Recent developments in light flavor hadron physics

H. Leutwyler

Universität Bern

Helmholtz-Institut für Strahlen- und Kernphysik

Universität Bonn, 13. Dezember 2007

Standard Model at low energies

- For $E \ll M_{\mathrm{W}} c^2 \simeq 80$ GeV:
  weak interactions are frozen, neutrini decouple
  Standard Model reduces to QED + QCD

Standard Model at low energies

- For $E \ll M_{\rm w} c^2 \simeq 80$ GeV:
  weak interactions are frozen, neutrini decouple
  Standard Model reduces to QED + QCD
- Precision theory for cold matter,
  atomic structure, solids, ...

  Bohr radius: $$a = \frac{4\pi}{e^2\, m_e}$$

Standard Model at low energies

- For $E \ll M_{\rm W} c^2 \simeq 80$ GeV:
  weak interactions are frozen, neutrini decouple
  Standard Model reduces to QED + QCD
- Precision theory for cold matter,
  atomic structure, solids, $\ldots$

  Bohr radius: $$a = \frac{4\pi}{e^2\, m_e}$$
- Effective theory, dynamical variables are:
  gauge fields for photons and gluons
  Fermi fields for leptons and quarks

Standard Model at low energies

- For $E \ll M_{\text{w}} c^2 \simeq 80\,$GeV:
  weak interactions are frozen, neutrini decouple
  Standard Model reduces to QED + QCD
- Precision theory for cold matter,
  atomic structure, solids, $\ldots$

  Bohr radius: $$a = \frac{4\pi}{e^2\, m_e}$$
- Effective theory, dynamical variables are:
  gauge fields for photons and gluons
  Fermi fields for leptons and quarks
- Interaction fully determined by group geometry
  Lagrangian contains 3 coupling constants:

$$\boxed{g,\ e,\ \theta}$$

Standard Model at low energies

- Quark and lepton mass matrices can be brought to diagonal form, eigenvalues real, positive

$$\boxed{m_e,\, m_\mu,\, m_\tau,\, m_u,\, m_d,\, m_s,\, m_c,\, m_b,\, m_t}$$

Standard Model at low energies

- Quark and lepton mass matrices can be brought to diagonal form, eigenvalues real, positive

$$\boxed{m_e,\; m_\mu,\; m_\tau,\; m_u,\; m_d,\; m_s,\; m_c,\; m_b,\; m_t}$$

- For bound states of quarks,
  e.m. interaction is a small perturbation
  Perturbation series in powers of e $\surd$

- Focus on the leading term, i.e. set $e = 0$

$\Rightarrow$ SM Lagrangian reduces to QCD

$$\boxed{g\,,\; \theta\,,\; m_u\,, m_d\,,\; m_s\,,\; m_c\,,\; m_b\,,\; m_t}$$

Standard Model at low energies

- Quark and lepton mass matrices can be brought to diagonal form, eigenvalues real, positive

$$\boxed{m_e,\ m_\mu,\ m_\tau,\ m_u,\ m_d,\ m_s,\ m_c,\ m_b,\ m_t}$$

- For bound states of quarks,
  e.m. interaction is a small perturbation
  Perturbation series in powers of e $\surd$
- Focus on the leading term, i.e. set $e = 0$

$\Rightarrow$ SM Lagrangian reduces to QCD

$$\boxed{g\ ,\ \theta\ ,\ m_u\ ,m_d\ ,\ m_s\ ,\ m_c\ ,\ m_b\ ,\ m_t}$$

- vacuum angle θ breaks CP
  Neutron dipole moment is very small

$\Rightarrow$ strong upper limit, $\theta \simeq 0$

Energy gap of QCD

- Main characteristic of QCD at low energies:
  Energy gap is very small, $M_\pi \simeq$ 140 MeV
- Part I of this talk:
  - This has to do with a hidden approximate symmetry
  - Symmetry becomes exact for $m_u, m_d \rightarrow 0$

  $\Rightarrow$ Energy gap disappears: pions become massless
  - In reality $m_u, m_d \neq 0$, but very small

  $\Rightarrow$ Symmetry is nearly perfect

Plan of talk

- Part II
  - Progress achieved on the lattice allows to simulate quarks with sufficiently small masses
  - Can establish contact with physics, using χPT
  - First meaningful determinations of effective coupling constants
- Part III
  - Puzzling results on $K_L \to \pi\mu\nu$ from NA48
  - Data are in conflict with a venerable low energy theorem: Callan-Treiman-relation
  - Physics beyond the Standard Model or incorrect measurement ?

Plan of talk

- Part IV
  - Significant progress in understanding the interaction among the pions at low energies

$\Rightarrow$ Currently, theory is even ahead of experiment ...

  - Masses and widths of the lowest resonances can now be calculated in a reliable manner

$\Rightarrow$ σ turns out to be lighter than the ρ

  - Precision experiments start testing the theory

Theoretical paradise

$$m_u = m_d = m_s = 0 \qquad\qquad m_c = m_b = m_t = \infty$$

QCD with 3 massless quarks

- Interactions of u, d, s are identical
  If the masses are the same, there is no difference at all

Theoretical paradise

$$m_u = m_d = m_s = 0 \qquad\qquad m_c = m_b = m_t = \infty$$

QCD with 3 massless quarks

- Interactions of u, d, s are identical
  If the masses are the same, there is no difference at all
- More symmetry: for massless fermions,
  right and left do not communicate

⇒ Lagrangian of massless QCD is invariant under independent rotations of the right– and left–handed quark fields $\quad \mathrm{SU(3)_R \times SU(3)_L}$

⇒ 16 conserved “charges”

$Q_1^{\mathrm{V}}, \ldots, Q_8^{\mathrm{V}}$ (vector currents)

$Q_1^{\mathrm{A}}, \ldots, Q_8^{\mathrm{A}}$ (axial currents)

Chiral symmetry

- Charges commute with the Hamiltonian:

$$[Q_i^{\mathrm{V}}, H_0] = 0 \qquad [Q_i^{\mathrm{A}}, H_0] = 0$$

  "Chiral symmetry" of massless QCD

- Consequence of anomaly in singlet axial current: θ only enters in the combination $\det m \times e^{i\theta}$

$\Rightarrow$ Massless theory is independent of θ

Massless QCD

- Lagrangian contains a single parameter: g
- g is net colour of a quark, depends on radius of the region considered. Colour contained within radius r:

$$\frac{g^2}{4\pi} = \frac{2\pi}{9\,|\ln(r\,\Lambda_{\text{QCD}})|}$$

Massless QCD

- Lagrangian contains a single parameter: g
- g is net colour of a quark, depends on radius of the region considered. Colour contained within radius r:

$$\frac{g^2}{4\pi} = \frac{2\pi}{9\,|\ln(r\,\Lambda_{\text{QCD}})|}$$

- Intrinsic scale Λ_{QCD} carries dimension

$\Rightarrow$ No dimensionless free parameter

All dimensionless physical quantities are pure numbers, determined by the theory

Cross sections can be expressed in terms of Λ_{QCD} or in the mass of the proton

- It would be great if nature could be described in this way

Asymmetry of ground state

- The state of lowest energy must be asymmetric with respect to chiral rotations: $Q_i^{\mathrm{A}}\,|0\rangle \neq 0$ Nambu 1960

$\Rightarrow$ Chiral symmetry is hidden, “spontaneously broken”

- Very strong experimental evidence $\surd$
  Very strong evidence from lattice calculations $\surd$
  Analytic understanding of the ground state still poor

- If $|0\rangle$ were invariant under chiral rotations, then
  $\langle 0|\,\bar{u}u\,|0\rangle = \langle 0|\,\bar{u}_{\mathrm{R}}u_{\mathrm{L}}\,|0\rangle + c.c. = 0$
  Since $|0\rangle$ is not symmetric $\Rightarrow$ no reason for $\langle 0|\,\bar{u}u\,|0\rangle$ to vanish, order parameter of lowest dimension

- State of lowest energy is invariant under the vector charges, $Q_i^{\mathrm{V}}\,|0\rangle = 0$ Vafa and Witten 1984

$\Rightarrow$ $\langle 0|\,\bar{u}u\,|0\rangle = \langle 0|\,\bar{d}d\,|0\rangle = \langle 0|\,\bar{s}s\,|0\rangle$

Goldstone bosons

- Consequence of $Q_i^{\mathsf{A}}\,|0\rangle \neq 0$:

  $$H_0\,Q_i^{\mathsf{A}}\,|0\rangle = Q_i^{\mathsf{A}}\,H_0\,|0\rangle = 0$$

  Spectrum must contain 8 states $Q_1^{\mathsf{A}}\,|0\rangle,\ \ldots\ ,\ Q_8^{\mathsf{A}}\,|0\rangle$
  with $E = 0$, spin 0, negative parity, octet of $\mathrm{SU(3)}_{\mathrm{V}}$
  "Goldstone bosons"

- The 8 lightest mesons do have these quantum numbers: $\pi^+,\pi^0,\pi^-,K^+,K^0,\bar{K}^0,K^-,\eta$
  But massless they are not

Back to earth

- world $\neq$ paradise

Back to earth

- world $\neq$ paradise

  $m_u, m_d, m_s \neq 0$ $\qquad\qquad$ $m_c, m_b, m_t \neq \infty$

  Light quark masses break chiral symmetry,
  allow the left to talk to the right

Back to earth

- world $\neq$ paradise

  $m_u, m_d, m_s \neq 0$ $\qquad$ $m_c, m_b, m_t \neq \infty$

  Light quark masses break chiral symmetry,
  allow the left to talk to the right

- Chiral symmetry broken in two ways:
  spontaneously $\qquad$ $\langle 0|\,\bar{q}_{\mathsf{R}}\,q_{\mathsf{L}}\,|0\rangle \neq 0$
  explicitly $\qquad$ $m_u\,,\,m_d\,,\,m_s \neq 0$

- Only the diagonal vector currents are strictly conserved in QCD: $N_u, N_d, N_s, N_c, N_b, N_t \;\longrightarrow$ baryon number, electric charge, strangeness, charm, ...

- It so happens that m_u, m_d, m_s are small

$\Rightarrow$ H_{QCD} has an approximate SU(3)$_{\mathsf{L}}\times$SU(3)$_{\mathsf{R}}$ symmetry

Light quark masses as perturbations

- Masses of the light quarks enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d + m_s \, \bar{s}s\}$$

  H_0 describes u, d, s as massless, c, b, t as massive

  H_0 is invariant under $\text{SU(3)}_\text{L} \times \text{SU(3)}_\text{R}$

Expansion in powers of m_u, m_d, m_s	$\Longleftrightarrow$	Perturbation series in powers of H_1

- H_0 treats π, K, η as massless particles

  H_1 gives them a mass

Gell-Mann-Oakes-Renner formula

- First order perturbation theory yields:

$$M_\pi^2 = (m_u + m_d) \times |\langle 0|\,\bar{u}u\,|0\rangle| \times \frac{1}{F_\pi^2}$$

$$\Uparrow \qquad\qquad \Uparrow$$

explicit spontaneous

Gell-Mann, Oakes & Renner 1968

Coefficient: decay constant F_π

$$\langle 0|\,\bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\,p^\mu\sqrt{2}\,F_\pi$$

Value of F_π is known from $\pi^+ \rightarrow \mu^+\nu$

$\Rightarrow$ The main low energy properties of QCD can be understood on the basis of this formula

Pattern of lowest levels

- $M_\pi^2 = (m_u + m_d)\, B + O(m^2)$

$\Rightarrow$ The energy gap of QCD is small because m_u, m_d happen to be small

Pattern of lowest levels

- $M_\pi^2 = (m_u + m_d)\,B + O(m^2)$

$\Rightarrow$ The energy gap of QCD is small because m_u, m_d happen to be small

- $M_{K^+}^2 = (m_u + m_s)\,B + O(m^2)$
  $M_{K^0}^2 = (m_d + m_s)\,B + O(m^2)$

$\Rightarrow$ M_K^2 is much larger than M_π^2, because m_s happens to be large compared to m_u, m_d

- Goldstone boson masses measure the strength of symmetry breaking $\Rightarrow$ strongly violate SU(3)

- Check: first order perturbation theory also yields
  $M_\eta^2 = \frac{1}{3}\,(m_u + m_d + 4m_s)B + O(m^2)$

$\Rightarrow$ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 $\surd$

Magnitude of the perturbations due to m_u, m_d, m_s

- $\langle 0|\,\bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\,p^\mu\sqrt{2}\,F_\pi$
  $\langle 0|\,\bar{s}\gamma^\mu\gamma_5 u|K^+\rangle = i\,p^\mu\sqrt{2}\,F_K$
  Difference between F_K and F_π comes from $m_s \neq m_d$
- Observed ratio: $\dfrac{F_K}{F_\pi} = 1.19 \pm 0.01$

  Branching fraction of $K \to \pi e \nu$ changed by $> 3\,\sigma$ in 2004 ! $1.22 \to 1.19$

$\Rightarrow$ $m_s - m_d$ generates correction of order 20%

- $m_u, m_d \ll m_s \Rightarrow$ correction mainly comes from m_s
- effects from m_u, m_d are tiny

Conclusions for part I

$G =$ SU(3)$_\text{L} \times$SU(3)$_\text{R}$ is an approximate symmetry of H_QCD

$|0\rangle$ approximately symmetric only under SU(3) $\subset G$

- World we live in is close to the paradise
- Light quark masses amount to a perturbation

Conclusions for part I

$G = \mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$ is an approximate symmetry of H_{QCD}

$|0\rangle$ approximately symmetric only under $\mathrm{SU(3)} \subset G$

- World we live in is close to the paradise
- Light quark masses amount to a perturbation
- Chiral part of the symmetry is hidden

$\Rightarrow$ Only the subgroup $\mathrm{SU(3)} \subset G$ is an approximate symmetry of the spectrum and of the matrix elements "Eightfold way", $u \leftrightarrow d \leftrightarrow s$

- m_u, m_d are particularly small

$\Rightarrow$ $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ is a nearly exact symmetry of H_{QCD}

$\Rightarrow$ Expansion in powers of m_u, m_d converges very rapidly

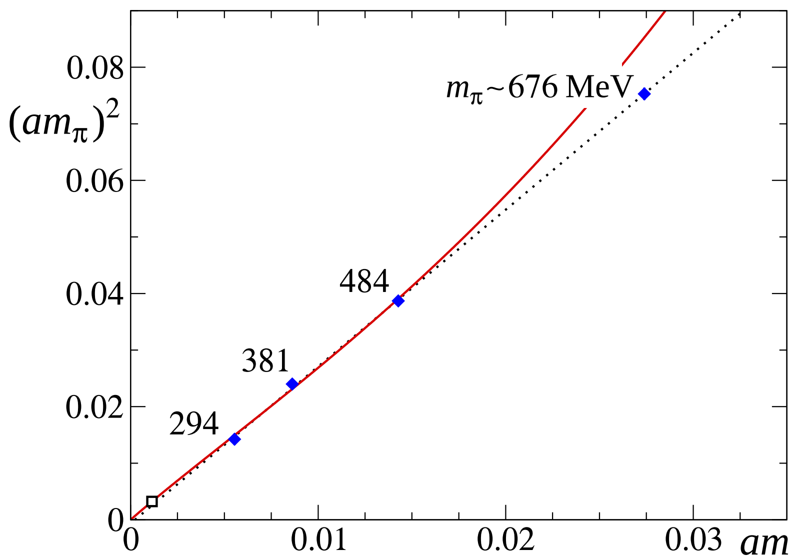
Part II: Lattice results for light quarks

- Considerable progress with light dynamical fermions
- QCD with two light flavours: quenching no more needed
- Several groups have data of impressive quality for QCD with $N_f = 2$, in isospin limit $m_u = m_d = m$
  Reach pion masses of 300 MeV or even lower

