Looking back at 50 years of particle physics

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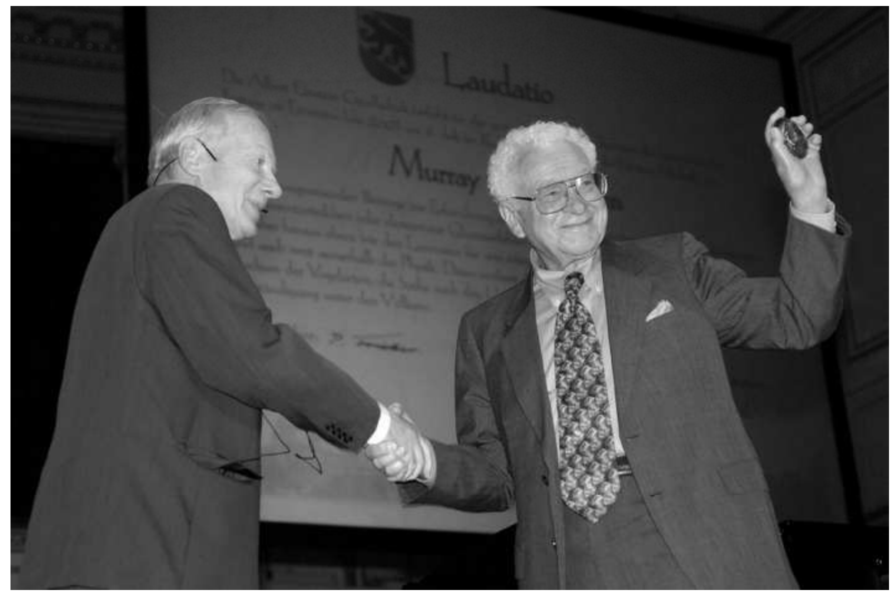
Biased account, my perspective, not a historian

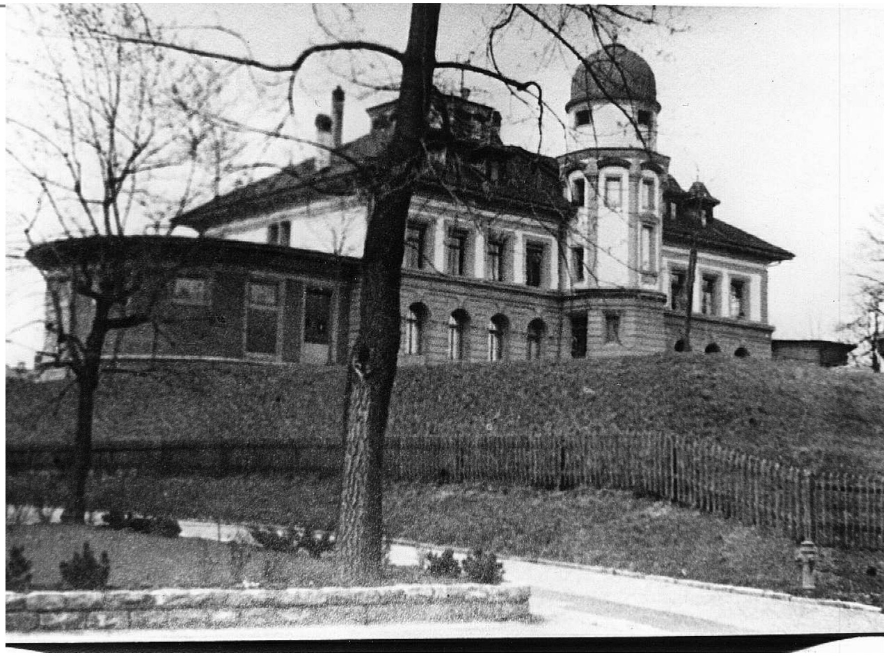
Wisdom possibly grows with age, but memory does not . . .

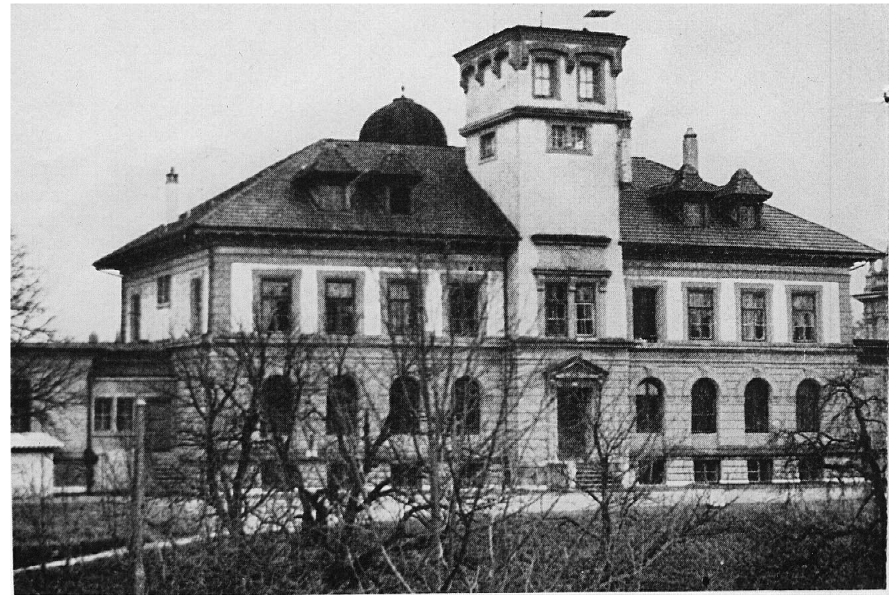
Bern 1955: 50 years of Relativity



Bern 2005: 100 years of Relativity

How the institute looked at the end of the fifties





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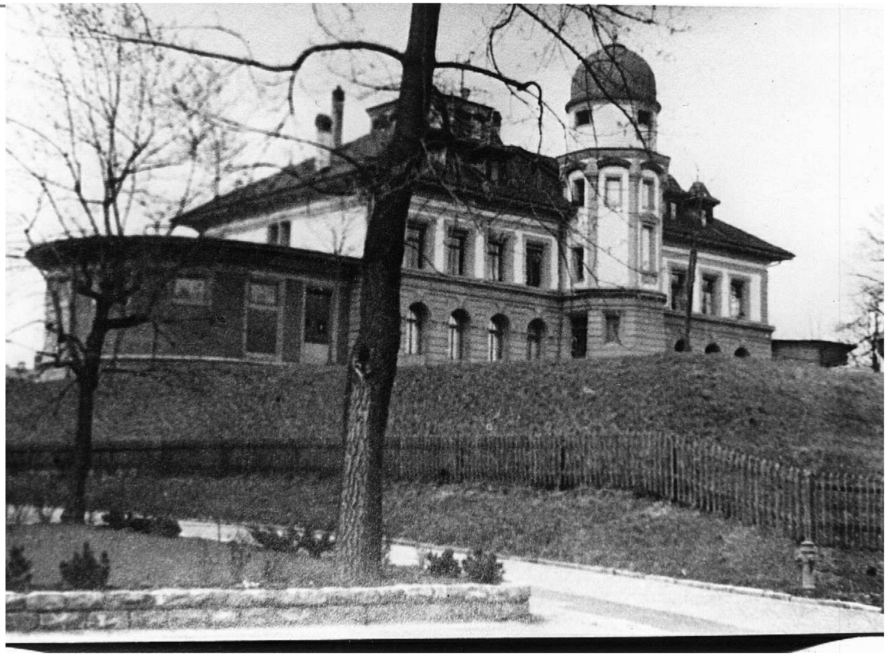
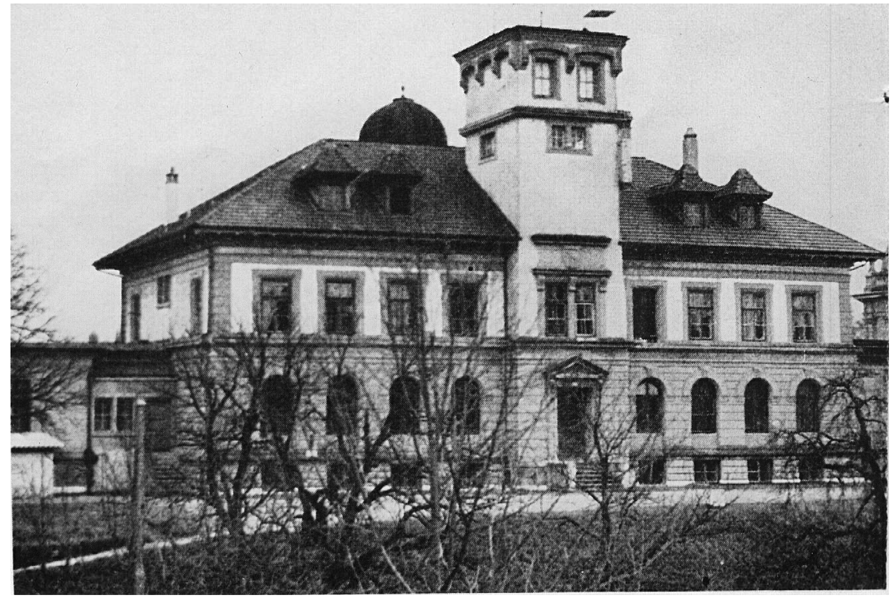
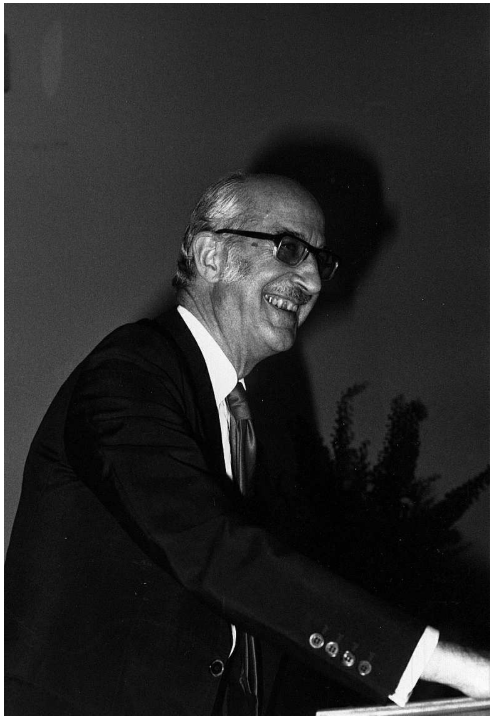




