Particle Physics: Low Energy, High Accuracy

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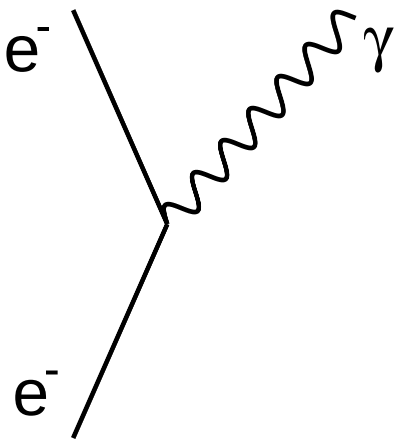
Qualitative aspects of the Standard Model

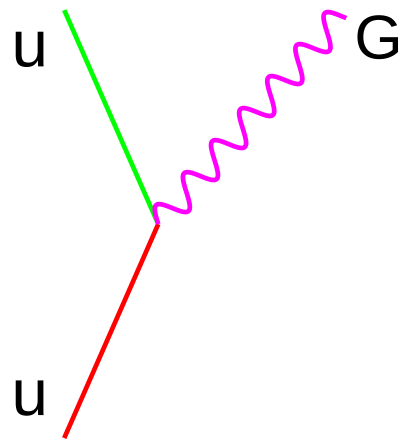
The Standard Model is a miracle:

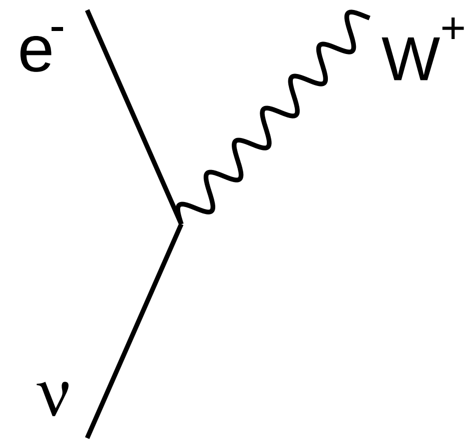
- Since a long time, we know that the microscopic world is governed by three types of interaction:

  strong, electromagnetic, weak
- These have qualitatively very different properties
- Nevertheless, they are all generated by gauge fields !

Gauge field interactions







electromagnetic	strong	weak
QED	QCD	QFD
charge	colour	flavour
photon	gluons	$W^{\pm}, Z$

Behaviour at short distance

- At short distances (1 TeV $\leftrightarrow 2 \cdot 10^{-19}$ m)
  all of the forces obey the inverse square law

  $$V = \text{constant} \times \frac{\hbar c}{r} \qquad \text{interaction energy}$$

- The constant is a pure number

$\Rightarrow$ Interaction strength is fixed by 3 pure numbers

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e.m.	strong	weak
$\frac{e^2}{4\pi}$	$\frac{g_{\text{s}}^2}{4\pi}$	$\frac{g_{\text{w}}^2}{4\pi}$

- Possibly, the strength of the three interactions even becomes the same at $r \sim 10^{-30}$ m (GUT)

Why are the three interactions so different ?

- strong $\simeq$ weak ??
- $\frac{1}{r}$ – law describes an interaction of long range
  strong and weak interactions are short range !
- Photons can be seen by eye, gluons not

etc. etc. etc.

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Two features are responsible for the difference :

- Properties of the vacuum
- Photons do not carry charge, but gluons carry colour

Standard Model

vacuum = condensate of Higgs particles

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⇒ Photons do not notice these