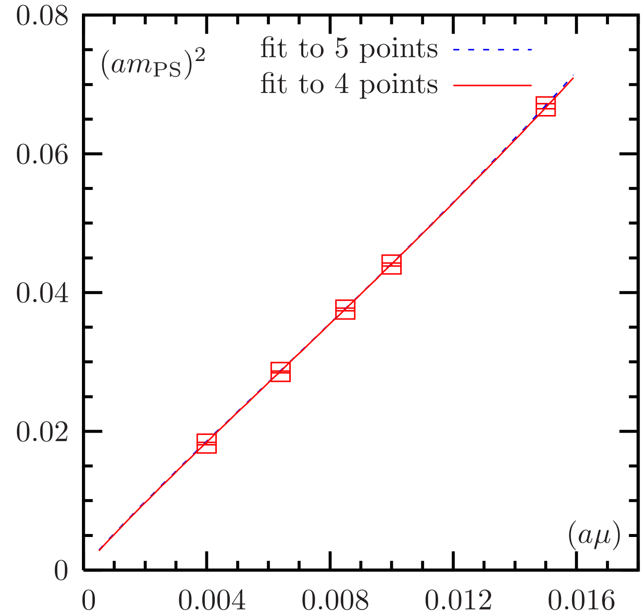
⇒ Legitimate to use χPT for the extrapolation to the value of interest, $m = \frac{1}{2}(m_u^{\text{phys}} + m_d^{\text{phys}})$

⇒ GMOR formula can now be checked on the lattice: Determine M_π as a function of m

Pion mass as a function of the quark mass



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- Proportionality of M_π^2 to the quark mass appears to hold out to values of $m_u = m_d$ that are an order of magnitude larger than in nature
- Main limitation: systematic uncertainties in particular: $N_f = 2 \rightarrow N_f = 3$

Expansion of M_π^2 in powers of the quark mass

- Consequences of hidden, approximate symmetry can be worked out by means of an effective field theory

  Weinberg 1979

- GMOR formula represents leading term of χPT
- At NLO, the expansion contains a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$

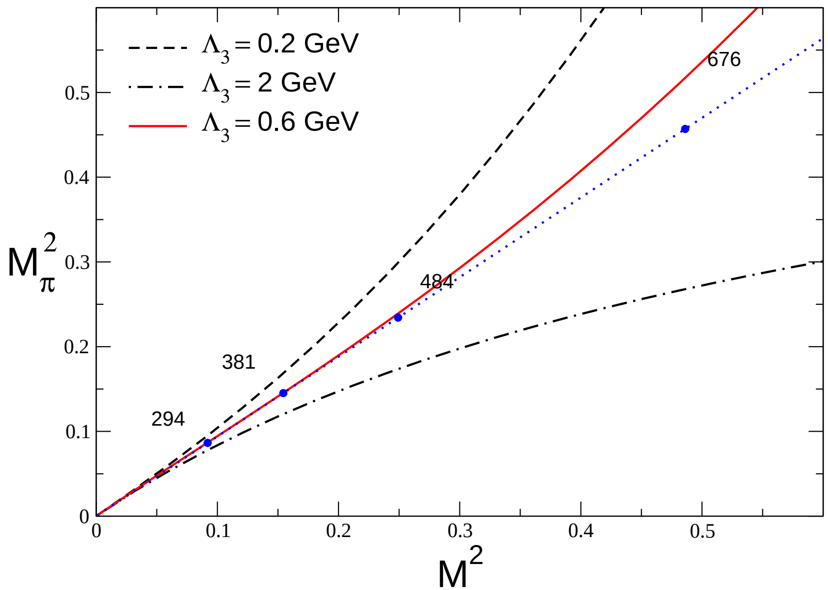
$$M^2 \equiv B(m_u + m_d)$$

Gasser & L. 1983

- Coefficient is determined by pion decay constant
  Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3) mass formulae:
  $0.2\,\text{GeV} < \Lambda_3 < 2\,\text{GeV}$

  Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



Express the result for Λ_3 in terms of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

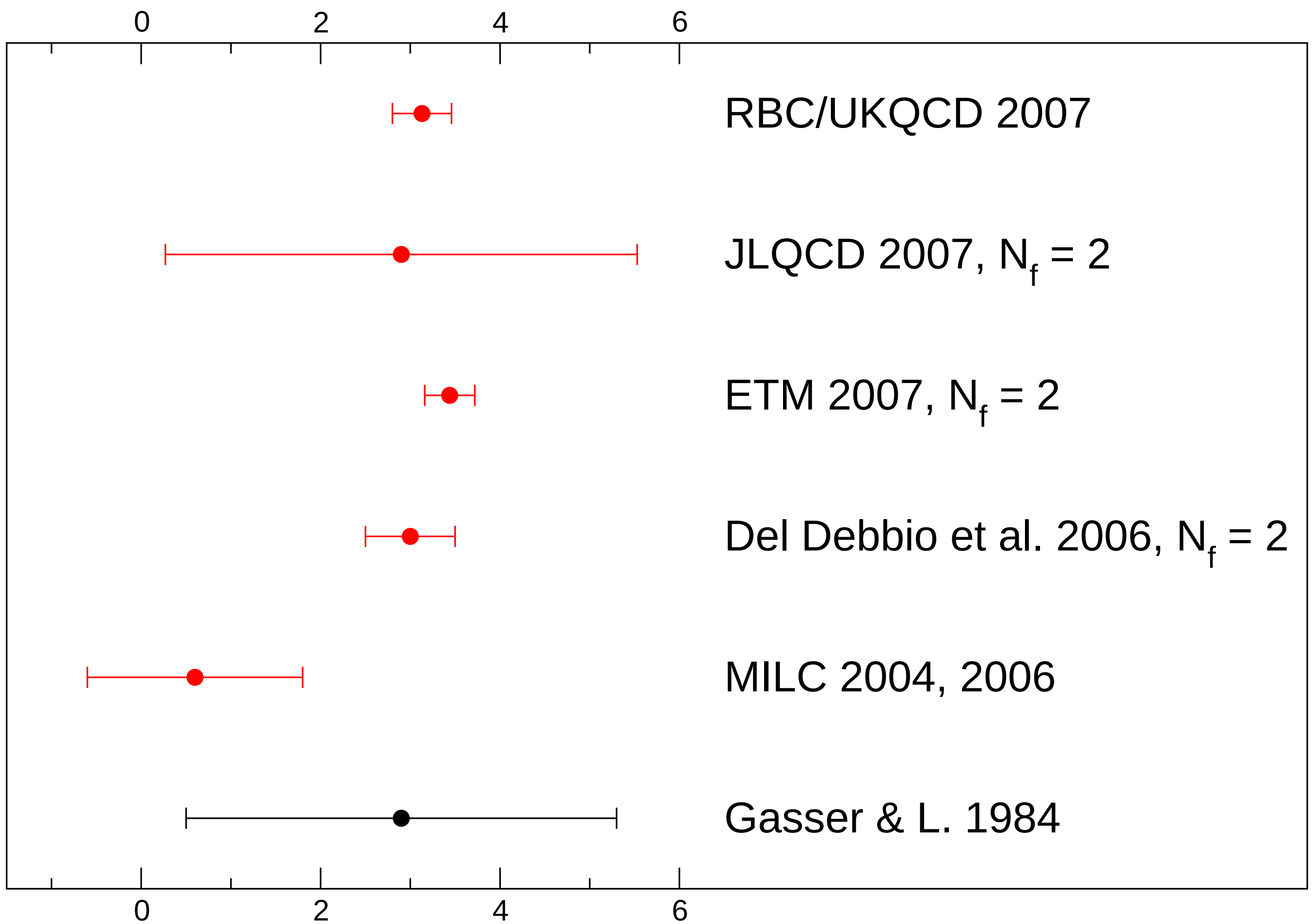
$\bar{\ell}_3 = 2.9 \pm 2.4 \quad \leftrightarrow \quad 0.2\,\text{GeV} < \Lambda_3 < 2\,\text{GeV}$ — Gasser & L. 1984

$\bar{\ell}_3 = 0.6 \pm 1.2$ — MILC 2004, 2006

$\bar{\ell}_3 = 3.0 \pm 0.5$ — Del Debbio et al. 2006

⋮

Lattice results for Λ_3



$$\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

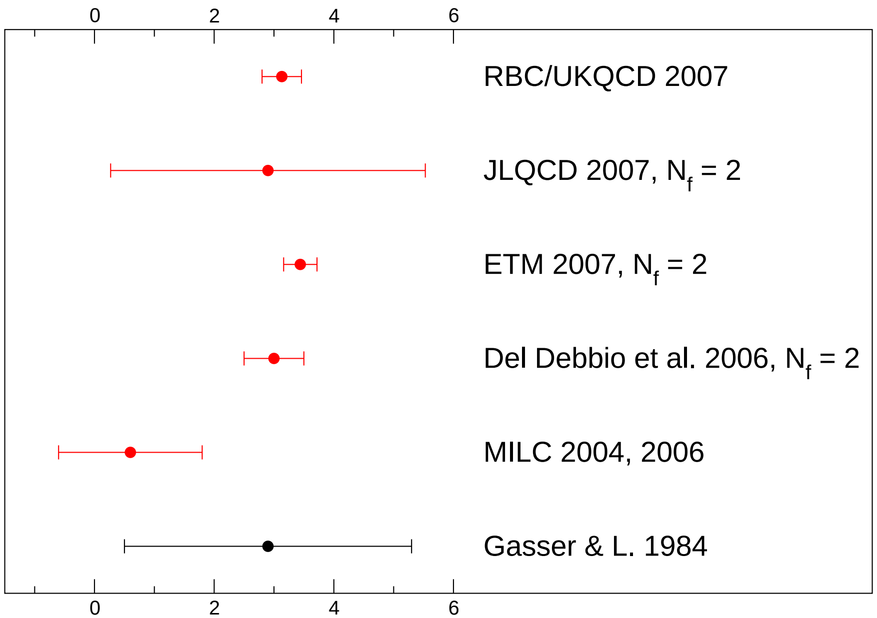
$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

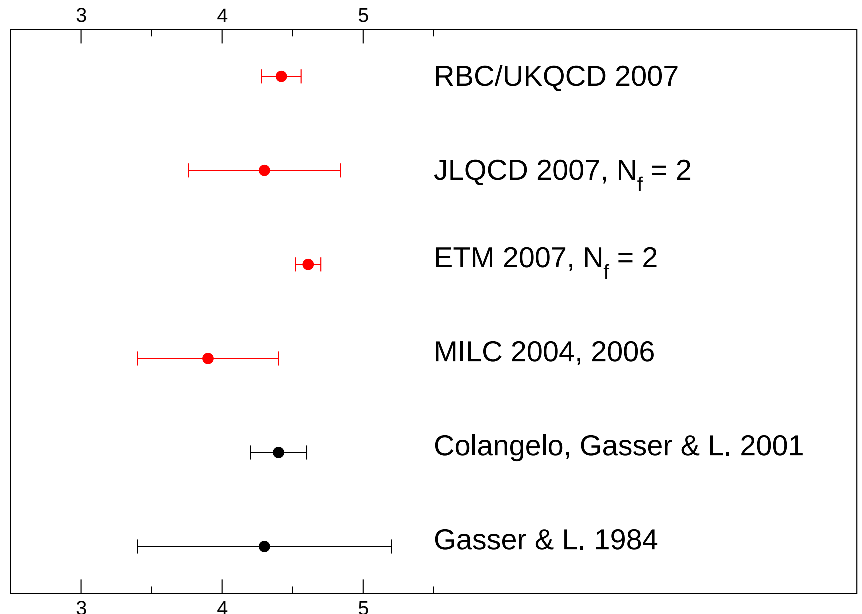
F is value of pion decay constant in limit $m_u, m_d \to 0$

- Structure is the same, coefficients and scale of logarithm are different
- Quark mass dependence of F_π can also be measured on the lattice $\Rightarrow$ measurement of Λ_4

Lattice results for Λ_3 and Λ_4



$$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M_\pi^2}$$



$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

NNLO

- The next order contains the square of a logarithm:

$$M_\pi^2 = M^2\left\{1+\frac{x}{2}\ln\frac{M^2}{\Lambda_3^2}+\frac{17x^2}{8}\left(\ln\frac{M^2}{\Lambda_M^2}\right)^2+x^2k_M+O(M^6)\right\}$$

$$F_\pi = F\left\{1-x\ln\frac{M^2}{\Lambda_4^2}-\frac{5x^2}{4}\left(\ln\frac{M^2}{\Lambda_F^2}\right)^2+x^2k_F+O(M^6)\right\}$$

$$x \equiv \left(\frac{M}{4\pi F}\right)^2$$

Colangelo 1995, Bijnens et al. 1996, Bürgi 1996

- For physical value of m_u, m_d, the NNLO terms are tiny

$\Rightarrow$ Size of $\Lambda_M, k_M, \Lambda_F, k_F$ barely known

- Must become clearly visible if m_u, m_d are made larger

Expansion in powers of m_s

- Theoretical reasoning
  - The eightfold way is an approximate symmetry
  - Only coherent way to understand this within QCD: $m_s - m_d$, $m_d - m_u$ can be treated as perturbations
  - Since $m_u, m_d \ll m_s$

$\Rightarrow$ m_s can be treated as a perturbation

$\Rightarrow$ Expect expansion in powers of m_s to work, but convergence to be comparatively slow

Expansion in powers of m_s

- Theoretical reasoning
  - The eightfold way is an approximate symmetry
  - Only coherent way to understand this within QCD: $m_s - m_d$, $m_d - m_u$ can be treated as perturbations
  - Since $m_u, m_d \ll m_s$

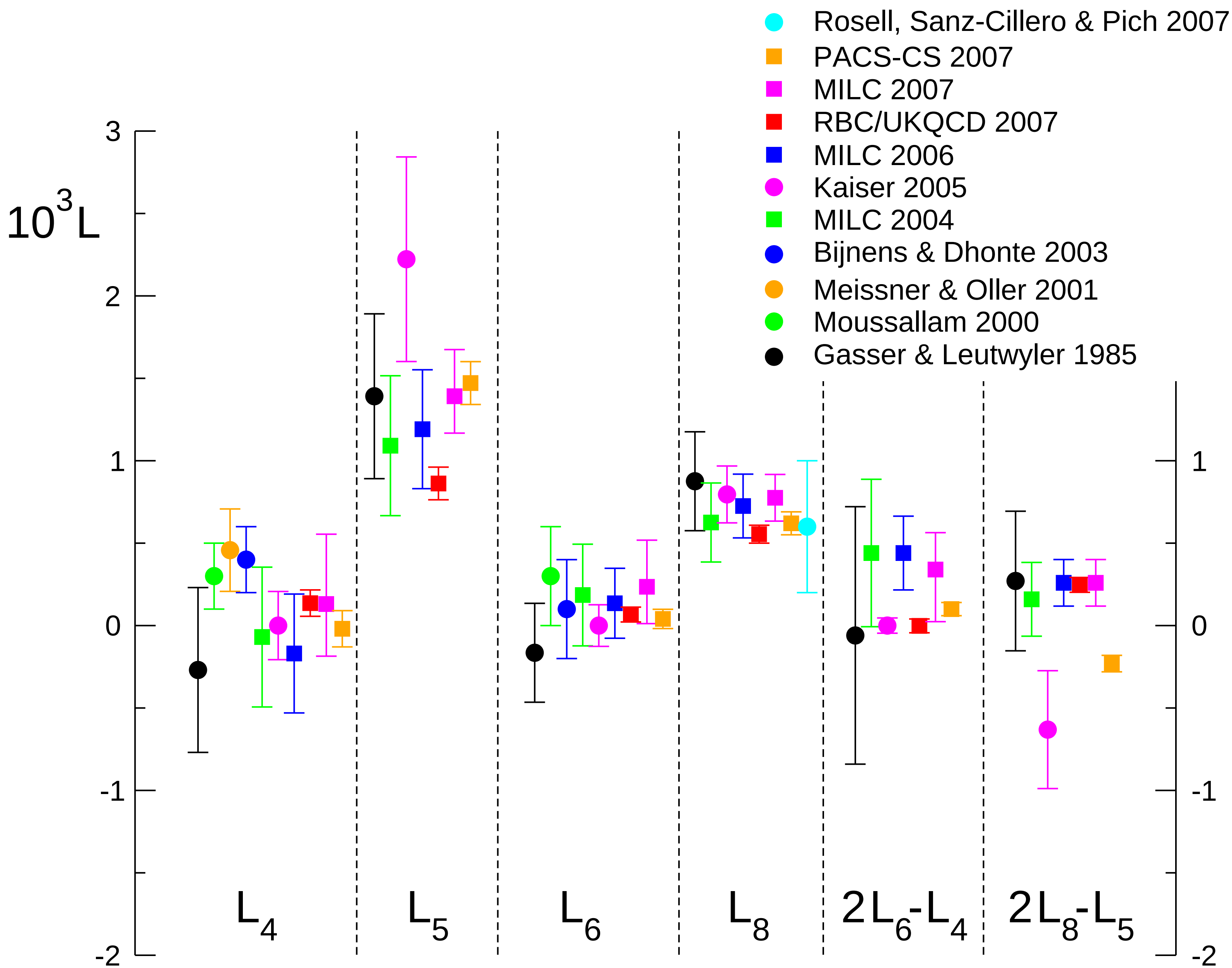
$\Rightarrow$ m_s can be treated as a perturbation

$\Rightarrow$ Expect expansion in powers of m_s to work, but convergence to be comparatively slow

- This can now also be checked on the lattice
  - Use preliminary results of RBC/UKQCD

    Lin & Scholz in Proc. Lattice 2007, arXiv : 0710.0536
  - Those effective coupling constants that are relevant for F_π, F_K, M_π, M_K have been determined on this basis, both for SU(2)×SU(2) and SU(3)×SU(3)
    LO: F, B, F_0, B_0 NLO: $\ell_3, \ell_4, L_4, L_5, L_6, L_8$

Effective coupling constants of SU(3)×SU(3)



Does the expansion in powers of m_s work ?

