard model quantities:

$$G_F = \frac{\hbar c}{4\sqrt{2}}\, g_w^2\, d^2 = \frac{\hbar^3}{4\sqrt{2}\, c}\, \frac{g_w^2}{M_W^2}$$

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$$\frac{g_w^2}{4\pi} = \frac{1}{29.461(21)}$$

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weak

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- Is about 5 times larger than the fine structure constant

$$\frac{e^2}{4\pi\epsilon_0\,\hbar c} = \frac{1}{137.03599911(46)}$$

electromagnetic

Consequence for leptons and quarks

- Leptons, quarks also feel the Higgs condensate

⇒ become massive like $W^{\pm}, Z$

- t-quark interacts very strongly with the Higgs particles
  electrons, neutrini barely interact with them

⇒ $m_t \gg m_e, m_\nu$

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$\Rightarrow$ $m_t \gg m_e, m_\nu$

- The observed mass pattern is bizarre:

$$m_t = 174.2(3.3)\ \text{GeV}$$

$$\vdots$$

$$m_e = 0.510998892(4)\ \text{MeV}$$

Neutrini even lighter, by orders of magnitude

- The pattern of the lepton and quark masses is not understood at all

Size of the atoms

$$a_{Bohr} = \frac{4\pi\epsilon_0\,\hbar^2}{e^2\,m_e} = \frac{4\pi\epsilon_0\,\hbar c}{e^2} \times \frac{\hbar}{m_e c}$$

- Electron-Higgs interaction happens to be weak

⇒ Electrons pick up only little mass

- e is one of the three basic coupling constants

$$\frac{e^2}{4\pi\epsilon_0\,\hbar c} \simeq \frac{1}{137}$$ happens to be small

⇒ Viewed from the SM: atoms are huge

- Muons interact 200 times more strongly

⇒ Muonic atoms are 200 times smaller,
but only live for 2 microseconds

Properties of the vacuum – summary

- W, Z sense the presence of Higgs particles in the vacuum, γ, G do not

$\Rightarrow$ W, Z are subject to the horror vacui

- Leptons and quarks also have the horror vacui
  some are more terrified than others
- The atoms are huge because the electrons are not much intimidated
- Puzzle: gravitons feel all forms of energy
  Why do they not take notice of the Higgses ?

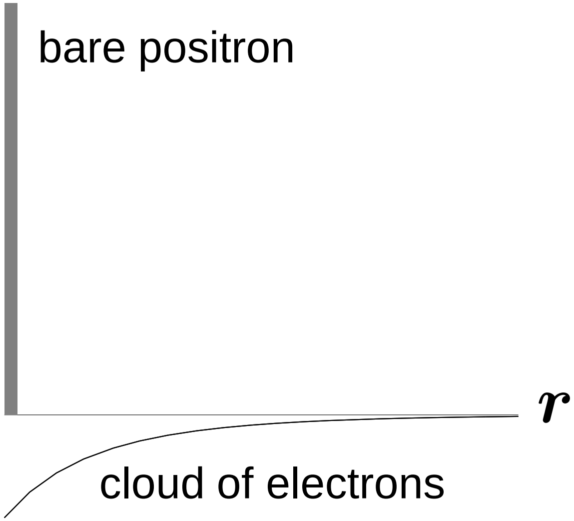
Compare the electromagnetic and strong interactions