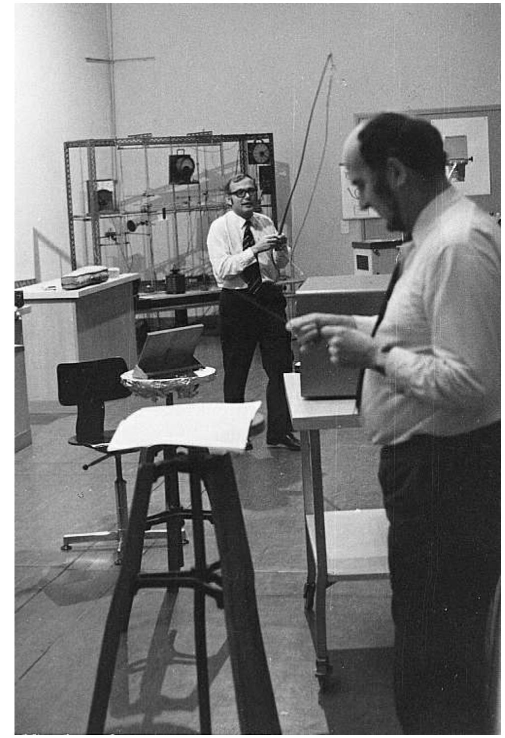
Hans Bebie and I started studying physics in 1957 (pictures below taken in 1964/65)

Our main physics teachers



A. Mercier

F. G. Houtermans



J. Geiss + H. Oeschger

Lecture hall

Institute for theoretical physics

- Until about 1950, this was a one man show, called "Seminar für theoretische Physik"
- 1 room + 1 bookshelf + 1 desk + 1 chair

  The bookshelf was big enough to host the entire library for theoretical physics
- All of the courses on theoretical physics were delivered by A. Mercier
- More on the history of the exact sciences at the UnIversity of Bern: V. Gorgé, in Hochschulgeschichte Berns 1528-1984, pp. 319-351
- W. Thirring was attracted here by Houtermans, acquired the "venia docendi" in 1955 and became Associate Professor in 1958

  Unfortunately, Thirring was offered the chair for theoretical physics in Vienna, accepted the offer in 1959
- By then, 2 or 3 assistant positions had become available

  Hans Bebie and I were hired as "Hilfsassistent"
- For a couple of years, Thirring's position then served to invite guest professors: E.C.G. Stueckelberg, S.A. Wouthuysen, M.A. Tonnelat, J.R. Klauder, H. Rollnik, G. Sudarshan . . .

What we were taught

- First semester: skipped part of the course on "Differential- und Integralrechnung" in favour of the one on mechancs, given by Schürer (Institute for Astronomy). Excellent course, genuine improvement over Maxwell's notation for vectors, but the calligraphy was awful: had to learn scribbling gothic letters . . .
- Lectures on electrodynamics, thermodynamics (Thirring), elasticity theory, special & general relativity, statistical mechanics, quantum mechanics, atomic physics, nuclear physics. Course on particle physics on Saturday mornings, given by Charles Peyrou (CERN, bubble chamber experiments)
  - nonrelativistic potential models for the nuclear forces, Blatt & Weisskopf
  - α-decay, γ decay $\surd$
  - heard about the neutrino, Fermi theory of β decay, parity violaton
  - nuclear reactions, stellar structure $\surd$
- Learned quantum field theory with the book on QED of Jauch & Rohrlich
  - magnetic moments of electron and muon $\surd$
  - Lamb shift $\surd$

Guests at the institute at the beginning of the sixties

- M. A. Tonnelat (Inst. Poincaré, Paris) was guest professor at the University of Bern around 1960/61, lectured on gravity, unification of gravity & electromagnetism, Einstein - Schrödinger, Kaluza - Klein, . . .

  She was enthusiastic about the progress in observational cosmology, expected the deceleration parameter q in Hubble's law to be measured within a year or two . . .
- We were asked us to give our seminars en français, but she made every effort to understand talks delivered in français fédéral fille
- J. R. Klauder (Bell Labs., Murray Hill, N.J.) was guest professor in 1961/62 fabulous teacher – taught us about path integrals

What I did at the time

- Wrote the diploma thesis under the guidance of M. A. Tonnelat (Kaluza-Klein, 1960)
- In France, a decent way to communicate scientific results was to submit these to a member of the Académie des Sciences

RELATIVITÉ. — *Sur une modification des théories pentadimensionnelles destinée à éviter certaines difficultés de la théorie de Jordan-Thiry.* Note (*) de M. **Heinrich Leutwyler**, présentée par M. Louis de Broglie.

Les équations du champ adoptées par Y. Thiry (1)

$$S_{\alpha\beta} = r u_\alpha u_\beta \qquad (\alpha, \beta = 0, 1, 2, 3, 4;\ i, k = 1, 2, 3, 4;\ x^4 = ct) \tag{1}$$

conduisent à des difficultés dans la définition d'une variation à symétrie sphérique (2) et aussi dans l'obtention d'équations approchées du mouvement (3).

En effet K. Just a montré que l'application de (1) au calcul du champ de gravitation créé par une masse neutre possédant la symétrie sphérique conduisait à modifier d'un facteur 5/4 la valeur prévue pour l'avance du périhélie de Mercure. Cette conclusion est indépendante du choix de la métrique quadridimensionnelle (métrique conforme).

Comptes rendus des séances de l'Académie des Sciences, séance du 21 novembre 1960

- Wrote the PhD thesis under the supervision of J. R. Klauder (fermions in curved space, 1962) and then started struggling with the quantum theory of gravity . . .

Most important aha-experience in particle physics since then

- Weak, e.m. and strong interaction phenomena are qualitatively very different. Nevertheless, they are all generated by the same type of fields !

IG Physik, Gesellschaft mit besonderer Haftung, advertisement ca. 1973

Im Falle eines Falles

klebt ein EICHFELD

wirklich alles !

Bezugsquellennachweis

H. Weyl, Z. Phys. 56 (1929) 330, C. N .Yang and R. Mills, Phys. Rev. 96 (1954) 191

Remarks concerning the history of the gauge field concept

- Final form of the laws obeyed by the electromagnetic field: Maxwell $\sim$ 1860 survived relativity and quantum theory, unharmed
- Schrödinger equation for electrons in electromagnetic field ($\hbar = c = 1$)

$$\frac{1}{i}\frac{\partial\psi}{\partial t} - \frac{1}{2m_e^2}(\vec{\nabla} + i\,e\vec{A})^2\psi - e\,\varphi\,\psi = 0$$

contains the potentials $\vec{A}$, φ

- Only $\vec{E} = -\vec{\nabla}\varphi - \dfrac{\partial\vec{A}}{\partial t}$ and $\vec{B} = \vec{\nabla}\times\vec{A}$ are of physical significance
- $\vec{A}\,' = \vec{A} + \vec{\nabla}\alpha\,,\ \varphi\,' = \varphi - \dfrac{\partial\alpha}{\partial t}$ describe the same physical situation as $\vec{A}, \varphi$

$\Rightarrow$ "gauge transformation" of the electromagnetic potentials, term coined by Weyl

- If $\psi(x)$ solves the Schrödinger equation for $\vec{A}, \varphi$,

  then $\psi(x)' = e^{-ie\alpha(x)}\,\psi(x)$ is a solution for $\vec{A}', \varphi'$

$\Rightarrow$ "gauge transformation" of the electron wave function

$\Rightarrow$ "gauge invariance" of the Schrödinger equation

History of the gauge field concept, ctd.

- Equivalence principle of the e.m. interaction:

  $\boxed{\psi \text{ physically equivalent to } e^{-ie\alpha}\,\psi}$

- $e^{-ie\alpha}$ is unitary 1×1 matrix, $e^{-ie\alpha} \in$ U(1)

  $\alpha = \alpha(\vec{x}, t)$ space-time dependent function

- gauge invariance $\Longleftrightarrow$ local U(1) symmetry

  electromagnetic field is gauge field of U(1) Weyl 1929

- gauge invariance is the crucial property of QED

- Illustrate the statement with the core of QED: photons + electrons.
  - Gauge invariance allows only 2 free parameters in the Lagrangian of this system: e, m_e.
  - Moreover, only one of these is dimensionless: $e^2/4\pi = 1/137.035\,999\,679\,(94)$.

  $\Rightarrow$ U(1) symmetry + renormalizability fully determine the properties of the e.m. interaction, except for this number, which, so far still remains unexplained.

History of the gauge field concept, ctd.

- Side remark. In QED, there is an additional operator of dimension ≤ 4: $F_{\mu\nu}\tilde{F}^{\mu\nu}$.
  The term represents a total derivative, $F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu f^\mu$.
  Since e.m. fields with a nontrivial behaviour at large distances do not appear to play a significant role (no instantons or the like), this term does not affect the physics.
- Kaluza (1921) & Klein (1926): 5-dimensional Riemann space with a metric that is independent of the fifth coordinate is equivalent to 4-dimensional world with

  gravity + U(1) gauge field + scalar field

  $x^{5'} = x^5 + f(\vec{x}, t) \iff$ gauge transformation
- Coordinate independent characterization: Riemann space that is invariant under translations in one direction, isometry, Killing vectors, isometry group: U(1)

  Fifth dimension can be compactified to a circle, U(1) generates motions on this circle.
- O. Klein (1938), W. Pauli (1953), generalization of the Kaluza-Klein scenario. Riemann spaces of dimension > 5 can admit *nonabelian* isometry groups that reduce the system to a 4-dimensional one:

  gravity + *nonabelian* gauge fields + several scalar fields

History of the gauge field concept, ctd.

- Pauli was motivated by the isospin symmetry of the meson-nucleon interaction, presumably considered a Riemann space with isometry group SU(2).
- He did not publish the idea, because he thought that the quanta of a gauge field were necessarily massless: gauge invariance does not allow one to put a mass term into the Lagrangian.

$\Rightarrow$ Concluded that the forces mediated by gauge fields would be of long range and hence in conflict with observation

- Description of Pauli's thoughts by P. Gulmanelli $\Rightarrow$ www.mink.itp.unibe.ch
- Excellent account of these developments: N. Straumann, arXiv:0810.2213
- Yang and Mills (1954)
- Ronald Shaw (student of A. Salam) independently formulated nonabelian gauge fields in his PhD thesis, Cambridge 1955

Breakthrough in the theory of the weak interaction

- Many people tried to arrive at a renormalizable form for the weak interaction by introducing intermediate bosons
- Also, it gradually became plausible that the weak interaction is of $\boldsymbol{V} - \boldsymbol{A}$ structure

$\Rightarrow$ If mediated by intermediate bosons, then these must have spin 1

- One of the main obstacles here was Pauli's objection: massless intermediate bosons could not possibly do the job
- Higgs (1964), Brout & Englert (1964) and Guralnik, Hagen & Kibble (1964): in the presence of scalar fields, gauge fields can pick up mass $\Rightarrow$ forces mediated by gauge fields can be of short range
- Glashow (1961), Weinberg (1967), Salam (1968) showed that this mechanism can be used to arrive at a realistic model for the electromagnetic and weak interactions:

$$\boxed{\text{gauge fields of SU(2)}\times\text{U(1) + four real scalar fields}}$$

- Salam gives a very good description of the developments that finally led to the G-W-S theory, in his Nobel lecture of 1979.