- RBC/UKQCD data concern QCD with 2 + 1 flavours:
  $m_u = m_d = m, \quad m \neq m_s$
- Leading coupling constants in SU(2)×SU(2):
  - F is value of F_π in limit $m \rightarrow 0$
  - B is value of $M_\pi^2/(2m)$ in limit $m \rightarrow 0$
  - F and B are independent of m, but depend on m_s

Does the expansion in powers of m_s work ?

- RBC/UKQCD data concern QCD with 2 + 1 flavours:
  $m_u = m_d = m, \quad m \neq m_s$
- Leading coupling constants in SU(2)×SU(2):
  - F is value of F_π in limit $m \rightarrow 0$
  - B is value of $M_\pi^2/(2m)$ in limit $m \rightarrow 0$
  - F and B are independent of m, but depend on m_s
- Expansion in powers of m_s:

$$F = F_0\{1 + m_s r_1^F + m_s^2 r_2^F + \ldots\}$$

$$B = B_0\{1 + m_s r_1^B + m_s^2 r_2^B + \ldots\}$$

Does the expansion in powers of m_s work ?

- RBC/UKQCD data concern QCD with 2 + 1 flavours:
  $m_u = m_d = m, \quad m \neq m_s$
- Leading coupling constants in SU(2)×SU(2):
  - F is value of F_π in limit $m \to 0$
  - B is value of $M_\pi^2/(2m)$ in limit $m \to 0$
  - F and B are independent of m, but depend on m_s
- Expansion in powers of m_s:

$$F = F_0\{1 + m_s r_1^F + m_s^2 r_2^F + \ldots\}$$

$$B = B_0\{1 + m_s r_1^B + m_s^2 r_2^B + \ldots\}$$

- The terms in the curly brackets violate the OZI-rule
  In large N_c limit, F and B become independent of m_s
- Expansion only useful if $1 \gg m_s r_1^F \gg m_s^2 r_2^F$

Does the expansion in powers of m_s work ?

- Expansion in powers of m_s:

$$F = F_0\{1 + m_s r_1^F + m_s^2 r_2^F + \ldots\}$$

$$B = B_0\{1 + m_s r_1^B + m_s^2 r_2^B + \ldots\}$$

- Numerical results
  - $$F/F_0 = 1.23 \pm 0.07$$

    $$B/B_0 = 1.03 \pm 0.07$$
  - Values quoted for L_4 and L_6 imply

    $$m_s r_1^F = 0.20 \pm 0.04$$

    $$m_s r_1^B = 0.043 \pm 0.033$$
  - For the central values, the expansion thus looks like

    $$F = F_0\{1 + 0.20 + 0.03\}$$

    $$B = B_0\{1 + 0.043 - 0.013\}$$

Consequence for quark condensate

- Quark condensate: $\Sigma \equiv |\langle 0|\,\bar{u}u\,|0\rangle|_{m\to 0}$
- Exact relation: $\Sigma = F^2 B$
- In large N_c limit, Σ is also independent of m_s

$\Rightarrow$ Results for F, B imply

$$\Sigma/\Sigma_0 = 1.55 \pm 0.15$$

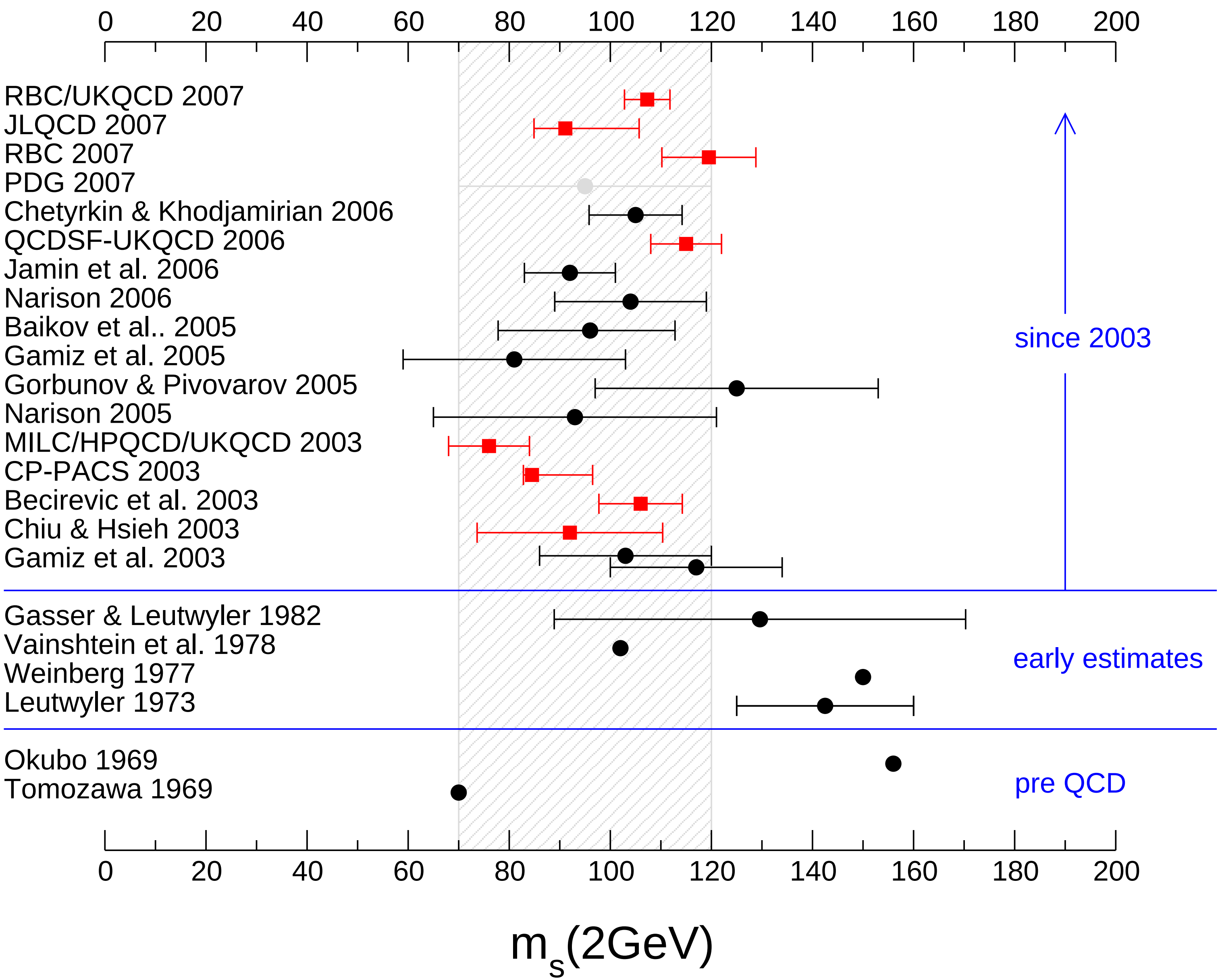
$$m_s r_1^{\Sigma} = 0.43 \pm 0.09$$

$\Rightarrow$ Expansion in powers of m_s for central values:

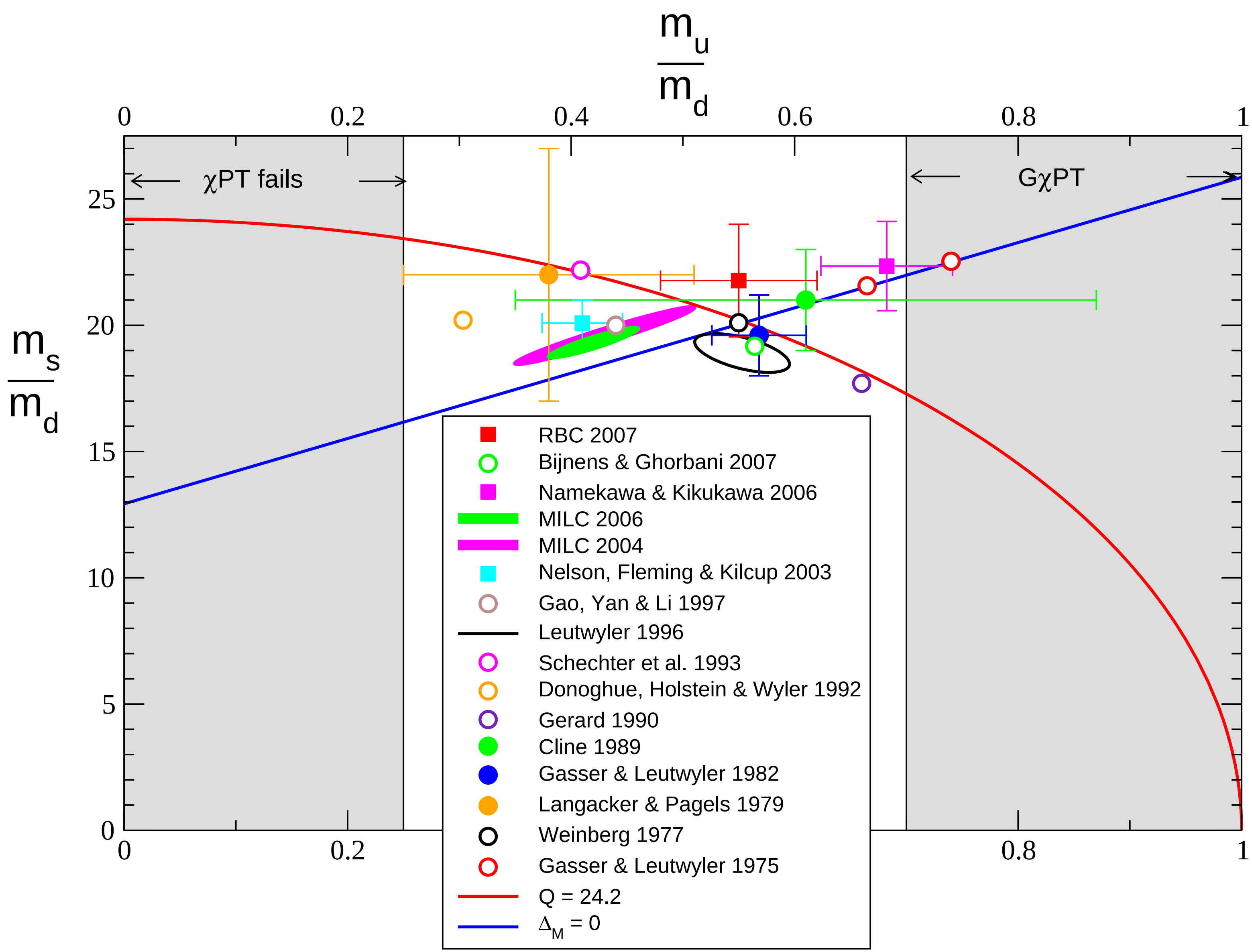
$$\Sigma = \Sigma_0\{1 + 0.43 + 0.12\}$$

- If the result persists: rather juicy violation of the OZI-rule in Σ, but expansion in powers of m_s looks OK
- Note: numbers are preliminary, systematic errors, continuum limit, finite size effects, ...

Mass of the strange quark



Results for quark mass ratios



Conclusions for part II

- Expansion in powers of m_u, m_d yields a very accurate low energy representation of QCD
- Lattice results clearly confirm the GMOR relation: M_π is proportional to $\sqrt{m_u + m_d}$

$\Rightarrow$ Energy gap of QCD is understood very well

- Lattice approach allows an accurate measurement of the effective coupling constant ℓ_3 already now
- Even for ℓ_4, the lattice starts becoming competitive with analytic methods
- Expect significant results for effective coupling constants of NNLO in SU(2)$\times$SU(2) very soon

Conclusions for part II, ctd.