- Photons do not have charge
- Gluons do have colour

⇒ The e.m. and strong interactions behave very differently at low energies

Compare structure of leptons and quarks

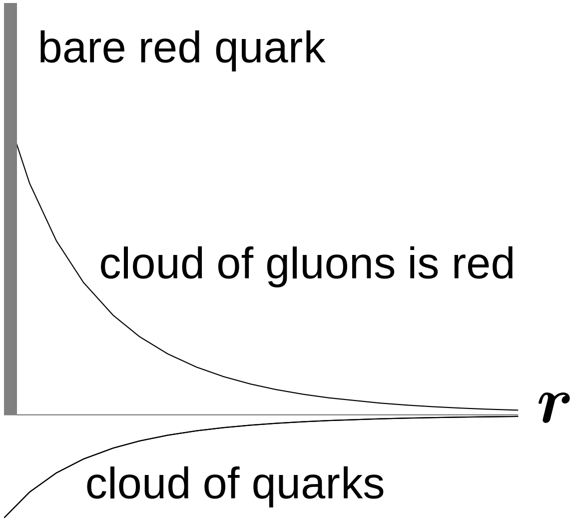
QED
density of charge



$$\boxed{e < e|_{\text{bare}}}$$

vacuum shields charge

QCD
density of colour



$$\boxed{g_s > g_s|_{\text{bare}}}$$

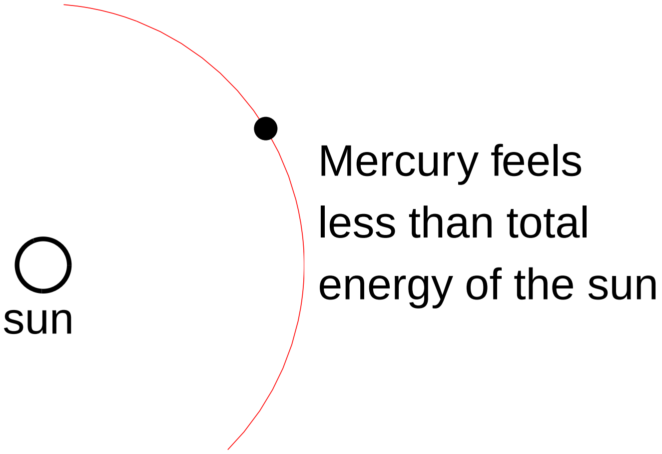
vacuum amplifies colour

⇒ The electromagnetic and strong interactions
polarize the vacuum very differently

Comparison with gravity

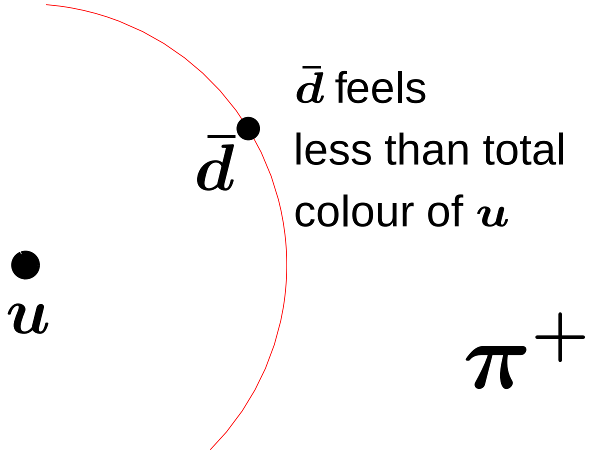
- Source of gravitational field: energy
  gravitational field does carry energy
- Source of e.m. field: charge
  e.m. field does not carry charge
- Source of gluon field: colour
  gluon field does carry colour

gravity



Mercury feels
less than total
energy of the sun

strong interaction



$\bar{d}$ feels
less than total
colour of u

- Perihelion shift of Mercury: $43'' = 50'' - \underset{\Uparrow}{7''}$ per century

Consequence of shielding/amplification

- Vacuum amplifies gluonic field of a quark
  Vacuum shields electric field of a lepton
- The difference has dramatic consequences:
  although the Lagrangians of QCD and QED
  are very similar, the properties of the strong
  and electromagnetic interactions are totally different
- field energy surrounding isolated quark = ∞
  only colour neutral states have finite energy

$\Rightarrow$ colour is confined

nuclear forces = van der Waals forces of QCD

- field energy surrounding a charged particle is finite

$\Rightarrow$ charge is not confined

Interaction at large distances, low energies

QED remains weak

$$\frac{e^2}{4\pi\epsilon_0\,\hbar c} \simeq \frac{1}{137}$$

photons, leptons
nearly decouple

QCD becomes strong

$$\frac{g_s^2}{4\pi} \simeq 1$$

gluons, quarks
confined

⇒ In QED, perturbation theory works at low energies
Spectrum of states can be seen in the Lagrangian

⇒ In QCD, perturbation theory fails at low energies
Spectrum of states cannot be seen in the Lagrangian
Fields in the Lagrangian $\not\Leftrightarrow$ observed particles

This is why it took so long to realize that the strong interactions originate in a gauge field. Pauli had studied nonabelian gauge fields in the 1930's, but dropped the idea because the Fock space contains massless particles