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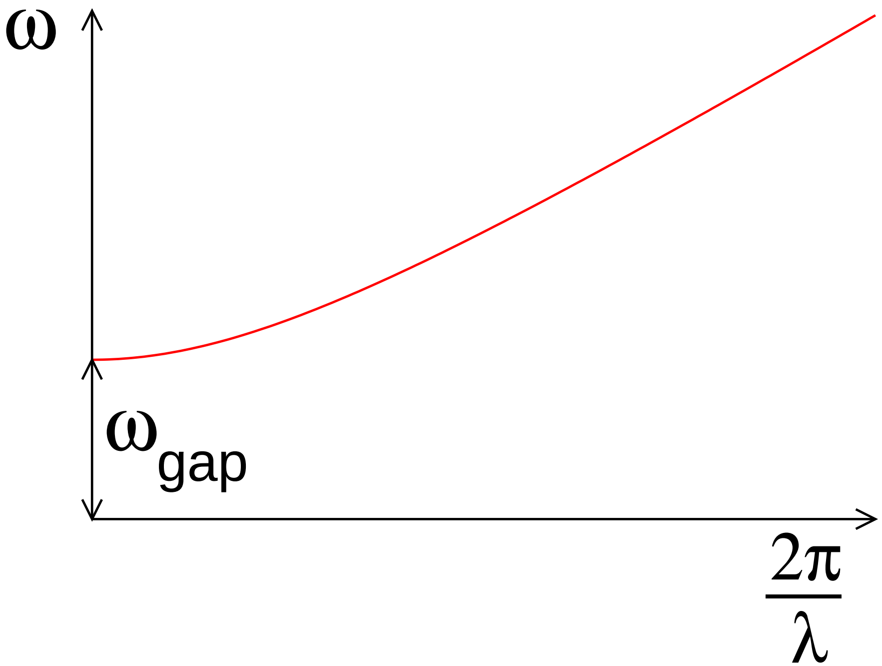
- The Higgs particles do carry flavour

$\Rightarrow$ W,Z do take notice

$\Rightarrow$ W,Z-waves of low frequency cannot propagate

$\Rightarrow$ For such waves, the vacuum is opaque

Frequency versus wave number



$$\hbar\omega_{\text{gap}} = E_{\text{gap}} = Mc^2$$

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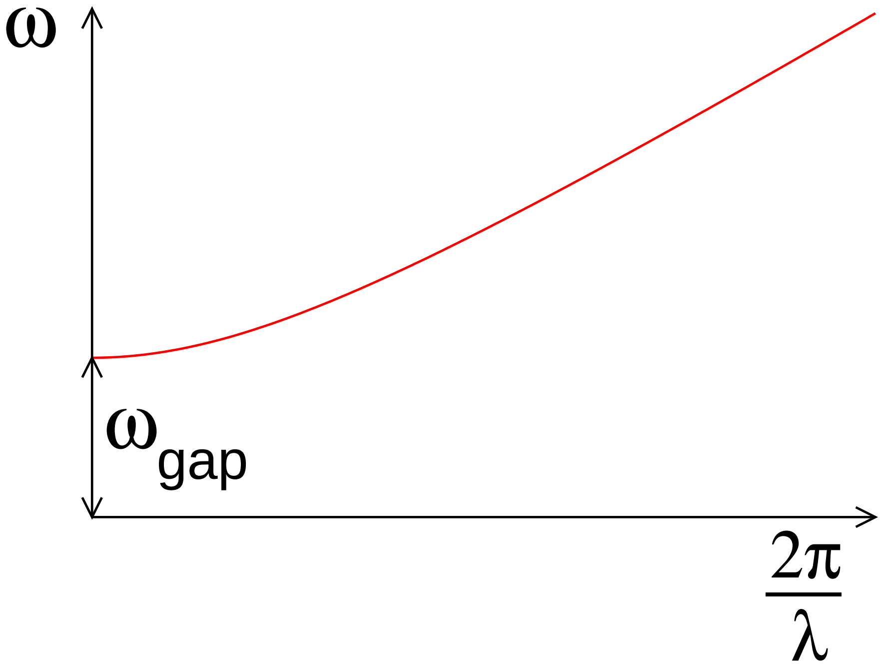

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- $M_\gamma, M_{\text{G}} = 0 \Rightarrow$ γ, G have $v = c$
- $M_{\text{W}}, M_{\text{Z}} \neq 0 \Rightarrow$ W, Z have $v < c$