- Lattice results for QCD with $N_f = 3$ are more shaky
- Difficult to have all 3 quarks light enough for the extrapolation to the physical values to be under control
- Significant progress with determination of m_s and with $L_4, \ldots, L_8$
- Lattice results for m_s/m_d confirm χPT estimates: values cluster around $m_s/m_d \simeq 20$
- Lattice results for m_u/m_d still scatter wildly, but none of these is consistent with $m_u = 0$

Part III: puzzling results for $K_L \longrightarrow \pi\mu\nu$

SEARCHING FOR THE 'TOTALLY UNEXPECTED'
IN THE LHC ERA

- Hadronic matrix element of weak current:
  $\langle K^0|\bar{u}\gamma^\mu s|\pi^-\rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$
- Scalar form factor $\sim \langle K^0|\partial_\mu(\bar{u}\gamma^\mu s)|\pi^-\rangle$

$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2}\, f_-(t)$$

Part III: puzzling results for $K_L \longrightarrow \pi\mu\nu$

SEARCHING FOR THE 'TOTALLY UNEXPECTED'
IN THE LHC ERA

- Hadronic matrix element of weak current:

$$\langle K^0|\bar{u}\gamma^\mu s|\pi^-\rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

- Scalar form factor $\sim \langle K^0|\partial_\mu(\bar{u}\gamma^\mu s)|\pi^-\rangle$

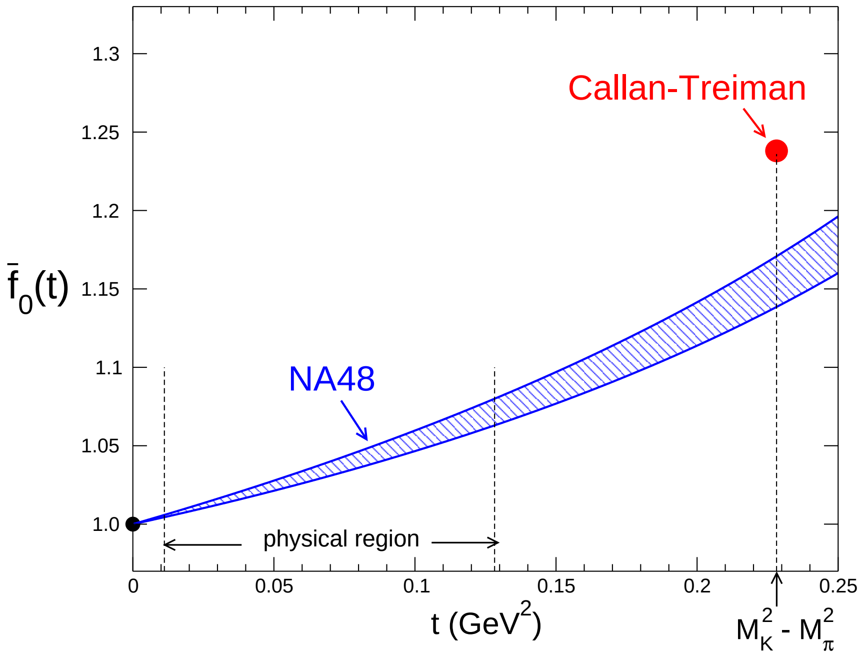
$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2}\, f_-(t)$$

- Low energy theorem of Callan and Treiman (1966):

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi}\left\{1 + O(m_u, m_d)\right\} \simeq 1.19$$

$f_0(0) = f_+(0) \simeq 0.96$ relevant for determination of V_{us}

Comparison with experiment



NA48, hep-ex/0703002

141 authors, 2.3×10^6 events

Plot shows normalized scalar form factor

$$\bar{f}_0(t) = \frac{f_0(t)}{f_0(0)}$$

- Callan-Treiman relation in this normalization:

$$\bar{f}_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi f_+(0)}$$

- Experimental value: $\dfrac{F_K}{F_\pi f_+(0)} = 1.2438 \pm 0.0040$

Passemar, Proc. KAON 2007, arXiv:0708.1235

Corrections, extrapolation

- Callan-Treiman-relation is exact only for $m_u, m_d \to 0$

  Corrections of NLO were worked out long ago, are tiny

  Gasser & L. 1985

  Form factor now known to NNLO Post & Schilcher 2002,

  Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005

  Including the uncertainties from $m_u, m_d \neq 0$:

  $\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$

  Bernard, Oertel, Passemar & Stern, preliminary

$\Rightarrow$ Cannot blame the discrepancy on the prediction

Corrections, extrapolation

- Callan-Treiman-relation is exact only for $m_u, m_d \to 0$

  Corrections of NLO were worked out long ago, are tiny

  Gasser & L. 1985

  Form factor now known to NNLO Post & Schilcher 2002, Bijnens & Talavera 2003, Cirigliano, Ecker, Eidemüller, Kaiser, Pich & Portoles 2005

  Including the uncertainties from $m_u, m_d \neq 0$:

  $$\bar{f}_0(M_K^2 - M_\pi^2) = 1.240 \pm 0.009$$

  Bernard, Oertel, Passemar & Stern, preliminary

$\Rightarrow$ Cannot blame the discrepancy on the prediction

- CT-point not in physical region, extrapolation needed

  Curvature can be calculated with dispersion theory

  Jamin, Oller & Pich 2004, Bernard, Oertel, Passemar & Stern 2006

$\Rightarrow$ Cannot blame the discrepancy on the extrapolation

Slope of the scalar form factor

- Definition of the slope $$\bar{f}_0(t) = 1 + \frac{\lambda_0\, t}{M_{\pi^+}^2} + O(t^2)$$
- Callan-Treiman-relation implies sharp prediction:

$$\lambda_0 = (16.0 \pm 1.0) \times 10^{-3}$$ Jamin, Oller & Pich 2004

- Update with current experimental information

$$\lambda_0 = (15.0 \pm 0.7) \times 10^{-3}$$

Bernard, Oertel, Passemar & Stern, preliminary

- To be compared with the result of NA48:

$$\lambda_0 = (8.9 \pm 1.2) \times 10^{-3}$$ Fit with dispersive representation of BOPS

$$\lambda_0 = (11.7 \pm 0.7_{\text{stat}} \pm 1.0_{\text{syst}}) \times 10^{-3}$$ Linear fit

Implications

- NA48 data on $K_L \to \pi\mu\nu$ disagree with SM

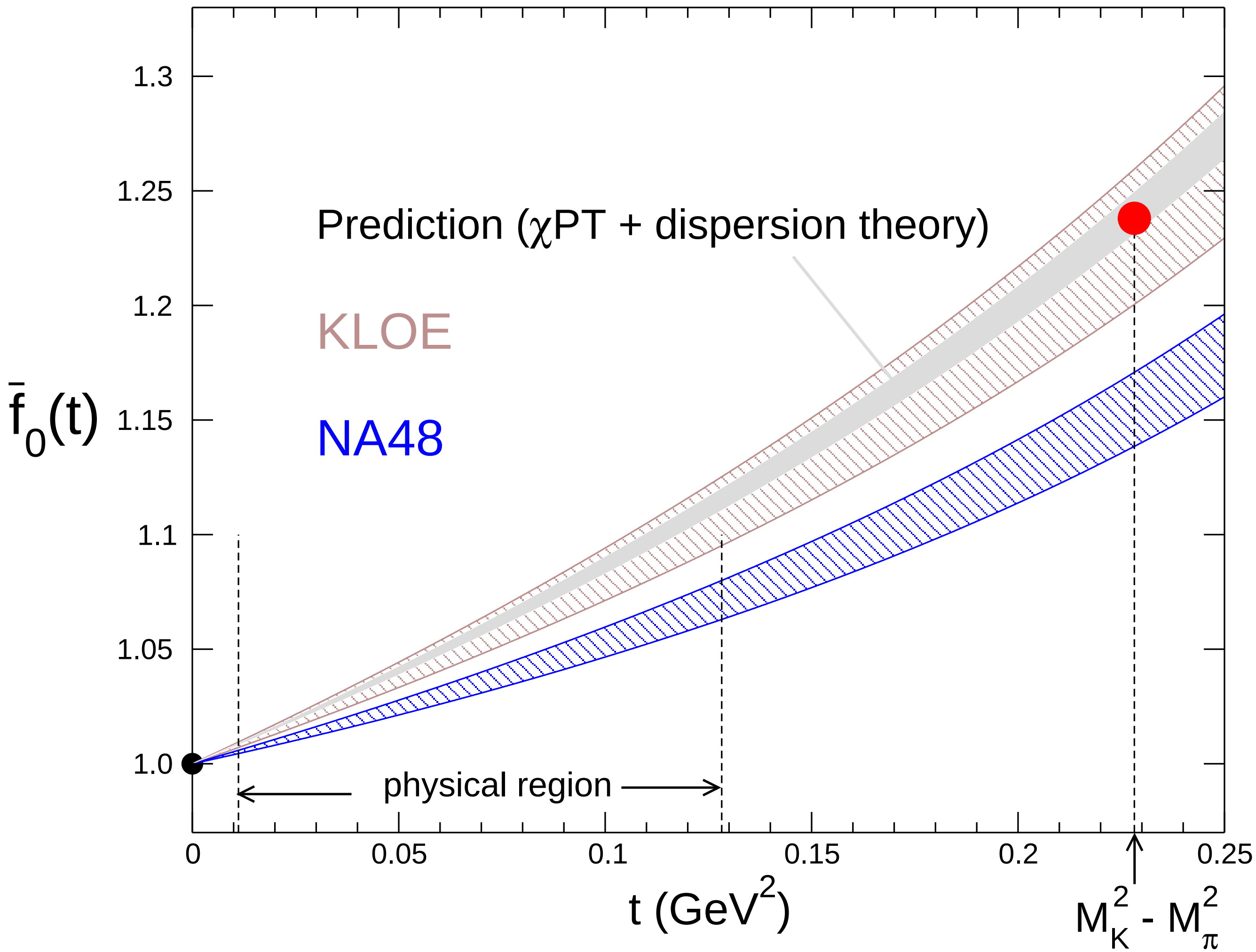
  If confirmed, the implications are dramatic:

$\Rightarrow$ Righthanded currents ? Bernard, Oertel, Passemar & Stern 2006

- In the meantime, new data from KLOE

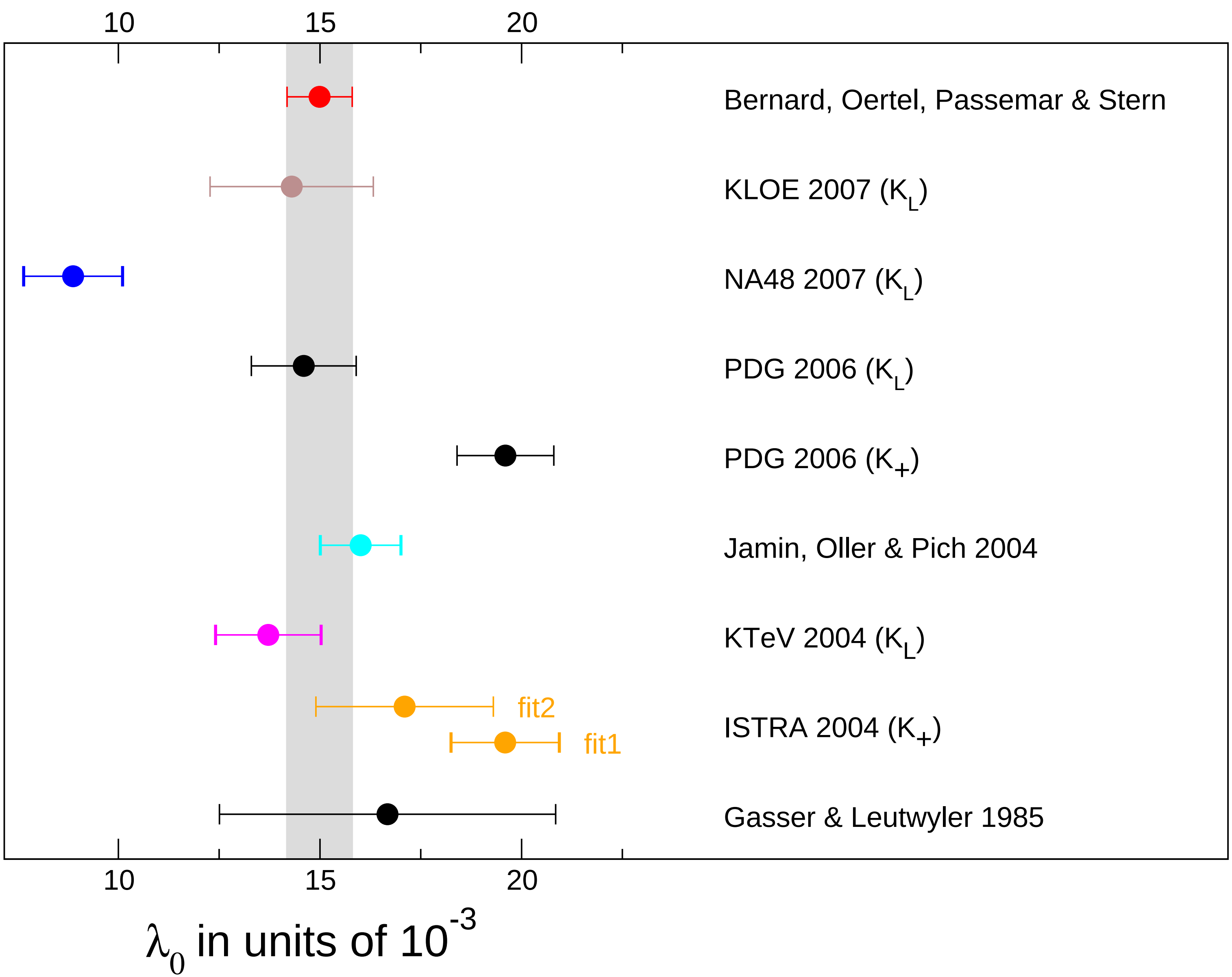
  Results are consistent with Callan-Treiman-relation

Comparison of theory and experiment



I thank Emilie Passemar for some of the material shown in this figure

Comparison of results for the slope



Conclusions for part III

- Experimental discrepancies need to be resolved
- Important to analyze existing data on charged K-decays (isospin breaking)
- Dispersion theory fixes the shape of the form factors

  The most recent analyses properly account for the curvature
  Publishing linear fits is useless, can even be misleading

- KTeV, ISTRA should be reanalyzed

  ISTRA: $\mathbf{0.54 \times 10^6}$ events

  KTeV: $\mathbf{1.9 \times 10^6}$ events

  NA48: $\mathbf{2.3 \times 10^6}$ events

- Excellent overview of current experimental situation: Antonelli, talk at Lepton-Photon 07 http://chep.knu.ac.kr/lp07

Part IV: $\pi\pi$ interaction

- Plays a crucial role whenever the strong interaction is involved at low energies

  Example: Standard model prediction for muon magnetic moment

- Main experiments on $\pi\pi$ scattering were done in the seventies. What's new ?
  - Significant theoretical progress, based on χPT + dispersion theory
  - New precision data:

$K \to \pi\pi e\nu$	E865	Brookhaven
	NA48/2	CERN
pionic atoms	DIRAC	CERN
$K \to 3\pi$	NA48/2	CERN

  - Lattice results on M_π, F_π, a_0^2, $\langle r^2 \rangle_s$

Model independent analysis

- $\pi\pi$ scattering is special: crossed channels are identical

$\Rightarrow$ Re $T(s,t)$ can be represented as a twice subtracted dispersion integral over Im $T(s,t)$ in physical region

S.M. Roy 1971

- The 2 subtraction constants can be identified with the S-wave scattering lengths:

$$a_0^0\;,\;\;a_0^2$$ ← isospin ← angular momentum

- Representation leads to dispersion relations for the individual partial waves: *Roy equations*

Roy equations

- Pioneering work on the physics of the Roy equations was done around the time when QCD was discovered

  Pennington & Protopopescu 1973, Basdevant, Froggatt & Petersen 1974

- Dispersion integrals converge rapidly (2 subtractions)

⇒ Crude phenomenological information on Im $T(s,t)$ for energies above 800 MeV suffices

⇒ Given a_0^0, a_0^2, the scattering amplitude can be calculated quite accurately

  Ananthanarayan, Colangelo, Gasser & L. 2001
  Descotes, Fuchs, Girlanda & Stern 2002

⇒ a_0^0, a_0^2 are the essential parameters at low energy

- Main problem in early work: a_0^0, a_0^2 poorly known
  Experimental information near threshold is meagre

Low energy theorems

- Chiral perturbation theory provides the missing piece: theoretical prediction for a_0^0, a_0^2