Short distances, high energies

- At short distances, perturbation theory works also in QCD: deep inelastic scattering, quark jets, gluon jets
- Leading order perturbation theory formula for potential between u and $\bar{u}$:

$$V = -\frac{4}{3}\,\frac{g_s^2}{4\pi r}\;, \text{ with } \;\frac{g_s^2}{4\pi} = \frac{2\pi}{7\,|\,\ln(r\Lambda_{\text{QCD}})|}$$

Only holds at short distances: $g_s \longrightarrow 0$ for $r \longrightarrow 0$
Logarithm must be large

- Experimental value at $r = \dfrac{\hbar}{M_{\text{Z}}\,c} \simeq 2 \cdot 10^{-18}$ m:

$$\frac{g_s^2}{4\pi} = \frac{1}{8.22 \pm 0.11} \qquad \Rightarrow |\,\ln(r\Lambda_{\text{QCD}})| \simeq 8$$

Comparison of strong and electromagnetic coupling constants

Take $$r = \frac{\hbar}{M_\mathrm{Z}\, c} \simeq 2 \cdot 10^{-18}\,\mathrm{m}$$

Strong coupling constant : $$\frac{g_s^2}{4\pi} = \frac{1}{8.43 \pm 0.14}$$

Fine structure constant : $$\frac{e^2}{4\pi} = \frac{1}{128.922 \pm 0.049}$$

Excellent experimental evidence for scale dependence, not only of g_s, but also of e and g_w

⇒ Vacuum polarization plays central role in our understanding of the laws of nature

Magnetic moment of the muon

$$\mu = \frac{e\hbar}{2\,m_\mu} \qquad \text{Dirac}$$

- Quantum fluctuations generate correction

$$\mu = \frac{e\hbar}{2\,m_\mu}\{1 + \text{a}\}$$

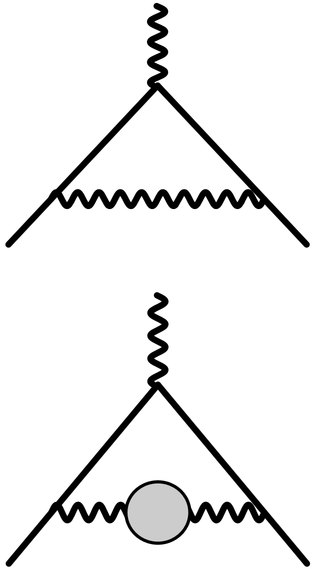
The term $2(1 + \text{a})$ is called the g-factor of the muon

- μ is measured to phantastic precision:

$$\text{a} = (11\,659\,203 \pm 8) \times 10^{-10}$$

- Standard Model predicts the value of a, but only an approximate evaluation is feasible

Contribution from vacuum polarization



- Leading contribution stems from triangle graph

  $$\text{a} = \frac{\alpha_{em}}{2\pi} + O(\alpha_{em}^2) \qquad \text{Schwinger}$$

- The terms of $O(\alpha_{em}^2)$ are also known explicitly, except for the effect due to hadronic vacuum polarization
- Contribution from hvp can be expressed in terms of the cross section for $e^+e^- \rightarrow$ hadrons

$$10^{10} \times \text{a}_{\text{hvp}} = 694.8 \pm 8.6$$

Theoretical result

- One recent analysis yields

$$10^{10} \times \mathrm{a}_{\mathrm{th}} = \mathbf{11659179.4} \pm \begin{cases} 8.6 & \text{hvp} \\ 3.5 & \text{lbl} \\ 0.4 & \text{ew} \end{cases}$$

⇒ Experiment agrees with theory to 8 digts !

- The difference between experiment and theory is

$$|\mathrm{a}_{\mathrm{exp}} - \mathrm{a}_{\mathrm{th}}| = \mathbf{23.6} \pm \mathbf{12.3} \times \mathbf{10^{-10}}$$

This amounts to 1.9 σ, shows up in the ninth digit

Some authors find somewhat larger discrepancies

Electromagnetic versus strong interactions – summary

- Photons do not carry charge
- Gluons are coloured

⇒ Photons and gluons polarize the vacuum quite differently

- Very strong experimental evidence for the vacuum to react exactly as field theory requires