What difference does it make ?

- Fermi theory of the weak interaction involves operators of dimension 6, such as $\bar{\mu}_L \gamma_\alpha \nu_L^\mu \, \bar{\nu}_L^e \gamma^\alpha e_L$ $\Rightarrow$ theory only makes sense at tree level. Contributions from loops become increasingly important as the energy grows, but we do not know how to cope with these. Fermi theory is not renormalizable.
- In contrast, the electroweak theory of G-W-S, Quantum Flavour Dynamics, is renormalizable. The crucial property is again gauge invariance.
- Illustrate the statement with the core of QFD: $\gamma, W^\pm, Z, e, \nu$
  - Lagrangian involves 2 dimensionless coupling constants g_1, g_2 and three mass parameters, for instance m_W, m_H, m_e. As compared to QED (e, m_e), an additional coupling constant and two new mass scales are needed, but this is it.

  $\Rightarrow$ Electromagnetic + weak interactions of electron and neutrino are fully determined by gauge invariance with respect to SU(2)$\times$U(1) and four dimensionless parameters. Unfortunately, we do not know where the numerical values of these parameters come from. The ratio $m_W/m_e \simeq 157\,335$ is particularly puzzling.
  - The Higgs mass is not yet well determined, but if the Higgs boson exists, then its mass will be measured at LHC

Strong interaction

- Investigation of the strong interaction started with the nuclear forces
- Discovery of the neutron Chadwick 1932

$\Rightarrow$ Nuclei = p + n

- Yukawa $\sim$ 1935

$$V_{e.m.} = -\frac{e^2}{4\pi r} \qquad\qquad V_s = -\frac{h^2}{4\pi r}\, e^{-\frac{r}{r_0}}$$

$$\frac{e^2}{4\pi} \simeq \frac{1}{137} \qquad\qquad \frac{h^2}{4\pi} \simeq 13$$

long range | short range

$$r_0 = \infty \qquad\qquad r_0 = \frac{\hbar}{M_\pi c} = 1.4 \cdot 10^{-15}\,\text{m}$$

$$M_\gamma = 0 \qquad\qquad M_\pi c^2 \simeq 140\,\text{MeV}$$

- Problems with Yukawa formula
  - p and n are extended objects, diameter comparable to range of force

  $\Rightarrow$ Formula only holds for $r \gg$ diameter
  - Yukawa formula does not explain the properties of the nuclei in a quantitative manner.

Strong interaction

- Nonrelativistic potentials are much more flexible than quantum field theories. Suitable potentials between two nucleons, with attraction at large distances, repulsion at short distances do yield a decent understanding of nuclear structure.

  Paris potential, Bonn potential, shell model of the nucleus

  $\Rightarrow$ Nuclear reactions, processes responsible for the luminosity of the sun, stellar structure, . . . were well understood already fifty years ago.

- These issues concern interactions among nucleons with small relative velocities. Experimentally, it had become possible to explore relativistic collisions, but the description in terms of nonrelativistic potentials did not cover these.
- When I completed my studies, the theory of the strong interaction was a genuine mess: models involving elementary fields for baryons and mesons, Yukawa interaction for the strong forces, perturbation theory with coupling constants of order 1, nuclear democracy, bootstrap . . .
- Absolutely nothing worked even half-way, beyond general principles like Lorentz invariance, causality, unitarity, crossing, dispersion relations
- Many people doubted that a local quantum field theory could do the job
- Smart people considered Regge theory very promising   Veneziano model 1968

Flavour symmetries, quarks

- Isospin — Heisenberg 1932
- Strangeness quantum number — Gell-Mann 1953

  Gell-Mann-Nishijima formula
- Eightfold way — Gell-Mann, Ne'eman 1961
- Pattern of symmetry breaking, $\boldsymbol{\Omega^-}$ — Gell-Mann, Okubo 1961/1962
- $\boxed{\text{Quark model}}$ — Gell-Mann, Zweig 1962
- Puzzle: why is the symmetry not exact ?

  Exact consequences of approximate properties ?

  Charges & currents form an exact algebra

  even if they do not commute with the Hamiltonian — Gell-Mann 1964
- Test of current algebra: size of $\langle N|A^\mu|N\rangle \sim g_A$ — Adler 1965, Weisberger 1966
- Prediction from current algebra: $\pi\pi$ scattering lengths — Weinberg 1966

Quark masses

- Even before the discovery of QCD, attempts at estimating the masses of the quarks were made
- Bound state models for mesons and baryons

  $m_u + m_u + m_d \simeq M_p \qquad\qquad m_u \simeq m_d \simeq 300$ MeV

  "constituent masses"
- Remarkably simple and successful picture, explains the pattern of energy levels without QCD, but $\boxed{\text{why do the quarks not show up in experiment ?}}$

$\Rightarrow$ The existence of quarks was considered doubtful

- Quarks were treated like the veal used to prepare a pheasant in the royal french cuisine
- The deep inelastic scattering experiments at SLAC in 1969 offered the first direct experimental evidence for point-like constituents within the proton. Feynman called these "partons", leaving it open whether they were the quarks or something else.
- Model for spontaneous symmetry breakdown requires much smaller fermion masses

  Nambu & Jona-Lasinio 1961
- Same conclusion from sum rules for currents Okubo 1969
- Conceptual basis of the royal french cuisine ?

Colour

- Protons, neutrons are composed of quarks

  p $= \boldsymbol{uud}$ n $= \boldsymbol{udd}$

- Quarks carry internal quantum number

$$\boldsymbol{u} = \begin{pmatrix} \boldsymbol{u_1} \\ \boldsymbol{u_2} \\ \boldsymbol{u_3} \end{pmatrix} \qquad \boldsymbol{d} = \begin{pmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_2} \\ \boldsymbol{d_3} \end{pmatrix}$$

- Occur in 3 “colours”, term coined by M. Gell-Mann, H. Fritzsch.
  O.W. Greenberg had introduced an internal degree of this type already in 1964, referring to this as "parastatistics".

- Fritzsch and Gell-Mann discussed the option that the strong interaction is mediated by a gauge field coupled to colour on the occasion of the High Energy Physics Conference at Fermilab, in 1972.

- Met Gell-Mann at Schladming in 1972, invited me to visit Caltech.
  Did that during the spring break in 1973 (mid-February to mid-May).
  Spent an extremely interesting period there (Feynman, Gell-Mann).
  Had the chance of participating in the discussion of the possibility that the strong interaction is mediated by a gauge field.

QCD

- Is the strong interaction mediated by a gauge field coupled to colour ?
- Rotations in colour space a local symmetry ?

$$u' = U \cdot u \qquad d' = U \cdot d$$

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \in \mathrm{SU}(3) \qquad \text{nonabelian group}$$

- Strong interaction mediated by a nonabelian gauge field ?
- Gell-Mann gave it a decent name:

$$\boxed{\text{Quantum chromodynamics}}$$

- Main problem at the time: All quantum field theories encountered in nature so far had the spectrum of perturbation theory. This also applies to the electroweak theory.

$\Rightarrow$ Proposal does not look plausible. The spectrum of physical states neither contains quarks nor massless gluons. Pauli

- Arguments in favour of QCD were listed in Phys. Lett. **B47** (1973) 365. One of these was asymptotic freedom – the papers of Gross & Wilczek and Politzer are quoted. Also, some open questions were pointed out, in particular, the U(1) problem.

Asymptotic freedom

- To my knowledge, the term "asymptotic freedom" was coined by Coleman (see his Erice Lectures).
- The first correct calculation of the β-function of a nonabelian gauge field theory was carried out by J. Khriplovich (Novosibirsk), for the case of SU(2), relevant for the electroweak interaction. He found that β is negative and concluded that the interaction becomes weak at short distance.

  I.B. Khriplovich, Yad. Phys. 10 (1969) 409 [Sov. J. Nucl. Phys. 10 (1970) 235]
- In his PhD thesis, t'Hooft, who was not aware of the work of Khriplovich, performed the calculation of the β-function for an arbitrary gauge group, including the interaction with fermions and Higgs scalars. He demonstrated that the theory is renormalizable and confirmed that, unless there are too many fermions or scalars, the β-function is negative. G. t'Hooft, Nucl. Phys. B33, 173 (1971), B35, 167 (1971)
- Gross, Wilczek, Politzer discussed the consequences of a negative β function: asymptotic freedom, infrared slavery. They suggested that this might explain Bjorken scaling, which had been observed at SLAC in 1969. Nobel Prize 2004
- A detailed account of the history of the quantum theory of gauge fields can be found in the 1998 Erice lecture notes of G. 't Hooft:

  G. 't Hooft, "The glorious days of physics: Renormalization of gauge theories," arXiv:hep-th/9812203

QCD

- Many papers concerning gauge field theories that include the strong interaction were published around the same time, e.g.
  Pati and Salam, Phys. Rev. D **8** (1973) 1240, D **10** (1974) 275
  Weinberg, Phys. Rev. Lett. 31 (1973) 494
- Many considered QCD a wild speculation
- Can one prove that QCD is fundamentally different from the quantum field theories encountered earlier in that the spectrum of physical states does not show the local degrees of freedom ? Or is this wishful thinking ?
- Only gradually, particle physicists abandoned their outposts in no man's and no woman's land, returned to the quantum fields and resumed discussion in the good old Gasthaus zu Lagrange Jost

$\Rightarrow$ Standard Model, clarified the picture enormously

- Gauge fields are renormalizable in $d = 4$
- Paradigm has changed: SM cannot be the full truth
  No reason for an effective theory to be renormalizable

$\Rightarrow$ Why is the SM renormalizable ?