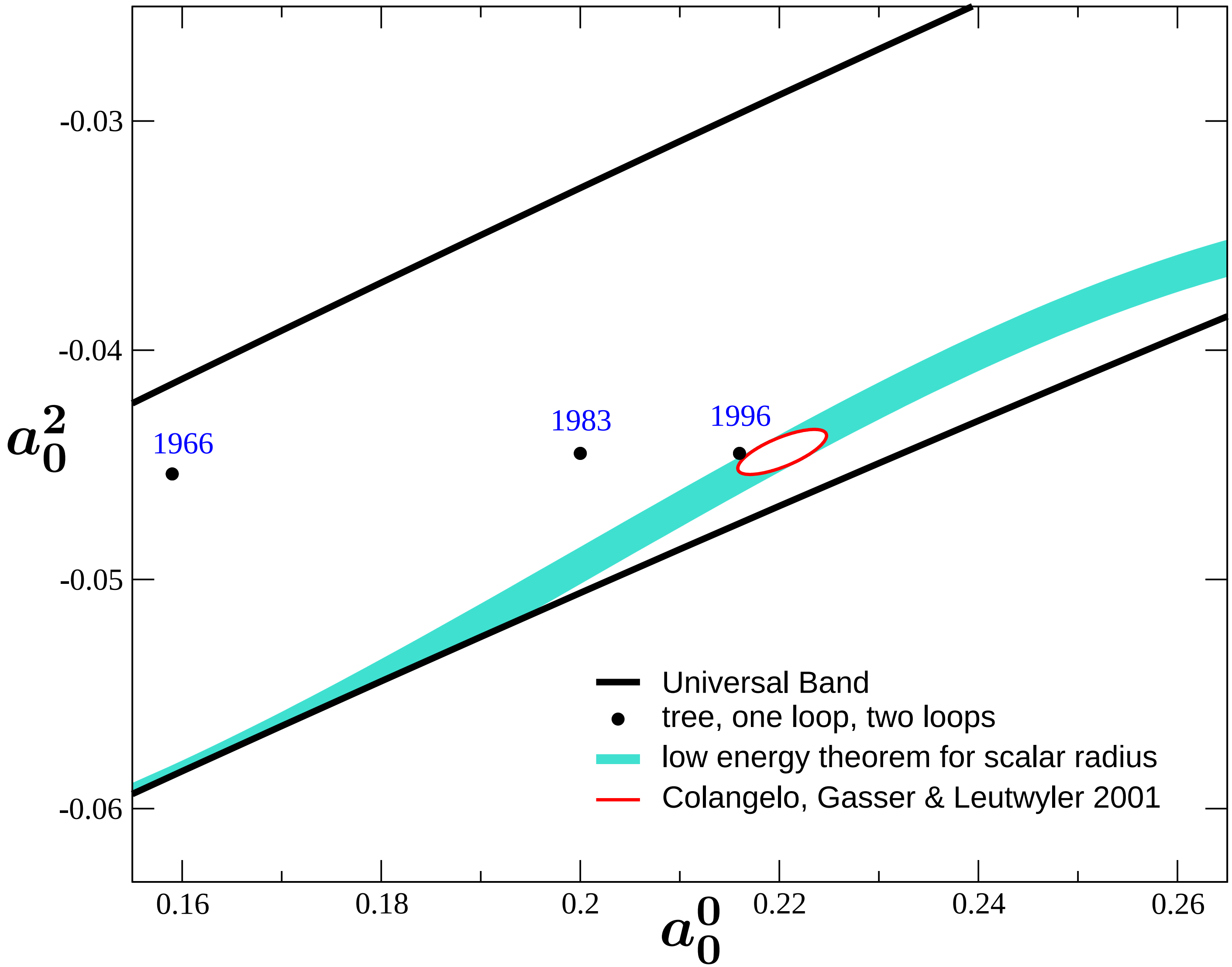
  Weinberg 1966, Gasser & L. 1983, Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

- Most accurate results for a_0^0, a_0^2 are obtained by matching the chiral and dispersive representations in the unphysical region below threshold

  Colangelo, Gasser & L. 2001

- In combination with the low energy theorems for a_0^0, a_0^2, the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to an incredible degree of accuracy

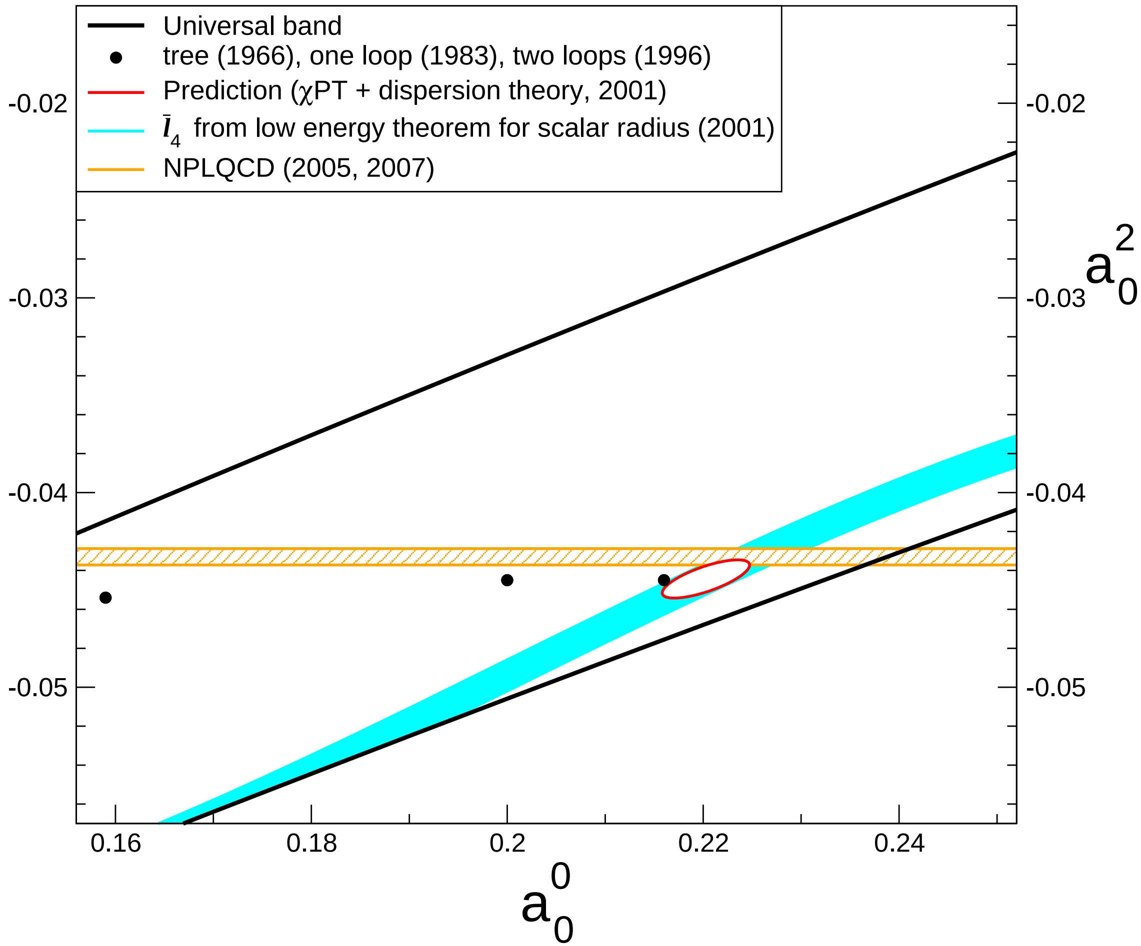
Predictions for the S-wave $\pi\pi$ scattering lengths



Sizeable corrections in a_0^0, while a_0^2 nearly stays put

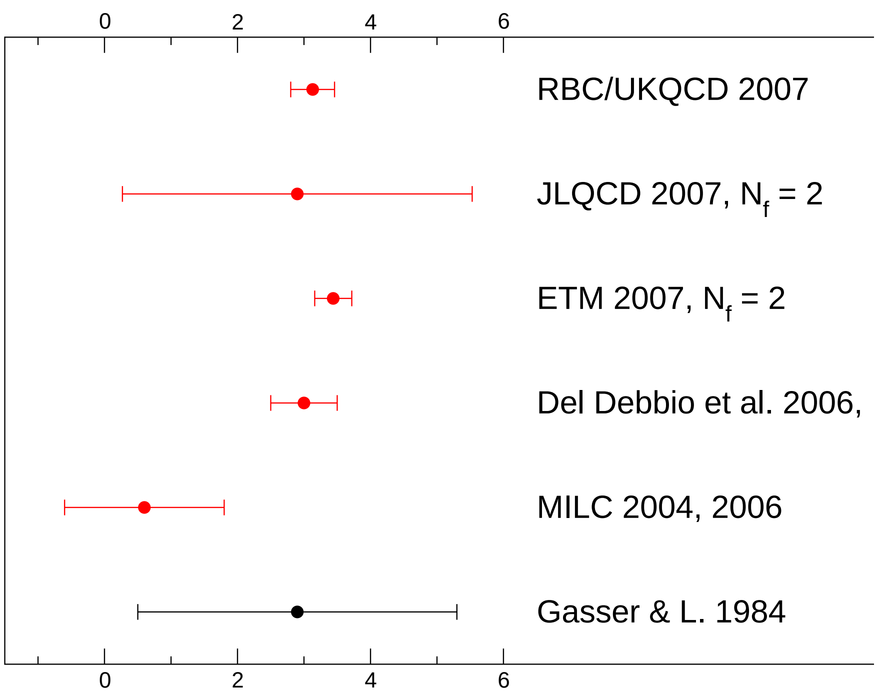
Lattice result for a_0^2

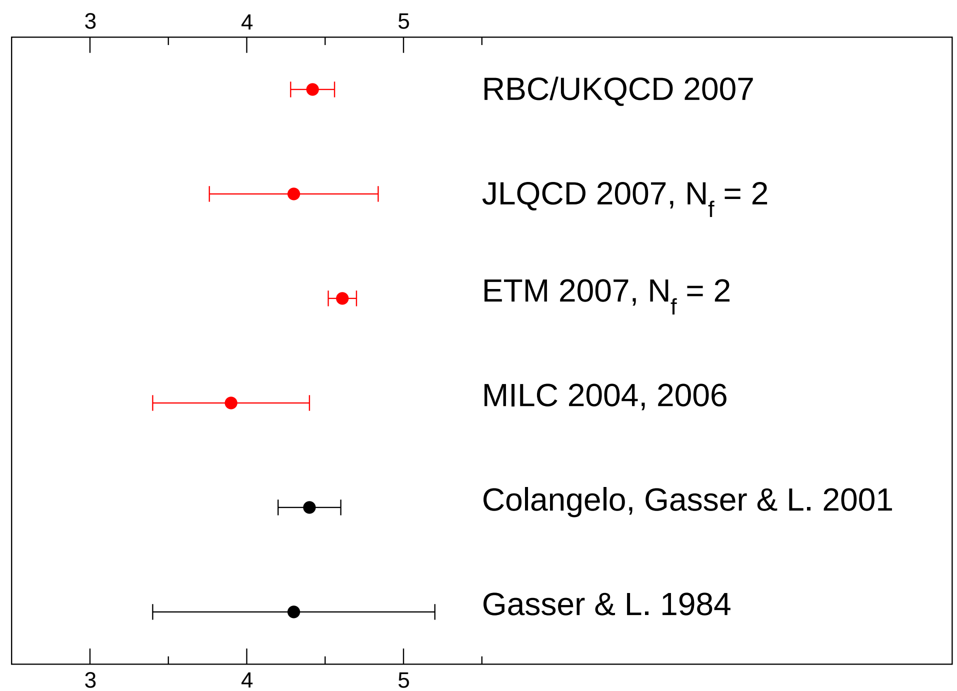
- Lattice allows direct measurement of a_0^2 via volume dependence of energy levels



Lattice results for ℓ_3, ℓ_4

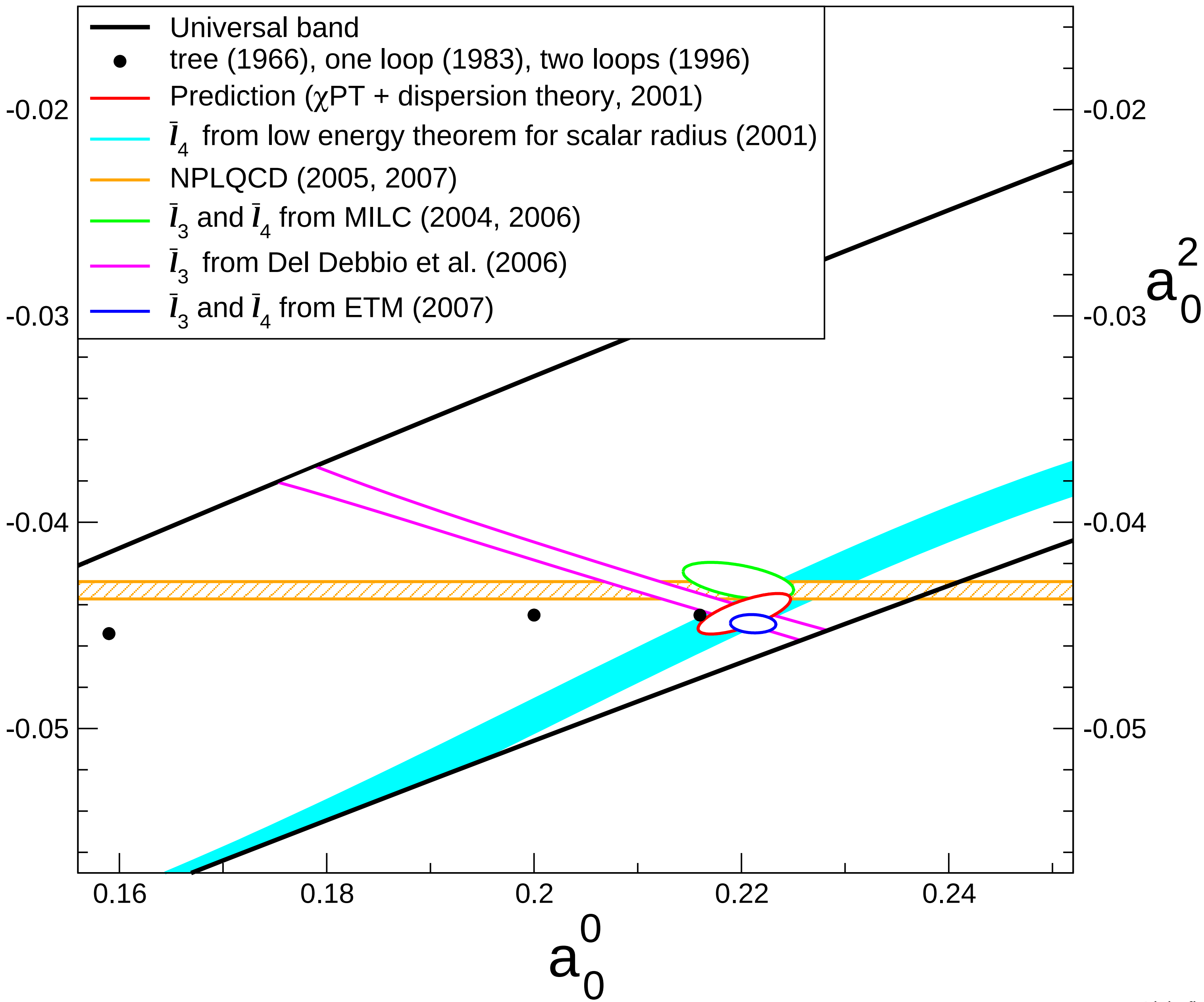
- Uncertainty in prediction for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4
- Can make use of the lattice results for these





$$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M_\pi^2} \qquad \bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

Consequence of lattice results for ℓ_3, ℓ_4



Experiments on light flavours at low energy

- Production experiments $\pi N \to \pi\pi N$, $\psi \to \pi\pi\omega$ ...
  Problem: pions are not produced in vacuo

$\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple
  Accuracy rather limited

- $\pi^+\pi^-$ atoms, DIRAC
- $K^\pm \to \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $K^\pm \to \pi^+\pi^- e^\pm \nu$ precision data from E865, NA48/2

Pionic atoms

- $\pi^+\pi^-$ atoms provide an ideal laboratory
- Decay through the strong interaction $\pi^+\pi^- \to \pi^0\pi^0$
  Decay rate $\propto (a_0^0 - a_0^2)^2$
- Interference of e.m. and strong interactions in bound state and decay is well understood

$\Rightarrow$ Can reliably measure low energy properties of the $\pi\pi$ scattering amplitude in this way

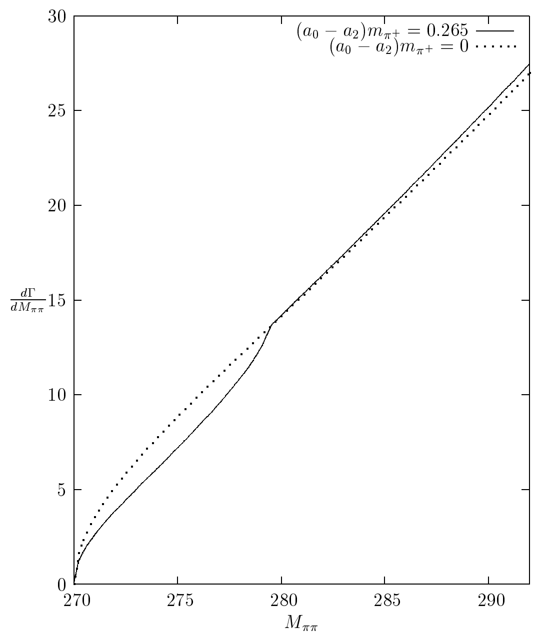
- Prediction for the lifetime: $\tau = 2.9 \pm 0.1$ fs

Gasser, Lyubovitskij, Rusetsky & Gall 2001

- Experimental result: $\tau = 2.91\,^{+0.49}_{-0.62}$ fs DIRAC 2005
- Experiment on πK-atoms is under way $\Rightarrow$ fabulous tool to explore strange quarks at low energy

Cusp in $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$

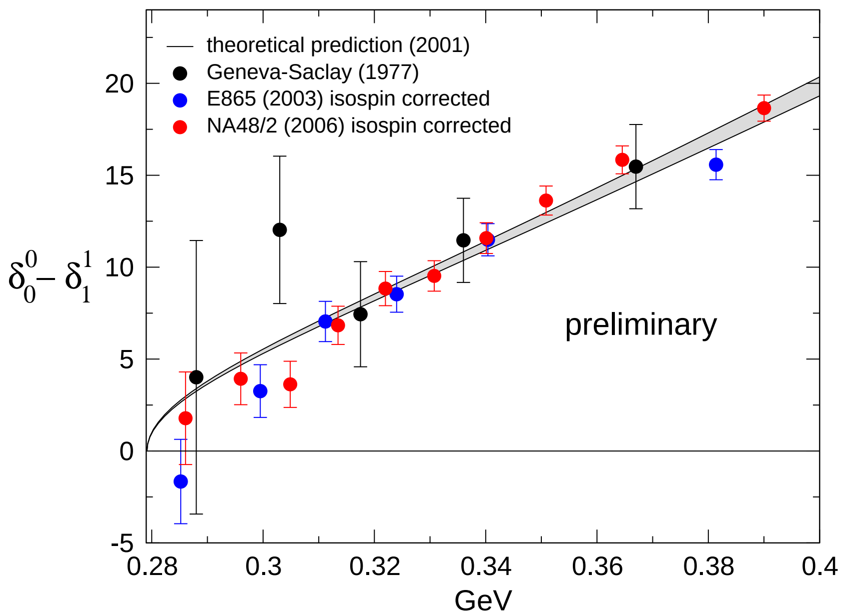
- Accurate data in the threshold region of the decay $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ allow a determination of $a_0^0 - a_0^2$
- NA48/2 has collected $\sim 10^8$ decays in this channel !



taken from N. Cabibbo, hep-ph/0405001

K_{e4} decay

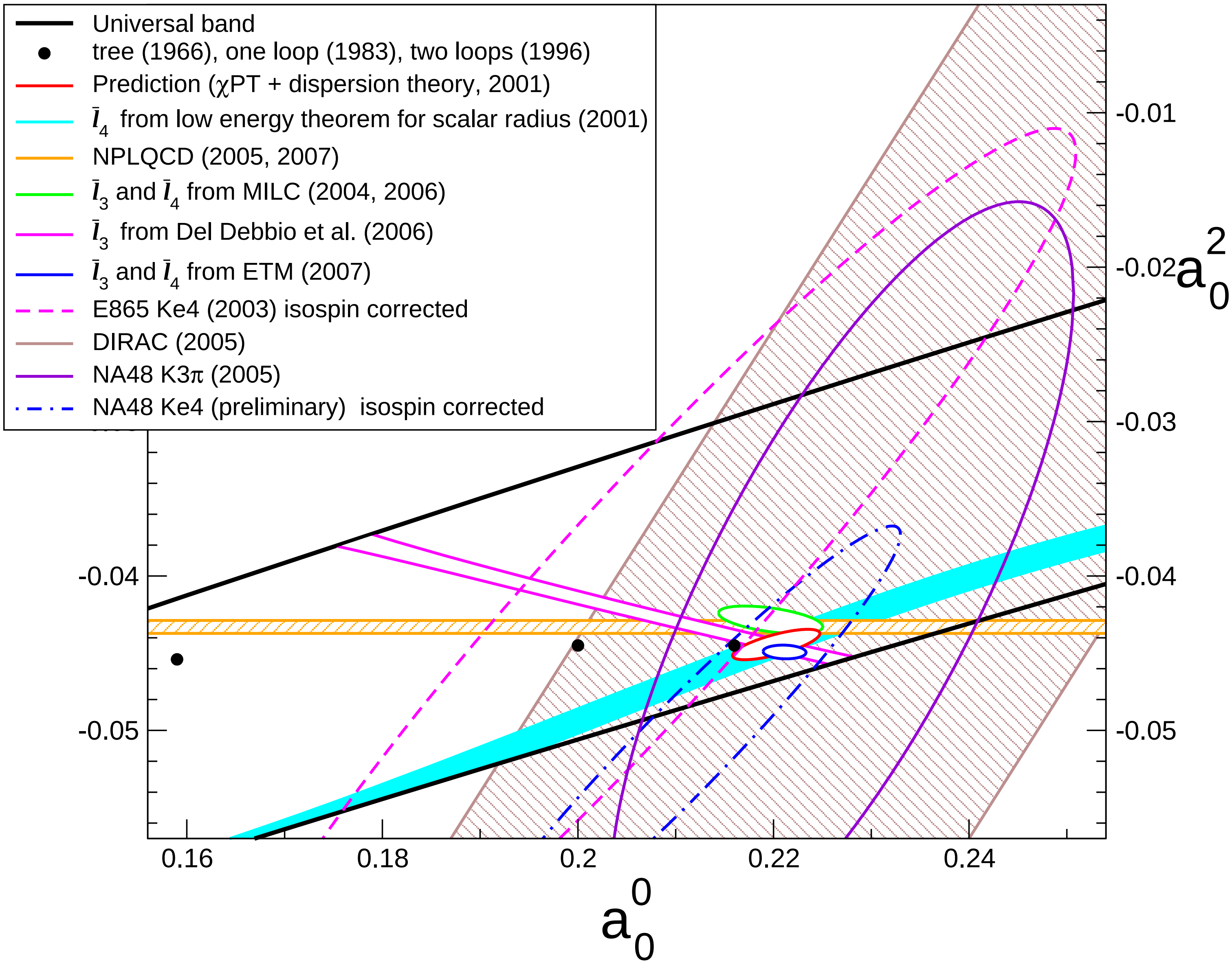
- $K \rightarrow \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from $K \rightarrow \pi^0\pi^0 e\nu \rightarrow \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Brigitte Bloch-Devaux 2007

Tests of the predictions for a_0^0, a_0^2: experiment and lattice



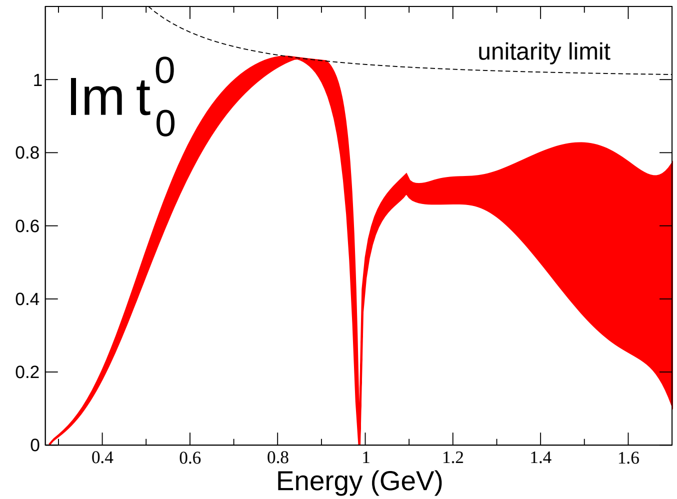
Where is the lowest resonance of QCD ?

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001

- Does QCD have a resonance near threshold ?
- Why care ?
  - Concerns the nonperturbative domain of QCD
  - Quark and gluon degrees of freedom useless there

  $\Rightarrow$ Understanding very poor, pattern of energy levels ?
  - Lowest resonance: σ ? ρ ?
- Resonance $\leftrightarrow$ pole on second sheet
  - Poles are universal
  - Pole position is unambiguous, even if width is large
  - Where is the pole closest to the origin ?

The red dragon



There is the broad object seen in $\pi\pi$ scattering, often called "background", which extends from about 400 MeV up to about 1700 MeV. This object we consider as a single broad resonance[2] *which we identify as the lightest glueball with quantum numbers* $J^{PC} = 0^{++}\dots$

[2] *we refer to it as red dragon*

P. Minkowski and W. Ochs, Eur. Phys. J. C9 (1999) 283

Model independent determination of the pole

- All of the results quoted by the PDG are obtained by
  (a) parametrizing the data for real values of s
  (b) continuing this parametrization analytically in s

  $\Rightarrow$ Result is sensitive to the parametrization used

- We found a model independent method:
  1. Poles on second sheet are zeros on first sheet
  2. The Roy equations are valid for complex values of s, in a limited region of the first sheet

  $\Rightarrow$ Exact representation of the partial waves in terms of observable quantities, valid for complex values of s

  3. Can evaluate this representation to good precision and determine the zeros numerically

Roy equation for the isoscalar S-wave

- S-matrix element $S_0^0(s) = \eta_0^0(s) e^{2\, i\, \delta_0^0(s)}$
- Relation to partial wave amplitude $t_0^0(s)$:

$$S_0^0(s) = 1 + 2\, i \rho\, t_0^0(s) \qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

Roy equation for the isoscalar S-wave

$$S_0^0(s) = 1 + 2\, i\rho\, t_0^0(s) \qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2)\, b + \int_{4M_\pi^2}^{\infty} ds' \,\{K_0(s,s')\, \mathrm{Im}\, t_0^0(s')$$

$$+ K_1(s,s')\, \mathrm{Im}\, t_1^1(s') + K_2(s,s')\, \mathrm{Im}\, t_0^2(s')\}$$

$+$ higher partial waves

Roy equation for the isoscalar S-wave

$$S_0^0(s) = 1 + 2\,i\rho\,t_0^0(s) \qquad\qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2)\,b + \int_{4M_\pi^2}^{\infty} ds'\,\{K_0(s,s')\,\mathrm{Im}\,t_0^0(s')$$

$$+ K_1(s,s')\,\mathrm{Im}\,t_1^1(s') + K_2(s,s')\,\mathrm{Im}\,t_0^2(s')\}$$

$+$ higher partial waves

- The subtraction constants are determined by a_0^0, a_0^2:
$$a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$$

Roy equation for the isoscalar S-wave

$$S_0^0(s) = 1 + 2\, i\rho\, t_0^0(s) \qquad\qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2)\, b + \int_{4M_\pi^2}^\infty ds' \,\{K_0(s,s')\,\mathrm{Im}\, t_0^0(s')$$

$$+ K_1(s,s')\,\mathrm{Im}\, t_1^1(s') + K_2(s,s')\,\mathrm{Im}\, t_0^2(s')\}$$

$+$ higher partial waves

- The subtraction constants are determined by a_0^0, a_0^2:
  $$a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$$
- The kernels are elementary functions, e.g.
  $$K_0(s,s') = \underbrace{\frac{1}{\pi(s'-s)}}_{r.h.cut} + \underbrace{\frac{2\, ln\{(s+s'-4M_\pi^2)/s'\}}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}}_{l.h.cut}$$

Roy equation for the isoscalar S-wave

$$S_0^0(s) = 1 + 2\,i\rho\,t_0^0(s) \qquad \rho = \sqrt{1 - 4M_\pi^2/s}$$

$$t_0^0(s) = a + (s - 4M_\pi^2)\,b + \int_{4M_\pi^2}^\infty ds'\,\{K_0(s,s')\,\mathrm{Im}\,t_0^0(s')$$

$$+ K_1(s,s')\,\mathrm{Im}\,t_1^1(s') + K_2(s,s')\,\mathrm{Im}\,t_0^2(s')\}$$

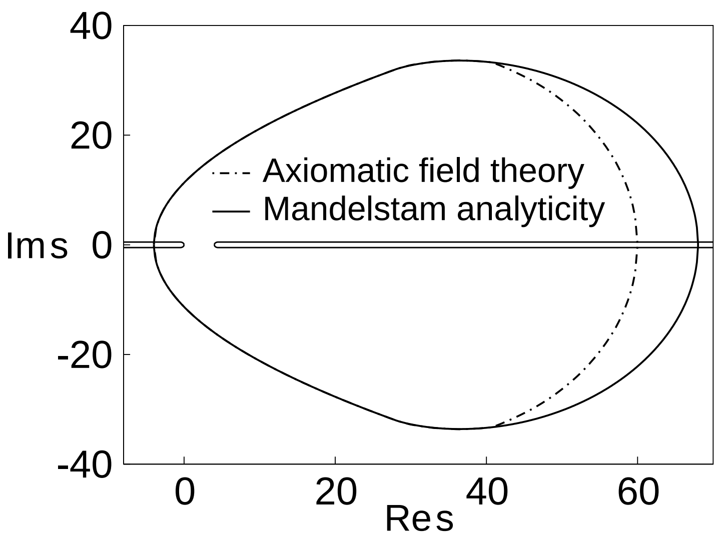
$+$ higher partial waves

- The subtraction constants are determined by a_0^0, a_0^2:
  $a = a_0^0\,, \qquad b = (2a_0^0 - 5a_0^2)/(12M_\pi^2)$
- The kernels are elementary functions, e.g.
  $$K_0(s,s') = \underbrace{\frac{1}{\pi(s'-s)}}_{r.h.cut} + \underbrace{\frac{2\,ln\{(s+s'-4M_\pi^2)/s'\}}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}}_{l.h.cut}$$
- Left hand cut is essential for convergence:
  $K_0(s,s') \sim 1/{s'}^3$ for large s'

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$
- Equations are valid for complex s in a limited region of the first sheet

I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001



s in units of M_π^2

- Proof is based on first principles, general quantum field theory

A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing*, Lecture Notes in Physics, vol. 3, 1969.

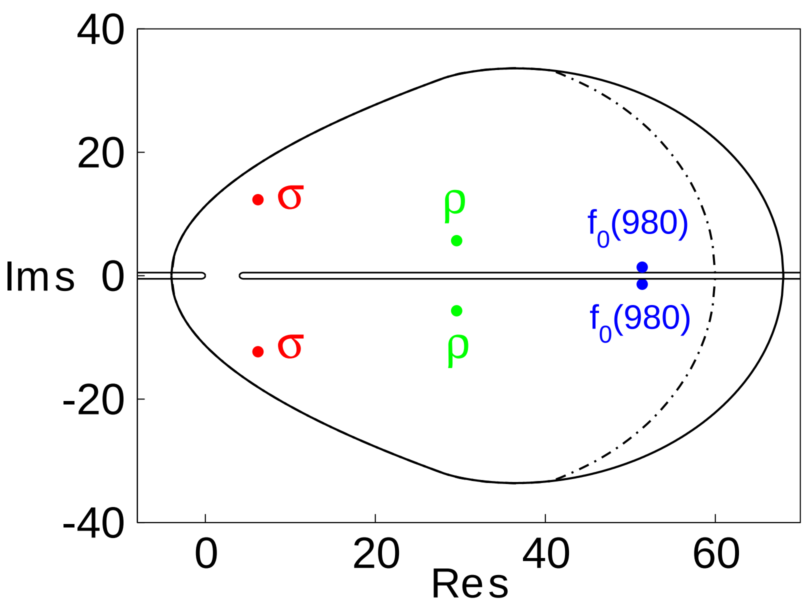
G. Mahoux, S. M. Roy and G. Wanders, Nucl. Phys. B70 (1974) 297.