Electromagnetic versus strong interaction

	QED	QCD
symmetry	U(1)	SU(3)
gauge field	$\vec{A}\,,\ \varphi$	gluon field
particles	photons	gluons
source	charge	colour
coupling constant	e	g

- All charged particles generate e.m. field
- All coloured particles generate gluon field
- Leptons do not interact strongly because they do not carry colour
- Equivalence principle of the strong interaction:

$$\boxed{U\cdot\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}\ \text{physically equivalent to}\ \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}}$$

QED+QCD

- Effective theory for $E \ll M_{\mathrm{W}} c^2 \simeq 80\,$GeV

  Symmetry U(1) $\times$ SU(3)
  Lagrangian QED $+$ QCD

- Dynamical variables:
  gauge fields for photons and gluons
  Fermi fields for leptons and quarks

- Interaction determined by group geometry, in terms of 3 coupling constants

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4g^2} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} + \ldots$$

- Quark and lepton mass matrices can be brought to diagonal form, eigenvalues real, positive

$$\boxed{m_e\,,\; m_\mu\,,\; m_\tau\,,\; m_u\,,\; m_d\,,\; m_s\,,\; m_c\,,\; m_b\,,\; m_t}$$

$\Rightarrow$ Precision theory for cold matter, atomic structure, solids, $\ldots$

Bohr radius: $$a = \frac{4\pi}{e^2\, m_e}$$

Qualitative difference between QED and QCD

- Photons do not have charge, gluons do have colour

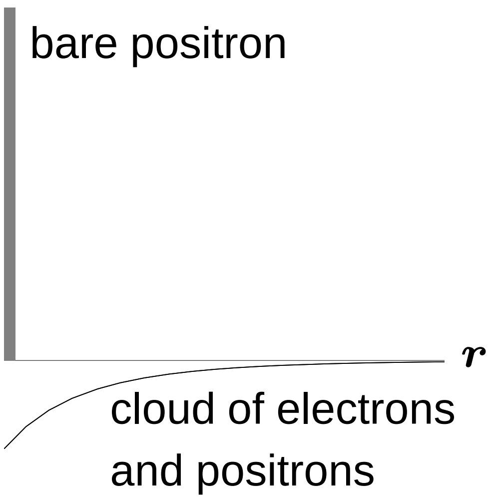
$\mathrm{x}_1 \cdot \mathrm{x}_2 = \mathrm{x}_2 \cdot \mathrm{x}_1$ for $\mathrm{x}_1, \mathrm{x}_2 \in$ U(1) abelian

$\mathrm{x}_1 \cdot \mathrm{x}_2 \neq \mathrm{x}_2 \cdot \mathrm{x}_1$ for $\mathrm{x}_1, \mathrm{x}_2 \in$ SU(3)

$\Rightarrow$ Consequence for vacuum polarization:

QED

Density of charge

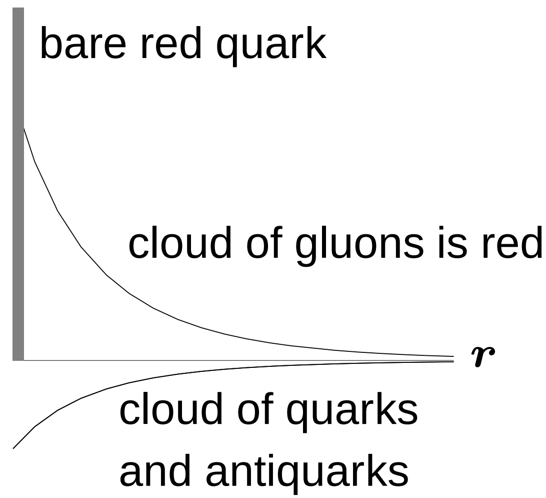


$\boxed{e < e_{\mathrm{bare}}}$

vacuum
shields charge

QCD

Density of colour



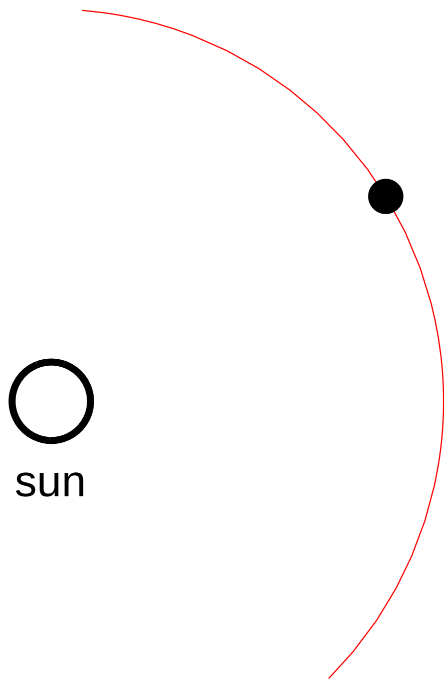
$\boxed{g > g_{\mathrm{bare}}}$

vacuum
amplifies colour

Comparison with gravity

- source of gravitational field: energy gravitational field <u>does</u> carry energy
- source of e.m. field: charge e.m. field <u>does not</u> carry charge
- source of gluon field: colour gluon field <u>does</u> carry colour

gravity

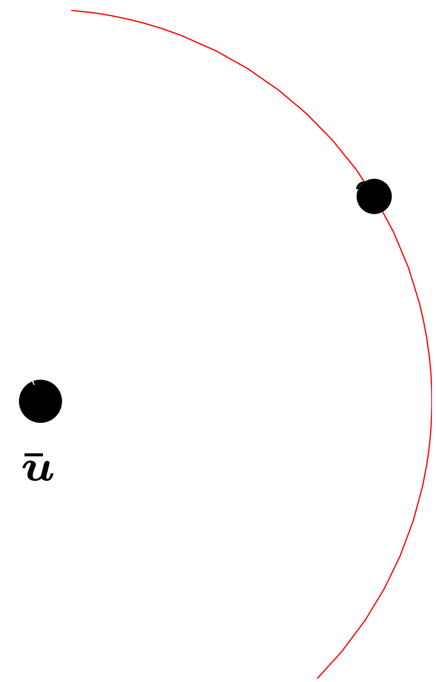


planet feels

less than total

energy of the sun

strong interaction



u *quark feels*

less than total

colour of $\bar{u}$

Perihelion shift of Mercury:

$$\mathbf{43'' = 50'' - 7''} \text{ per century}$$
$$\Uparrow$$

Force between u and $\bar{u}$

$$V_s = -\frac{4}{3}\,\frac{g^2}{4\pi r}\,, \qquad g \to 0 \quad \text{for} \quad r \to 0$$

$$\frac{g^2}{4\pi} = \frac{6\pi}{(11N_c - 2N_f)\,|\ln(r\,\Lambda_{\mathrm{QCD}})|}$$

$$|\ln(r\,\Lambda_{\mathrm{QCD}})| \simeq 7 \quad \text{for} \;\; r = \frac{\hbar}{M_Z\, c} \simeq 2\cdot 10^{-18}\ \text{m}$$

- Vacuum amplifies gluonic field of a bare quark
- Field energy surrounding isolated quark = ∞
  Only colour neutral states have finite energy

$\Rightarrow$ Confinement of colour ?

- Experimental evidence still much more convincing than the theoretical one

QED: interaction weak at low energies
QCD: interaction strong at low energies

$\dfrac{e^2}{4\pi} \simeq \dfrac{1}{137}$	$\dfrac{g^2}{4\pi} \simeq 1$
photons, leptons nearly decouple	gluons, quarks confined

- Nuclear forces = van der Waals forces of QCD

Chiral symmetry

- Main characteristic of the strong interaction at low energies:

  energy gap is small, $M_\pi \simeq$ 140 MeV
- In 1960, Nambu found out why that is so: Nobel Prize 2008
  - Has to do with a hidden approximate symmetry
  - Symmetry becomes exact for $m_u, m_d \longrightarrow 0$
  - For dynamical reasons, the state of lowest energy must be asymmetric

  $\Rightarrow$ Chiral symmetry is hidden, “spontaneously broken”
  - Nambu realized that the spontaneous breakdown of a continuous symmetry entails massless particles and concluded that the pions must play this role:

  $\Rightarrow$ No gap at all if $m_u, m_d \longrightarrow 0$: pions become massless
  - "Goldstone theorem" came later: Goldstone, Salam, Weinberg 1962
  - In reality $m_u, m_d \neq 0$, but very small

  $\Rightarrow$ Symmetry is not perfect, but nearly so

  $\Rightarrow$ There is an energy gap, but it is small

Compare magnets

- Spontaneous symmetry breakdown was discovered in condensed matter physics
- Heisenberg model of a magnet
  - Hamiltonian is invariant under rotation of the spins
  - Spontaneous magnetization selects direction

  $\Rightarrow$ Rotation symmetry is spontaneously broken
  - Goldstone bosons = spin waves, magnons
- Nambu pointed out that internal symmetries of particle physics can also break down spontaneously

Is there an approximate chiral symmetry in QCD ?

- $\boxed{m_u\,, m_d\,, m_s \text{ happen to be light}}$
- Theoretical paradise:

  $m_u = m_d = m_s = 0$

  $m_c = m_b = m_t = \infty$

  $\boxed{\text{QCD with 3 massless quarks}}$
- Lagrangian contains a single parameter: g, net colour of a quark depends on radius of the region considered
- Colour contained within radius r

$$\frac{g^2}{4\pi} = \frac{2\pi}{9\,|\ln(r\,\Lambda_{\text{QCD}})|}$$

- Intrinsic scale Λ_{QCD} is meaningful, but not dimensionless

$\Rightarrow$ No dimensionless free parameter

All dimensionless physical quantities are pure numbers, determined by the theory. Cross sections can be expressed in terms of Λ_{QCD} or in the mass of the proton.

Symmetries of massless QCD

- Interactions of u, d, s are identical. If the masses are set equal to zero, there is no difference at all: Lagrangian symmetric under $u \leftrightarrow d \leftrightarrow s$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$q' = V \cdot q \qquad V \in \mathrm{SU(3)}$$

V acts on quark flavour, mixes u, d, s

- More symmetry: For massless fermions, right and left do not communicate

$\Rightarrow$ Lagrangian of massless QCD is invariant under independent rotations of the right– and left–handed quark fields

$$q_{\mathrm{R}} = \tfrac{1}{2}(1+\gamma_5)\, q\,, \quad q_{\mathrm{L}} = \tfrac{1}{2}(1-\gamma_5)\, q$$

$$q'_{\mathrm{R}} = V_{\mathrm{R}} \cdot q_{\mathrm{R}} \qquad\qquad q'_{\mathrm{L}} = V_{\mathrm{L}} \cdot q_{\mathrm{L}}$$

$$G = \mathrm{SU(3)}_{\mathrm{R}} \times \mathrm{SU(3)}_{\mathrm{L}}$$

- Massless QCD has an exact chiral symmetry

Massless QCD

- Massless QCD invariant under $G = \text{SU(3)}_\text{R} \times \text{SU(3)}_\text{L}$

  SU(3) has 8 parameters

  $\Rightarrow$ Symmetry under Lie group with 16 parameters

  $\Rightarrow$ 16 conserved “charges”

  $Q_1^\text{V}, \ldots, Q_8^\text{V}$ (vector currents)

  $Q_1^\text{A}, \ldots, Q_8^\text{A}$ (axial currents)

  commute with the Hamiltonian:

$$[Q_i^\text{V}, H_0] = 0 \qquad [Q_i^\text{A}, H_0] = 0$$

- Vafa and Witten 1984: state of lowest energy is invariant under the vector charges

  $Q_i^\text{V} \, |0\rangle = 0$

- Axial charges ? $Q_i^\text{A} \, |0\rangle = ?$

Two alternatives for axial charges

$Q_i^{\mathsf{A}}\,\|0\rangle = 0$
Wigner-Weyl realization of G
ground state is symmetric
$\langle 0\|\,\bar{q}_{\mathsf{R}}\, q_{\mathsf{L}}\,\|0\rangle = 0$
ordinary symmetry
spectrum contains parity partners
degenerate multiplets of G

$Q_i^{\mathsf{A}}\,\|0\rangle \neq 0$
Nambu-Goldstone realization of G
ground state is asymmetric
$\langle 0\|\,\bar{q}_{\mathsf{R}}\, q_{\mathsf{L}}\,\|0\rangle \neq 0$
"order parameter"
spontaneously broken symmetry
spectrum contains Goldstone bosons
degenerate multiplets of $\mathsf{SU(3)_V} \subset \mathsf{G}$

$$\mathsf{G} = \mathsf{SU(3)_R} \times \mathsf{SU(3)_L}$$

Quark condensate

- Analog of Magnetization ?

$$\bar{q}_{\rm R}\, q_{\rm L} = \begin{pmatrix} \bar{u}_{\rm R}\, u_{\rm L} & \bar{d}_{\rm R}\, u_{\rm L} & \bar{s}_{\rm R}\, u_{\rm L} \\ \bar{u}_{\rm R}\, d_{\rm L} & \bar{d}_{\rm R}\, d_{\rm L} & \bar{s}_{\rm R}\, d_{\rm L} \\ \bar{u}_{\rm R}\, s_{\rm L} & \bar{d}_{\rm R}\, s_{\rm L} & \bar{s}_{\rm R}\, s_{\rm L} \end{pmatrix}$$

Transforms like $(\bar{3}, 3)$ under $\mathrm{SU(3)_R} \times \mathrm{SU(3)_L}$

- If the ground state were symmetric, the matrix $\langle 0|\, \bar{q}_{\rm R}\, q_{\rm L}\, |0\rangle$ would have to vanish, because it singles out a direction in flavour space

$\langle 0|\, \bar{q}_{\rm R}\, q_{\rm L}\, |0\rangle$ is referred to as the "quark condensate", quantitative measure of the strength of spontaneous symmetry breaking, "order parameter"

$\langle 0|\, \bar{q}_{\rm R}\, q_{\rm L}\, |0\rangle \Leftrightarrow$ magnetization

- Ground state is invariant under $\mathrm{SU(3)_V}$

$\Rightarrow$ $\langle 0|\, \bar{q}_{\rm R}\, q_{\rm L}\, |0\rangle$ is proportional to unit matrix

$$\langle 0|\, \bar{u}_{\rm R}\, u_{\rm L}\, |0\rangle = \langle 0|\, \bar{d}_{\rm R}\, d_{\rm L}\, |0\rangle = \langle 0|\, \bar{s}_{\rm R}\, s_{\rm L}\, |0\rangle$$

$$\langle 0|\, \bar{u}_{\rm R}\, d_{\rm L}\, |0\rangle = \ldots = 0$$

Goldstone bosons of QCD

- QCD chooses the Nambu-Goldstone mode

  $Q_i^{\mathrm{A}}\,|0\rangle \neq 0$
- Immediate consequence: $H_0\{Q_i^A\,|0\rangle\} = Q_i^A H_0\,|0\rangle = 0$

$\Rightarrow$ massless QCD contains 8 physical states $Q_1^A\,|0\rangle, \,\ldots\,, Q_8^A\,|0\rangle$ with $E = 0$, spin 0, negative parity, octet of SU(3)$_V$

Indeed, the 8 lightest mesons do have these quantum numbers:

$\pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, \eta$

- All other one-particle states must form degenerate multiplets of SU(3)$_V$
- In the real world, the multiplets are split and the lightest mesons are not massless
- Real world $\neq$ paradise

  $m_u\,,\, m_d\,,\, m_s \neq 0$

  Quark masses break chiral symmetry, allow left to talk to right
- Chiral symmetry broken in two ways:

  spontaneously $\langle 0|\,\bar{q}_{\mathrm{R}}\, q_{\mathrm{L}}\,|0\rangle \neq 0$

  explicitly $m_u\,,\, m_d\,,\, m_s \neq 0$

Quark masses as perturbations

- Only the diagonal vector currents are strictly conserved in QCD: $N_u, N_d, N_s, N_c, N_b, N_t \;\; \longrightarrow$ baryon number, electric charge, strangeness, charm, ...
- It so happens that m_u, m_d, m_s are small

$\Rightarrow$ H_{QCD} has an approximate SU(3)$_L\times$SU(3)$_R$ symmetry

- Masses of the light quarks enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d + m_s \, \bar{s}s\}$$

H_0 describes u, d, s as massless, c, b, t as massive

H_0 is invariant under SU(3)$_L\times$SU(3)$_R$

Expansion in powers of m_u, m_d, m_s	$\Longleftrightarrow$	Perturbation series in powers of H_1

- H_0 treats π, K, η as massless particles, H_1 gives them a mass

Pattern of light quark masses

- With the discovery of QCD, the mass of the quarks became an unambiguous concept: quark masses occur in the Lagrangian of the theory.
- First crude estimate within QCD relied on a model for the wave functions of π, K, ρ, based on SU(6) (spin-flavour-symmetry)

$$\frac{(m_u + m_d)}{2} = \frac{F_\pi M_\pi^2}{3 F_\rho M_\rho} \simeq 5\,\text{MeV}, \qquad\qquad m_s \simeq 135\,\text{MeV}$$

"Is the quark mass as small as 5 MeV ?" 1974

- Not very different from the pattern found within the Nambu-Jona-Lasinio model (1961) or the one obtained from sum rules by Okubo (1969)
- Difference between m_u and m_d ?

  In 1975, Jürg Gasser and I reanalyzed the Cottingham formula

$\Rightarrow$ e.m. self energy of proton $>$ neutron

$\Rightarrow$ $M_p < M_n$ cannot be due to the e.m. interaction

$\Rightarrow$ $M_p < M_n$ must be due to $m_u < m_d$

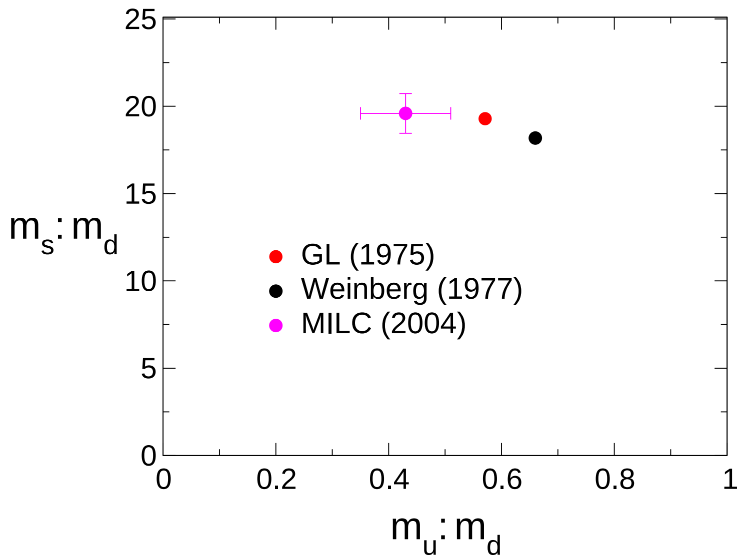
$\Rightarrow$ Isospin not a symmetry of the strong interaction ! In fact an apparently very strong breaking was needed:

$$m_u \simeq 4\,\text{MeV}, \quad m_d \simeq 7\,\text{MeV}$$

Crude picture for m_u, m_d, m_s

- $m_u \simeq 4\,\text{MeV},\;\; m_d \simeq 7\,\text{MeV},\;\; m_s \simeq 135\,\text{MeV}$ GL 1975
  - m_u and m_d are very different
  - m_u and m_d are small compared to m_s
  - “constituent masses” $\notin$ Lagrangian of QCD
- Took quite a while before this pattern was taken seriously extra muros

Weinberg 1977



Approximate symmetries are natural in QCD

- Why is isospin such a good quantum number ?

  (a) Dimensional transmutation, divergences of perturbation theory $\in$ physics $\Rightarrow$ QCD has an intrinsic scale

  (b) $m_d - m_u \ll$ scale of QCD, not $\ll m_u + m_d$
- Why is the eightfold way a decent approximate symmetry ?