$\Rightarrow$ Exact representation for $S_0^0(s)$ in this region
Do not need to parametrize the amplitude

Evaluation of the pole position

- Have an exact formula for the pole position in terms of physical quantities: $S_0^0(s) = 0$
- For the central solution of the Roy equations, $S_0^0(s)$ has two pairs of zeros in the region where the formula holds:

$$s = (6.2 \pm i\,12.3)\,M_\pi^2 \quad \sigma$$
$$s = (51.4 \pm i\,1.4)\,M_\pi^2 \quad f_0(980)$$



s in units of M_π^2

The eyes of the red dragon

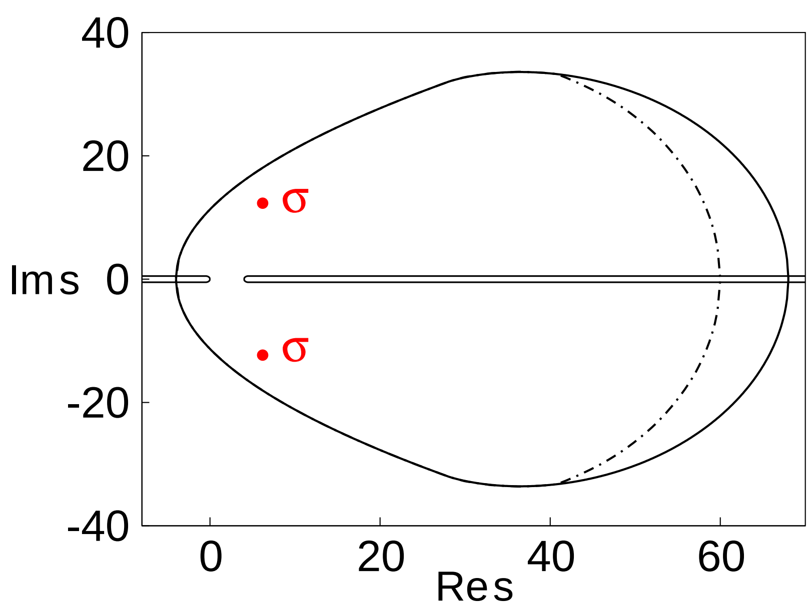
Tail at 1.7 GeV: $s \simeq 150\,M_\pi^2$

Result

- Lowest resonance of QCD has vacuum quantum numbers
- Pole on lower half of sheet II occurs in vicinity of

$$\sqrt{s} = 441\,^{+16}_{-8} \;-\; i\;272\,^{+9}_{-13}\ \text{MeV}$$

Qualitative discussion of our error analysis on request

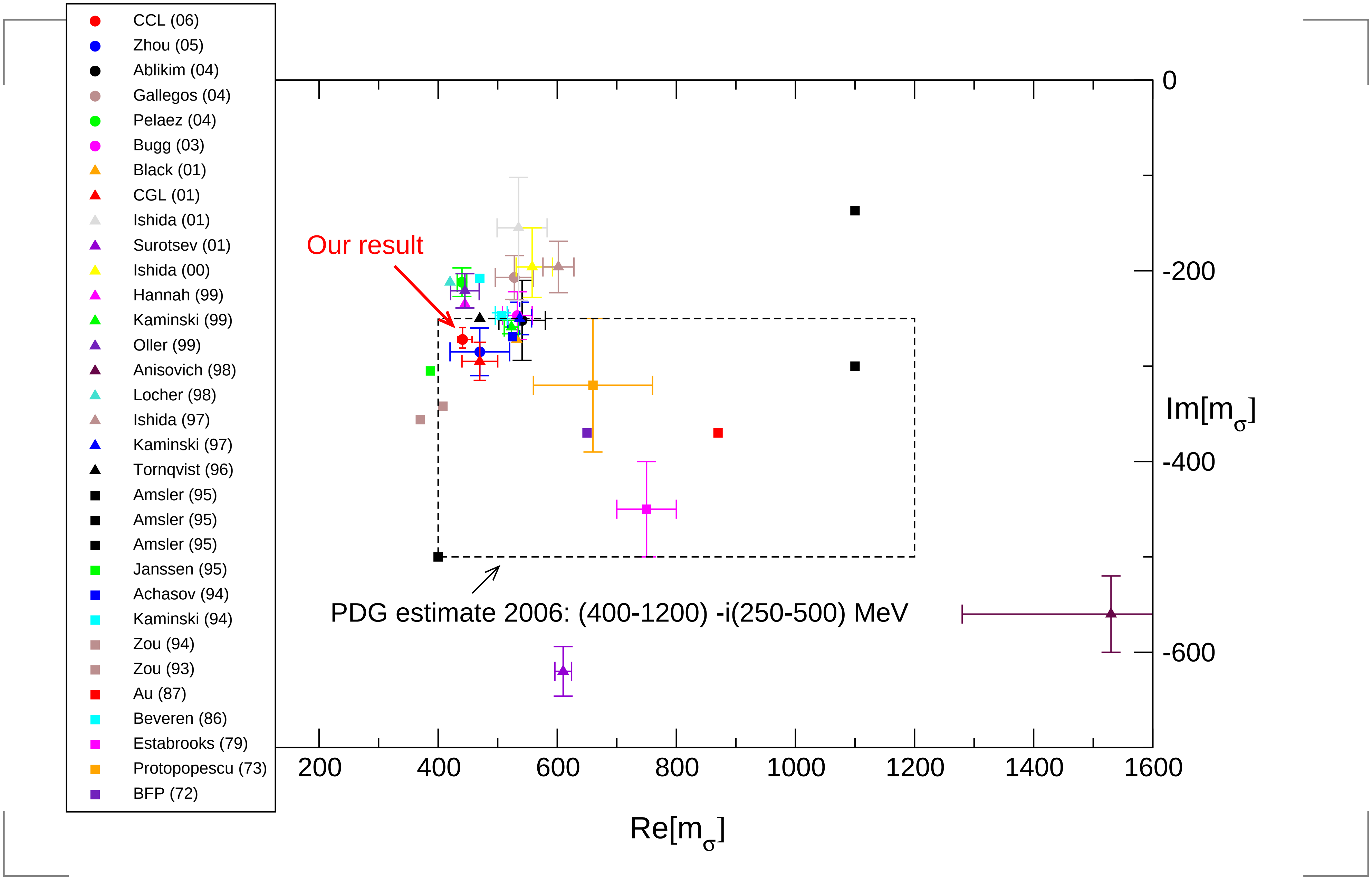


s in units of M_π^2

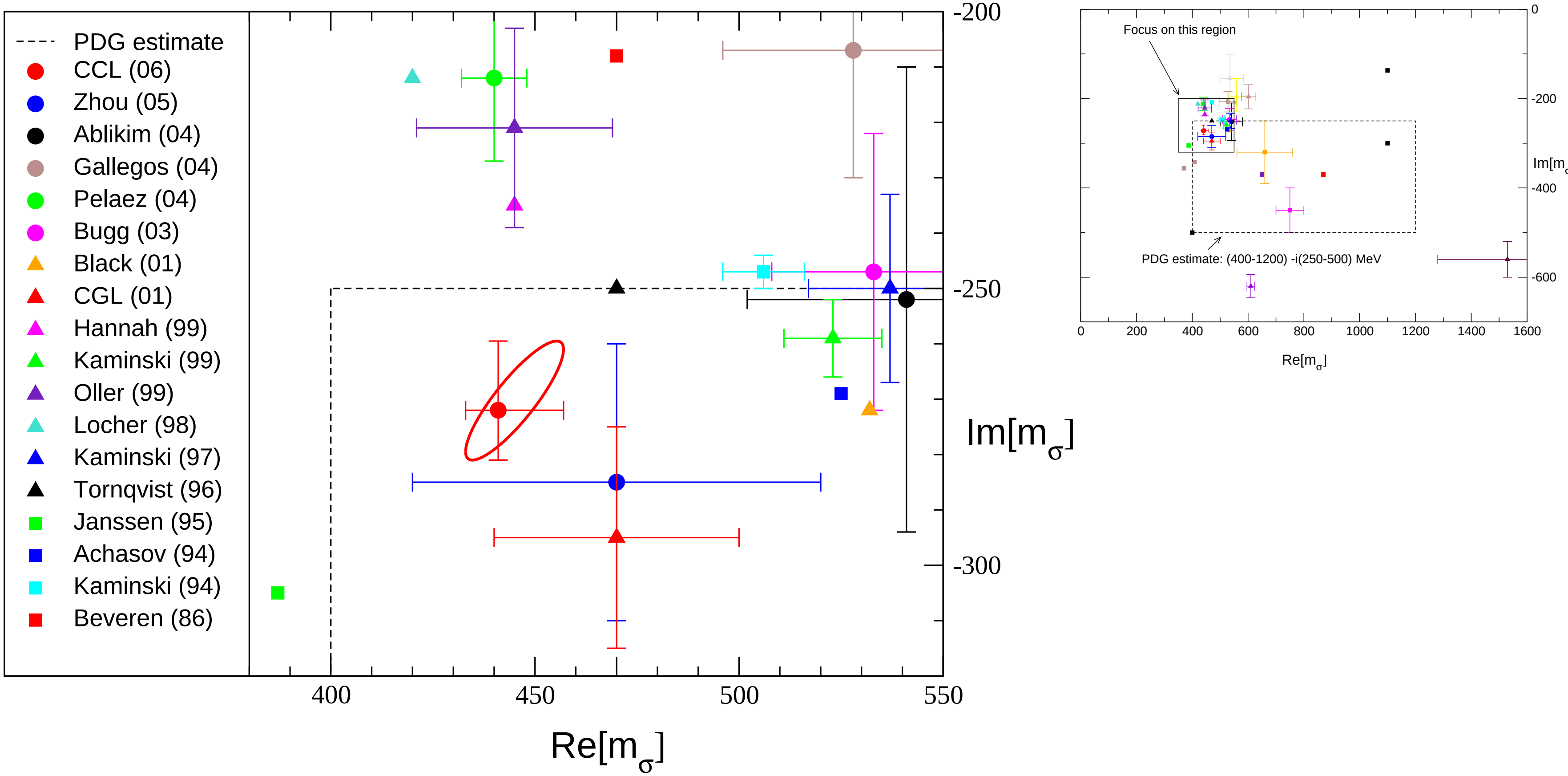
Loci Oculorum Draconis Rutili

T. Barnes, Theory summary, MESON 2006

Comparison with compilation of PDG



Vicinity of the pole



H. Leutwyler – Bern

Conclusions for part IV

- Low energy pion physics: theory ahead of experiment
  - Precision experiments carried out and under way
  - Lattice makes slow, but steady progress
  - So far, all tests confirm the theory
- Limitations of our approach:
  - Calculations cannot be done on back of an envelope
  - Analysis only covers low energies
    Extension to higher energies under way
    Experimental uncertainties near $K\bar{K}$ threshold, $f_0(980)$ currently represent the main limitation
  - Only a few applications have been worked out:
    $\pi\pi$ scattering, pion form factors, hadronic vacuum polarization in muon $g-2$
    $\gamma\gamma \to \pi^0\pi^0$ Pennington, hep-ph/0604212

Conclusions for part IV, ctd.

- Much is yet to be done: $J/\psi \to \omega\pi\pi$, $D \to 3\pi$, ... πK, πN, ...
- Model independent method for analytic continuation
  - The lowest resonance of QCD occurs at

    $$M_\sigma = 441\,^{+16}_{-8}\ \text{MeV} \qquad \Gamma_\sigma = 544\,^{+18}_{-25}\ \text{MeV}$$

    and carries vacuum quantum numbers
  - Crossing symmetry plays an essential role:
    Fixes contributions from left hand cut
    Ensures fast convergence, low energy dominance
  - Pole occurs at low value of s, closer to left hand cut than to singularities from $K\bar{K}$, $f_0(980)$
  - Result for Γ_σ relies on theory for a_0^2
    Experiments concerning a_0^2 would be most welcome

VISIT THE RED DRAGON

GENTLE ANIMAL
LOOK IN HIS EYES FROM CLOSE
SMELL HIS GOOD BREATH
BRING YOUR PIONS ALONG AND
FEED HIM YOURSELF

The management denies responsibility for incidents involving the dragon’s tail

SPARES

χPT, UχPT, IAM

- $\pi\pi$ scattering amplitude known to two loops of χPT

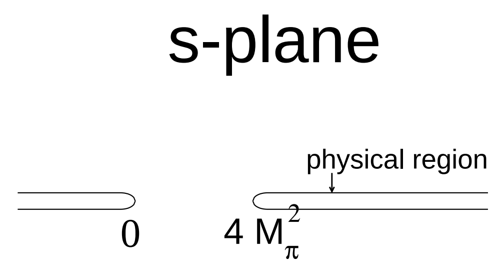
  Bijnens, Colangelo, Ecker, Gasser & Sainio 1996

- χPT works very well below threshold, but goes out of control long before the energy reaches M_ρ
- Range of validity of χPT can be extended by hand: “Unitarized χPT”, ”Inverse Amplitude Method”
- Padé: unitarity $\surd$ poles from ρ, σ $\surd$

  Truong, Dobado, Herrero, Peláez, Hannah, Oller, Guerrero, Ramos, Oset, Zheng, Xiao He, Qin, Deng, Nieves, Pavón Valderrama, Ruiz-Arriola, Gómez-Nicola Llanes-Estrada, Lähde, Meissner, Hanhart, Rios, . . .