  $m_s - m_u \ll$ scale of QCD
- Isospin is an even better symmetry because

  $m_d - m_u \ll m_s - m_u$
- $m_u \ll m_s \Rightarrow m_u, m_d, m_s \ll$ scale of QCD

$\Rightarrow$ Masses of the light quarks represent perturbations

Can neglect these in a first approximation

Magnitude of the perturbations due to m_u, m_d, m_s

- $\langle 0|\,\bar{d}\gamma^\mu\gamma_5\,u|\pi^+\rangle = i\,p^\mu\sqrt{2}\,F_\pi$

  $\langle 0|\,\bar{s}\gamma^\mu\gamma_5\,u|K^+\rangle = i\,p^\mu\sqrt{2}\,F_K$

  Difference between F_K and F_π comes from $m_s \neq m_d$

- Observed ratio: $\dfrac{F_K}{F_\pi} = 1.19 \pm 0.01$

  Branching fraction of $K \to \pi e \nu$ changed by $> 3\,\sigma$ in 2004 ! $1.22 \to 1.19$

$\Rightarrow$ $m_s - m_d$ generates correction of order 20%

- $m_u\,, m_d \ll m_s$ $\Rightarrow$ correction mainly comes from m_s
- effects from $m_u\,, m_d$ are tiny

Gell-Mann-Oakes-Renner formula

- First order perturbation theory yields:†

$$M_\pi^2 = \underset{\substack{\Uparrow\\ \text{explicit}}}{(m_u + m_d)} \times \underset{\substack{\Uparrow\\ \text{spontaneous}}}{|\langle 0|\,\bar{u}u\,|0\rangle|} \times \frac{1}{F_\pi^2}$$

Gell-Mann, Oakes & Renner 1968

Coefficient: decay constant F_π

$$\langle 0|\,\bar{d}\gamma^\mu\gamma_5 u|\pi^+\rangle = i\,p^\mu\sqrt{2}\,F_\pi$$

Value of F_π is known from $\pi^+ \to \mu^+\nu$

† formula does not appear like this in the paper

Consequences of GMOR formula

- $M_\pi^2 = (m_u + m_d)\, B + O(m^2)$

$\Rightarrow$ The energy gap of QCD is small because $m_u\,, m_d$ happen to be small

- $M_{K^+}^2 = (m_u + m_s)\, B + O(m^2)$

  $M_{K^0}^2 = (m_d + m_s)\, B + O(m^2)$

$\Rightarrow$ M_K^2 is much larger than M_π^2, because m_s happens to be large compared to $m_u\,, m_d$

- Goldstone boson masses measure the strength of symmetry breaking

$\Rightarrow$ strongly violate SU(3)

- Check: first order perturbation theory also yields

  $M_\eta^2 = \frac{1}{3}\,(m_u + m_d + 4m_s)B + O(m^2)$

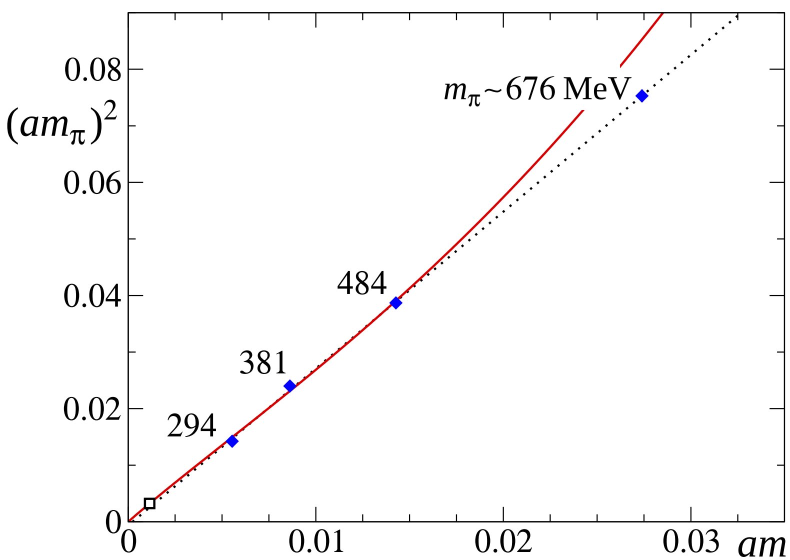
$\Rightarrow$ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 $\surd$

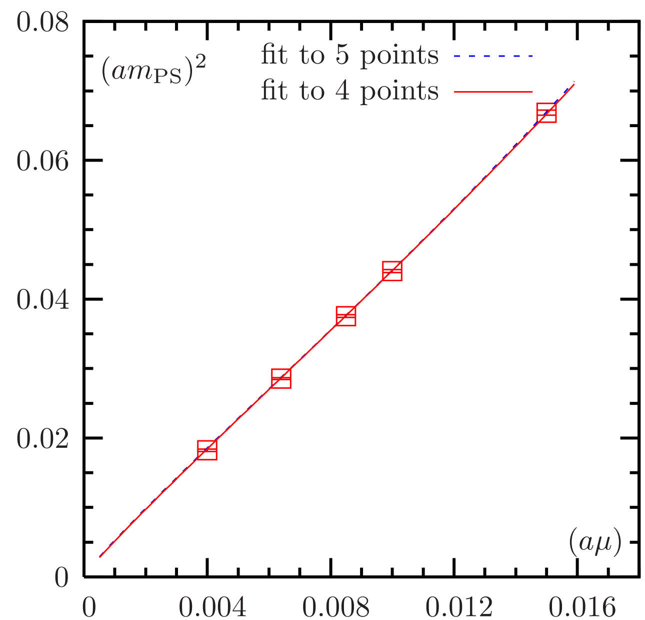
Lattice

- Simulations of QCD on a lattice now reach sufficiently small lattice spacings, sufficiently small quark masses to make contact with physics
- GMOR formula can now be checked on the lattice:

determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

Lattice

- Quality of data is impressive
- No quenching, quark masses are sufficiently light

$\Rightarrow$ Legitimate to use χPT for the extrapolation to the physical values of $m_u\,,m_d$

- Proportionality of M_π^2 to the quark mass appears to hold out to values of $m_u\,,m_d$ that are an order of magnitude larger than in nature
- Main limitation: systematic uncertainties
  in particular: $N_f = 2 \longrightarrow N_f = 3$

Net result

$G =$ SU(3)$_L \times$SU(3)$_R$ is an approximate symmetry of H_{QCD}

$|\mathbf{0}\rangle$ approximately symmetric only under SU(3)$_V \subset G$

- World we live in is close to the paradise
- Light quark masses amount to a small perturbation
- Chiral part of the symmetry is hidden

$\Rightarrow$ Only the subgroup SU(3) $\subset G$ is an approximate symmetry of the spectrum and of the matrix elements

"Eightfold way", $u \leftrightarrow d \leftrightarrow s$

- $m_u\,, m_d$ are particularly small

$\Rightarrow$ SU(2)$_L \times$SU(2)$_R$ is a nearly exact symmetry of H_{QCD}

$\Rightarrow$ Expansion in powers of $m_u\,, m_d$ converges very rapidly

Low energy expansion

- If the spectrum has an energy gap

$\Rightarrow$ no singularities in scattering amplitudes
or Green functions near $p = 0$

$\Rightarrow$ low energy behaviour may be analyzed with Taylor series expansion in powers of p

$$f(t) = 1 + \tfrac{1}{6}\langle r^2\rangle\, t + \ldots \quad \text{form factor}$$
$$T(p) = a + b\, p^2 + \ldots \quad \text{scattering amplitude}$$

Cross section dominated by
S–wave scattering length

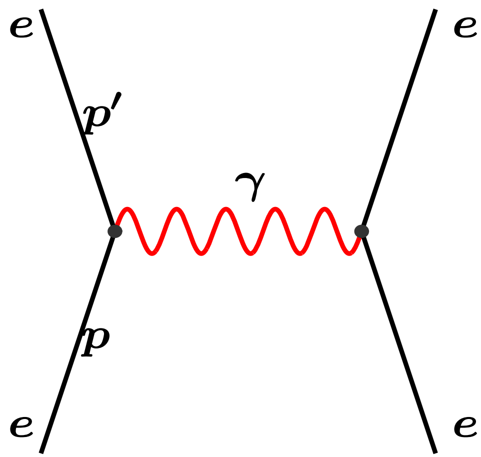
$$\frac{d\sigma}{d\Omega} \simeq |a|^2$$

- Expansion parameter: $\dfrac{p}{m} = \dfrac{\text{momentum}}{\text{energy gap}}$

- Taylor series only works if the spectrum has an energy gap, i.e. if there are

$\boxed{\text{no massless particles}}$

Illustration: Coulomb scattering



- Photon exchange $\Rightarrow$ pole at $t = 0$

$$T = \frac{e^2}{(p' - p)^2}$$

  Scattering amplitude does not admit Taylor series expansion in powers of p

- QCD does have an energy gap, but the gap is very small: M_π

$\Rightarrow$ Taylor series has very small radius of, convergence, useful only for $p < M_\pi$

Effective theory

- Massless QCD contains infrared singularities due to the Goldstone bosons
- For $m_u = m_d = 0$, pion exchange gives rise to poles and branch points at $p = 0$

$\Rightarrow$ Low energy expansion is not a Taylor series, contains logarithms

- Properties of the Goldstone bosons are governed by the hidden symmetry that is responsible for their occurrence

$\Rightarrow$ Goldstone bosons of low momentum interact only weakly: can treat the momenta as well as the quark masses as perturbations

"Chiral Perturbation Theory"

- Formulation in terms of an effective Lagrangian

Weinberg 1967, Coleman, Wess, Zumino, Callan, Dashen, Weinstein 1969

- Unperturbed Lagrangian describes massless Goldstone Bosons

$\Rightarrow$ Chiral perturbation series has infrared singularities

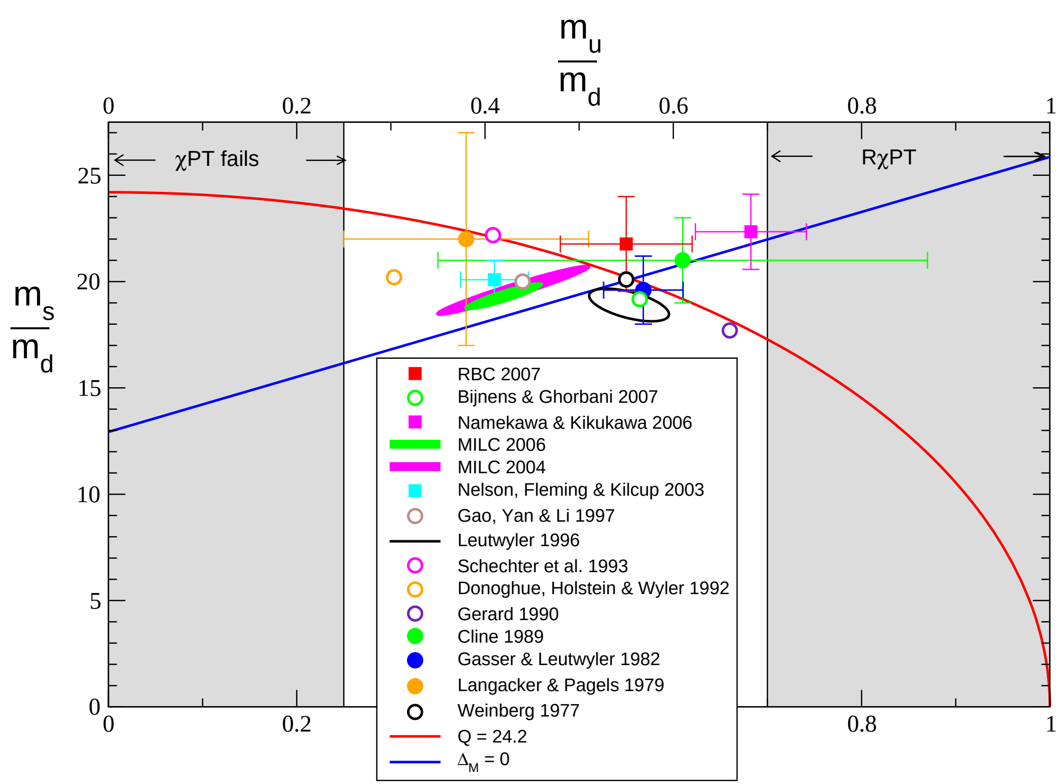
Li & Pagels 1971, Langacker & Pagels 1973, Gasser & Zepeda 1980, Gasser 1981