- Simple, useful approximation, also for form factors Improves chiral representation in physical region
- Enforces unitarity at the expense of crossing symmetry
- Main problem: model $\Rightarrow$ uncertainties not under control

Pole on second sheet $\leftrightarrow$ zero on first sheet



- $S_0^0(s) = \eta_0^0(s)\exp 2i\delta_0^0(s)$

  $S_0^0(s)$ is analytic in the cut plane

- For $0 < s < 4M_\pi^2$, $S_0^0(s)$ is real

$\Rightarrow$ $S_0^0(s^\star) = S_0^0(s)^\star$

x in elastic interval: $S_0^0(x \pm i\epsilon) = \exp \pm 2i\delta_0^0(x)$

- Second sheet is reached by continuation across the elastic interval of the right hand cut

$$S_0^0(x - i\epsilon)^{II} = S_0^0(x + i\epsilon)^{I} = 1/S_0^0(x - i\epsilon)^{I}$$

Analyticity $\Rightarrow$ $\boxed{S_0^0(s)^{II} = 1/S_0^0(s)^{I}}$ valid $\forall s$

Pole in $S_0^0(s)^{II}$ $\Longleftrightarrow$ zero in $S_0^0(s)^{I}$

Error analysis

- Results depend on phenomenological input used when solving the Roy equations, subject to uncertainties
  Can follow error propagation explicitly
- Pole position of $f_0(980)$ sensitive to input used for $\eta_0^0(s)$
- Pole position of σ mainly depends on 3 input variables:

  $a_0^0, a_0^2, \delta_A \equiv \delta_0^0$(800 MeV)

  - Information about a_0^0, a_0^2 is in good shape
  - Substantial uncertainties in phenomenology of δ_A
  - Use conservative range: $\delta_A = 82.3^\circ {}^{+10^\circ}_{-4^\circ}$

Error analysis

- Noise from remaining input variables is very small:

$$m_\sigma = (441 \pm 4) - i(272 \pm 6) \text{ MeV}$$

but the values of a_0^0, a_0^2, δ_A are crucial:

$$\begin{aligned} m_\sigma = & \;(441 \pm 4) - i(272 \pm 6) \\ & + \;(-2.4 + i\,3.8)\; \frac{a_0^0 - 0.22}{0.005} \\ & + \;(0.8 - i\,4.0)\; \frac{a_0^2 + 0.0444}{0.001} \\ & + \;(5.3 + i\,3.3)\; \frac{\delta_A - 82.3}{3.4} \end{aligned}$$

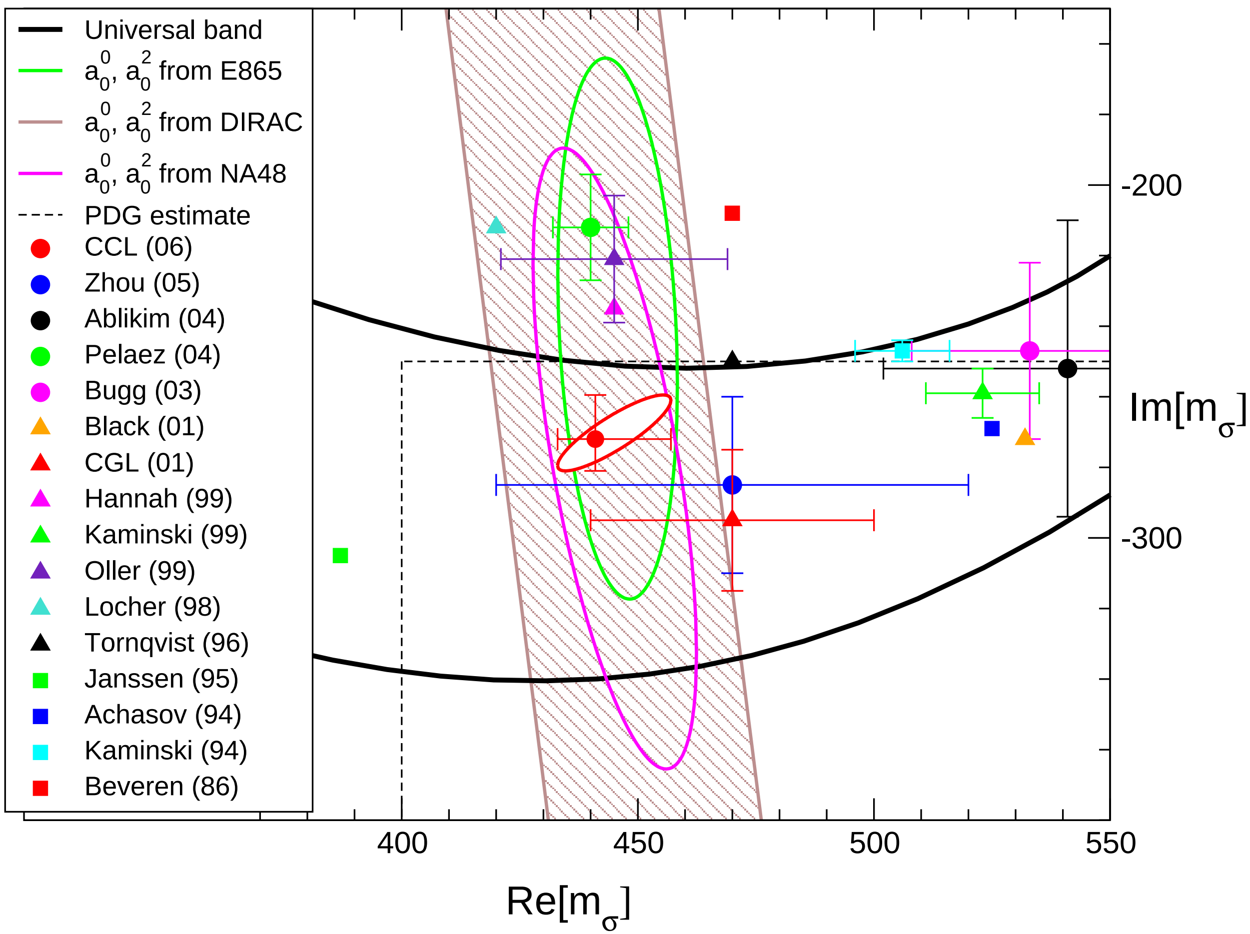
numbers in MeV

- Final result: insert the predictions for a_0^0, a_0^2, use the phenomenological range for δ_A and add errors up:

$$m_\sigma = 441\,{}^{+16}_{-8}\; - i\;272\,{}^{+9}_{-13} \text{ MeV}$$

Ignore the theoretical predictions for a_0^0, a_0^2

- Replace the low energy theorems for a_0^0, a_0^2 by the experimental results from E865, DIRAC and NA48
- $a_0^0, a_0^2 \in$ universal band



Why are our errors so incredibly small ?

- The σ occurs at low energies
- At low energies, the subtraction term dominates

$$t_0^0(s) \simeq a_0^0 + (2a_0^0 - 5a_0^2)\frac{(s-4M_\pi^2)}{12M_\pi^2}$$

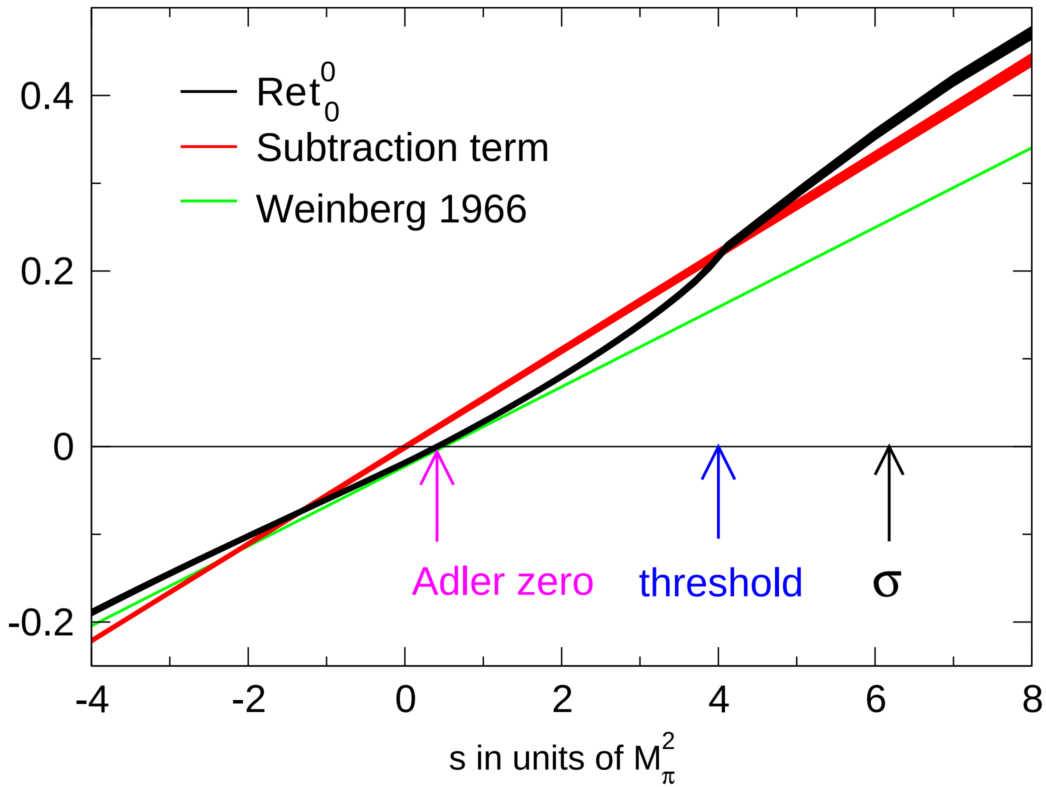
Insert low energy theorem for a_0^0, a_0^2

⇒ Roy equation reduces to Weinberg formula

$$t_0^0(s) \simeq \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Dispersion integrals only represent a correction

At low energies, the subtraction term dominates



$$s = (0.41 \pm 0.06)\, M_\pi^2 \quad \text{Adler zero}$$

$$s = (6.2 - i\,12.3)\, M_\pi^2 \quad \text{pole from } \sigma$$

Goldstone bosons of low energy interact only weakly

Estimate pole position on back of an envelope

- Approximate $t_0^0(s)$ with the Weinberg formula

$$t_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Where are the zeros of $S_0^0(s)$ in this approximation ?

Estimate pole position on back of an envelope

- Approximate $t_0^0(s)$ with the Weinberg formula

$$t_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Where are the zeros of $S_0^0(s)$ in this approximation ?

$$1 + 2\,i\,\sqrt{1 - 4M_\pi^2/s}\;t_0^0(s) = 0$$

Estimate pole position on back of an envelope

- Approximate $t_0^0(s)$ with the Weinberg formula

$$t_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Where are the zeros of $S_0^0(s)$ in this approximation ?

$$1 + 2\,i\,\sqrt{1 - 4M_\pi^2/s}\;t_0^0(s) = 0$$

$\Rightarrow$ Cubic equation for s

- Pair of complex zeros, $m_\sigma = 365 - i\,291$ MeV

  to be compared with $m_\sigma = 441\,{}^{+16}_{-8} \; - i\,272\,{}^{+9}_{-13}$ MeV

Estimate pole position on back of an envelope

- Approximate $t_0^0(s)$ with the Weinberg formula

$$t_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Where are the zeros of $S_0^0(s)$ in this approximation ?

$$1 + 2\,i\,\sqrt{1 - 4M_\pi^2/s}\; t_0^0(s) = 0$$

$\Rightarrow$ Cubic equation for s

- Pair of complex zeros, $m_\sigma = 365 - i\,291$ MeV

  to be compared with $m_\sigma = 441\,{}^{+16}_{-8} - i\,272\,{}^{+9}_{-13}$ MeV

$\Rightarrow$ Correction from higher orders amounts to

$$\Delta m_\sigma = 76\,{}^{+16}_{-8} + i\,19\,{}^{+9}_{-13}\ \text{MeV}$$

For the quantity that counts, the accuracy is modest

Estimate pole position on back of an envelope

- Approximate $t_0^0(s)$ with the Weinberg formula

$$t_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}$$

Where are the zeros of $S_0^0(s)$ in this approximation ?

$$1 + 2\,i\,\sqrt{1 - 4M_\pi^2/s}\; t_0^0(s) = 0$$

$\Rightarrow$ Cubic equation for s

- Pair of complex zeros, $m_\sigma = 365 - i\,291$ MeV

  to be compared with $m_\sigma = 441\,{}^{+16}_{-8} - i\,272\,{}^{+9}_{-13}$ MeV

- Real zero on sheet II, near $s = 0$ (full amplitude has kinematic singularity: vanishes on sheet II at $s = 0$)

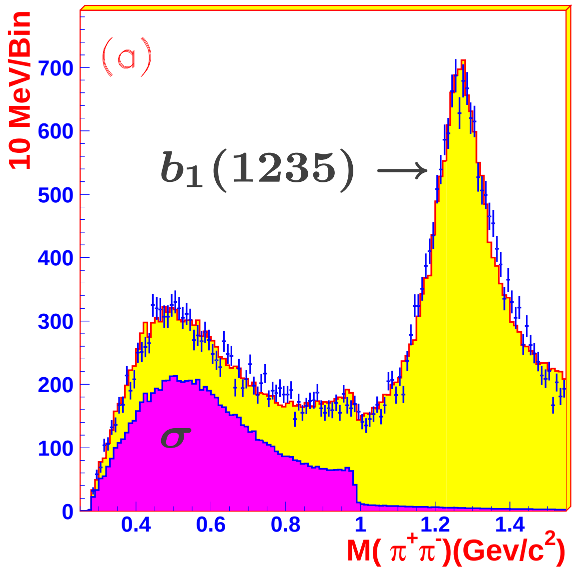
Curvature due to the left hand cut

- Left hand cut generates curvature
  Main contribution on the left stems from the ρ
- Most pole determinations neglect the left hand cut
  Pole from σ is too close for this to be justified
- Can estimate contributions from left hand cut with χPT

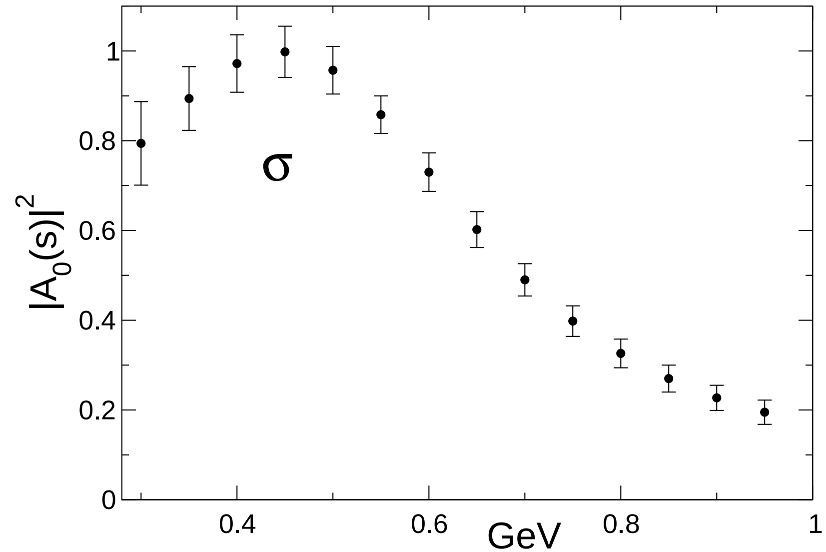
  Zhou, Qin, Zhang, Xiao, Zheng, Wu, JHEP 0502 (2005) 043

  Estimate is crude $\Rightarrow$ sizeable uncertainties
  Outcome for pole position agrees with our result

BES data on $J/\psi \to \omega\pi\pi$



BES, Phys. Lett. B598 (2004) 149



S-wave projection (D. Bugg, priv. comm.)

Outcome for pole position:

$$m_\sigma = (541 \pm 39) - i\,(252 \pm 42) \text{ MeV} \qquad \text{BES 2004}$$

(simple parametrization à la Breit-Wigner, $K\bar{K}$ and $\eta\eta$ final states neglected)

$$m_\sigma = (472 \pm 30) - i\,(271 \pm 30) \text{ MeV} \qquad \text{Bugg hep-ph/0608081}$$

(reanalysis based on a more complicated model)

Revised result differs from ours by less than $1\,\sigma$

Physical interpretation of the σ

- The head of the dragon is not made of glue
- The dragon likes flavoured food, pions in particular

Markushin & Locher 1999

$\Rightarrow$ Physics of the $\sigma \in$ Goldstone boson dynamics

$\Rightarrow$ Wave function has large tetra-quark component

Jaffe 1977

- Physics of the $f_0(980) \in$ Goldstone boson dynamics
  Interaction among two kaons is relevant

Hanhart hep-ph/0609136

- These states are very sensitive to SU(3) breaking
- Multiplet pattern ? $a_0(980)$?

Xiao, Zheng, Zhou hep-ph/0609009

The κ

- $K\pi$ scattering amplitude obeys an analog of the Roy equations. Pole from κ can be calculated on this basis

$$m_\kappa = (658 \pm 13) - i\,(278.5 \pm 12)\,\text{MeV}$$

Descotes-Genon and Moussallam 2006

- Confirms an earlier calculation, where the l.h. cut was estimated with χPT Zhou and Zheng, hep-ph/0603062
- Back-of-the-envelope calculation for $K\pi$ gives

$$m_\kappa = 671 - i\,262\,\text{MeV}$$

$\Rightarrow$ Physics of σ and κ is very similar