Singularities due to Goldstone bosons can be worked out with an effective field theory “Chiral Perturbation Theory”

- Chiral perturbation theory correctly reproduces the infrared singularities of QCD

Quark mass ratios

- Jürg Gasser analyzed the infrared singularities occurring in the expansion of the meson and baryon masses in powers of m_u, m_d, m_s, habilitation thesis,1981
- Together, we then used his results to get an improved determination of the quark masses, in particular of the ratios m_u/m_d and m_s/m_d, 1982



Expansion of M_π^2 in powers of the quark masses

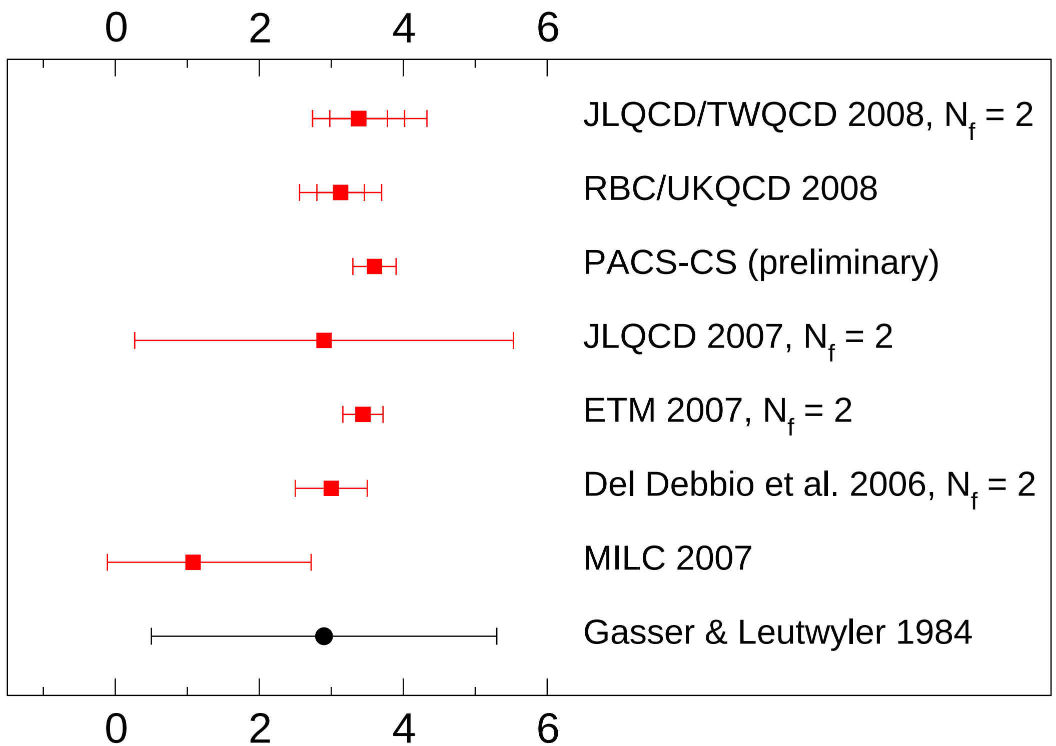
- Gell-Mann-Oakes-Renner is leading term of the chiral perturbation series
- Disregard isospin breaking, set $m_u = m_d = m$
- Expand in powers of m, keeping m_s fixed
- At NLO, the expansion contains a logarithm

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F_\pi^2} \ln \frac{M^2}{\Lambda_3^2} + O(M^4) \right\}$$
$$M^2 \equiv 2Bm$$

- Coefficient is determined by the pion decay constant
  Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3)$\times$SU(3):

0.2 GeV $\lesssim \Lambda_3 \lesssim 2$ GeV   Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



Horizontal axis shows the value of $\bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2}$

Range for Λ_3 obtained in 1984 corresponds to $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of RBC/UKQCD 2008: $\bar{\ell}_3 = 3.13 \underset{stat}{\pm 0.33} \underset{syst}{\pm 0.24}$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

$$F_\pi = F\left\{1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\Lambda_4^2} + O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1 + \frac{M^2}{32\pi^2 F^2}\ln\frac{M^2}{\Lambda_3^2} + O(M^4)\right\}$$

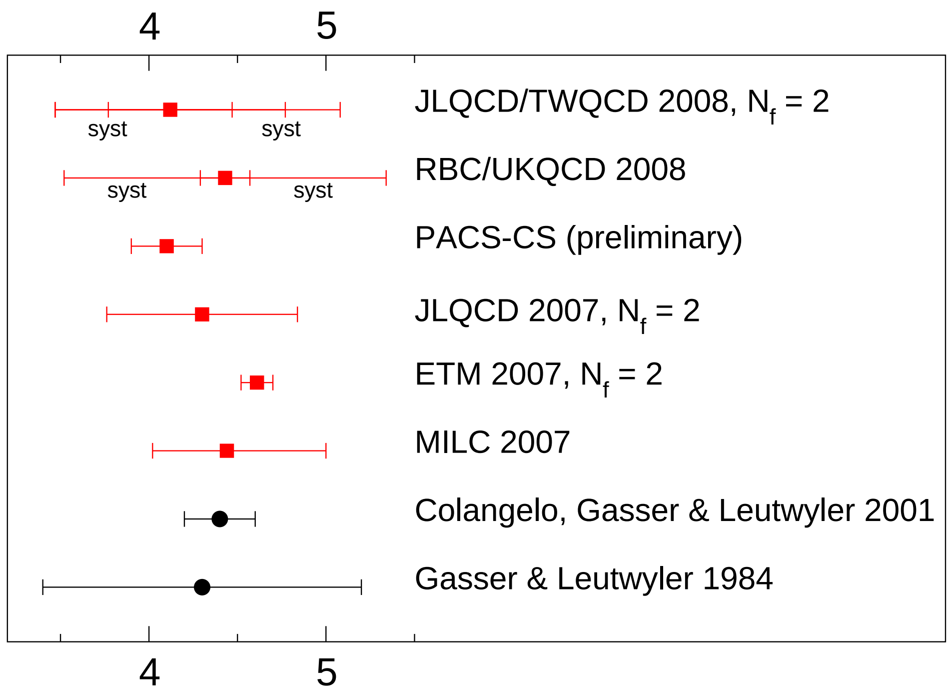
F is value of pion decay constant in limit $m_u, m_d \rightarrow 0$

- Structure is the same, coefficients and scale of logarithm are different
- Quark mass dependence of F_π can also be measured on the lattice

$\Rightarrow$ measurement of Λ_4

- Alternative method: determine the scalar form factor of the pion, radius $\langle r^2\rangle_s \Leftrightarrow \bar{\ell}_4$

Lattice results for Λ_4

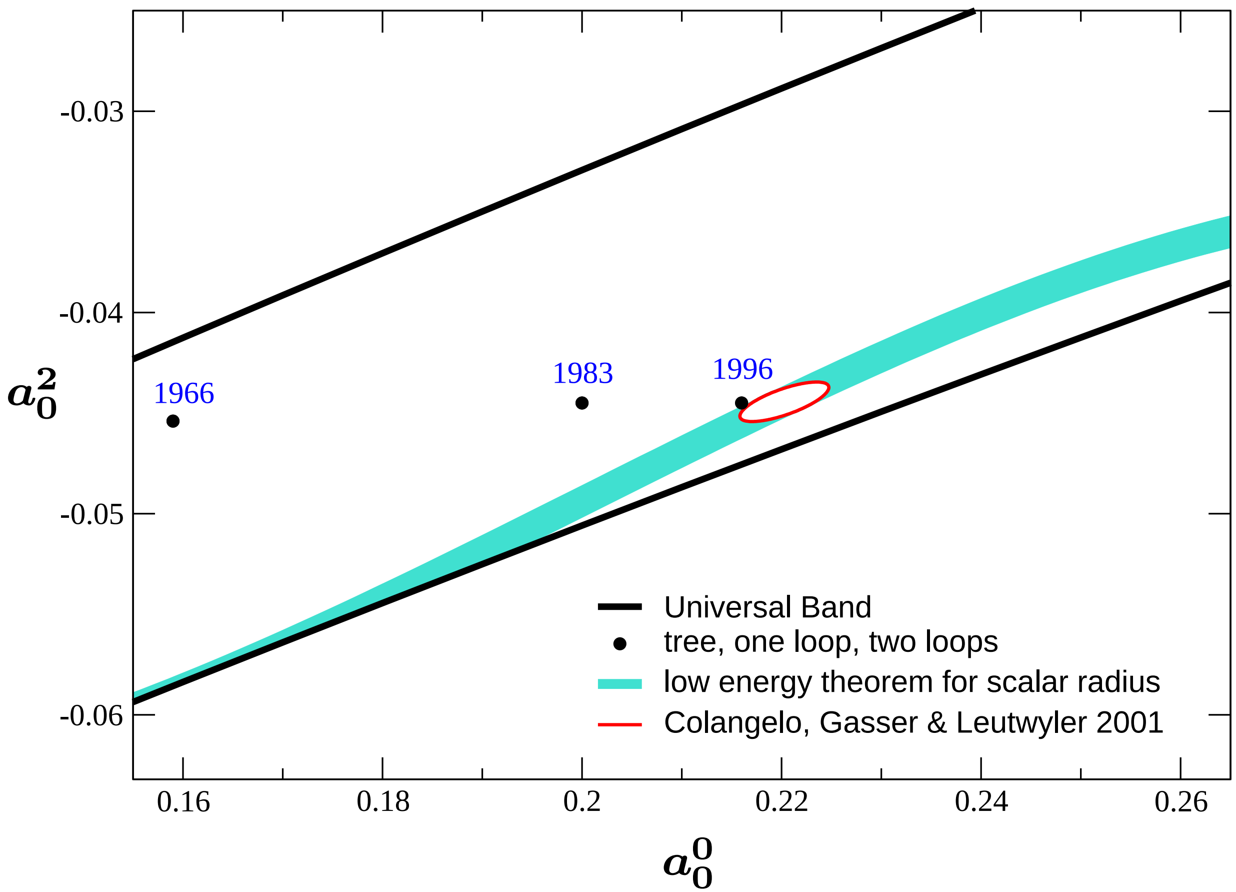


$$\bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

- Lattice results beautifully confirm the prediction for the sensitivity of F_π to m_u, m_d:

$$\frac{F_\pi}{F} = 1.072 \pm 0.007$$ Colangelo and Dürr 2004

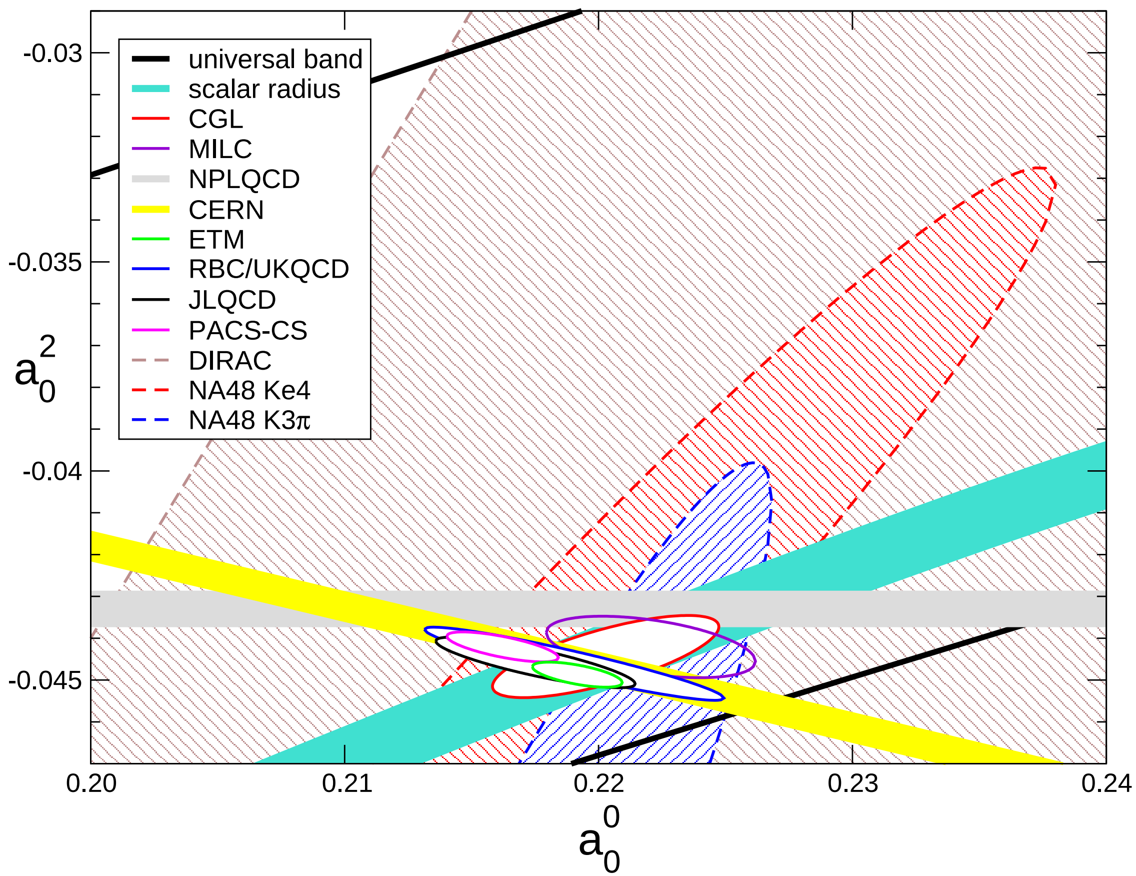
Predictions for the S-wave $\pi\pi$ scattering lengths



Sizable corrections in a_0^0, while a_0^2 nearly stays put

Consequence of lattice results for ℓ_3, ℓ_4

- Uncertainty in prediction for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4
- Can make use of the lattice results for these



Experiments on light flavours at low energy

- Production experiments $\pi N \longrightarrow \pi\pi N$, $\psi \longrightarrow \pi\pi\omega \ldots$

  Problem: pions are not produced in vacuo

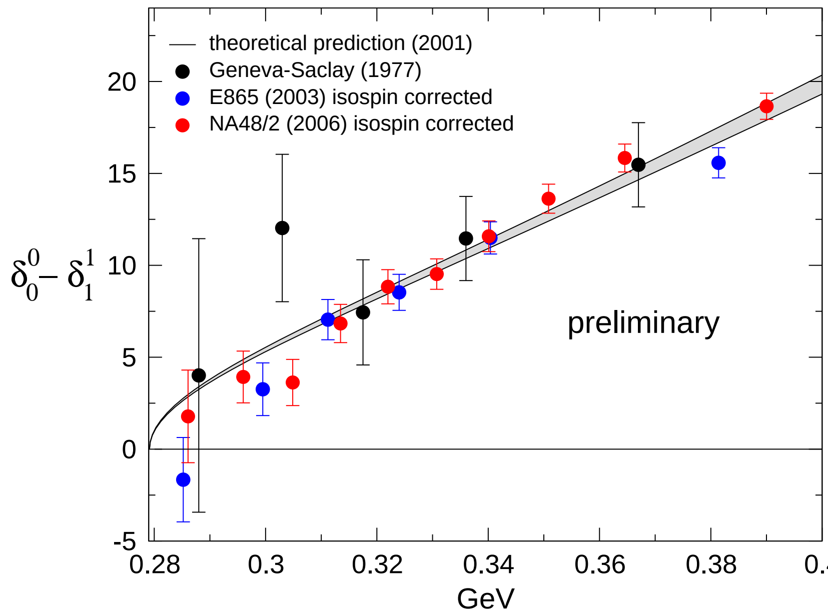
  $\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple

  Accuracy rather limited

- $K^\pm \longrightarrow \pi^+\pi^- e^\pm \nu$ data: CERN-Saclay, E865, NA48/2
- $K^\pm \longrightarrow \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $\pi^+\pi^-$ atoms, DIRAC

K_{e4} decay

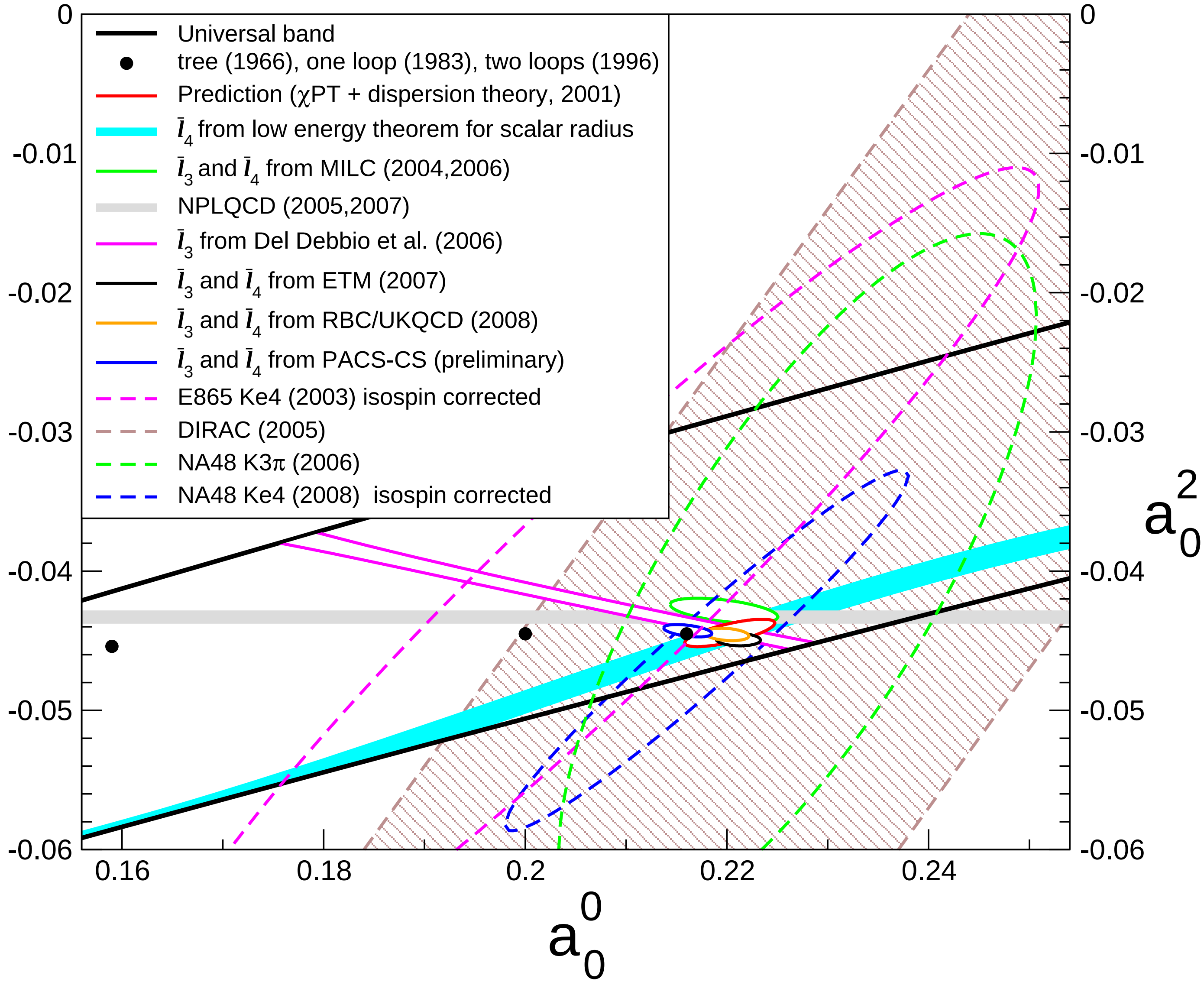
- $K \to \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007

a_0^0, a_0^2 : prediction, lattice & experiment



Many other topics encountered during the past 50 years

b- & t-quarks, heavy quark effective theory
anomalies
summing up perturbative QCD, Regge behaviour
soft collinear effective theories
large N_c, Zweig rule
phase structure of QCD
neutrini
GUT
SUSY
SUGRA
unparticles
dark matter
dark energy, is there a cosmological constant, after all ?
quantum theory of gravity
strings
extra dimensions, membranes
anthropic principle
⋮