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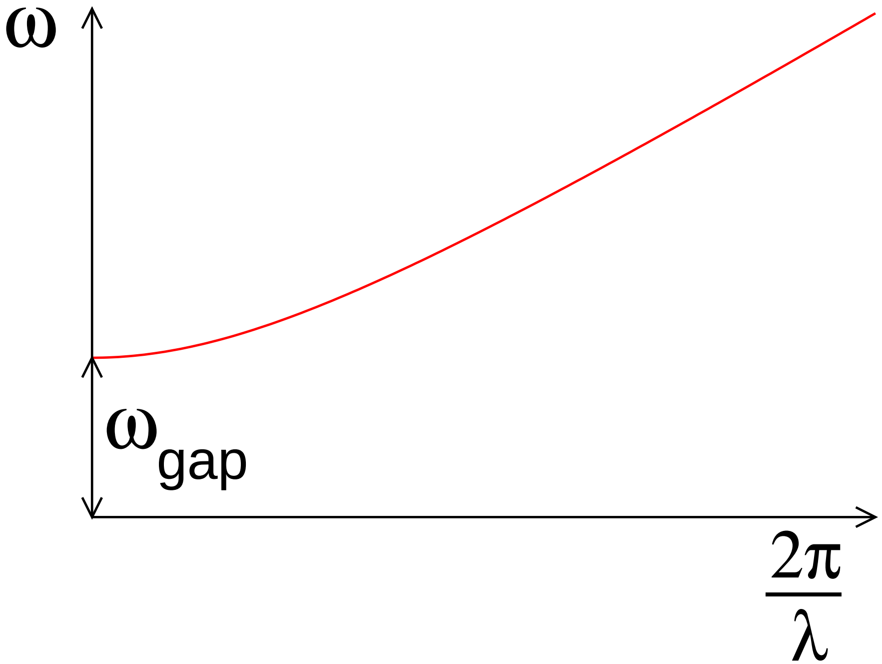

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- $M_{\text{W}}, M_{\text{Z}} \neq 0 \;\Rightarrow\;$ W, Z have $v < c$
- Penetration depth for small frequencies:

$$d = \frac{\hbar}{Mc} \qquad d_{\text{W}}, d_{\text{Z}} \sim 2 \cdot 10^{-18}\,\text{m}$$

Consequence for strength of weak interaction

- Interaction energy is reduced for $r \gtrsim d$:

$$\frac{g_{\mathrm{W}}^2}{4\pi r} \Rightarrow \frac{g_{\mathrm{W}}^2}{4\pi r} \cdot e^{-\frac{r}{d}}$$

Penetration depth of the weak interaction is small:

$$d = \frac{\hbar}{M_{\mathrm{W}} c} = 2.4542(9) \times 10^{-18}\ \mathrm{m}$$

$\Rightarrow$ Weak interaction is of short range

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- Effective strength at low energies:

$$\int d^3r \, \frac{g_{\text{W}}^2}{4\pi r} \cdot e^{-\frac{r}{d}} \;=\; g_{\text{W}}^2 d^2$$

⇒ At low energies, the weak interaction is weak

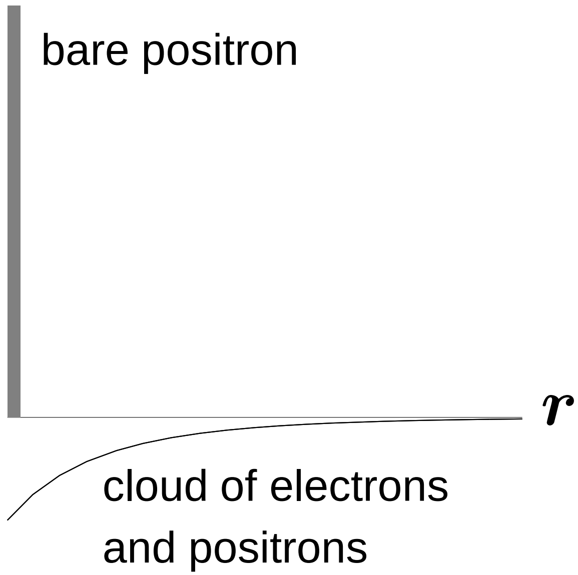
What makes the difference between QED and QCD ?

- Photons do not have charge
- Gluons do have colour

$\Rightarrow$ The e.m. and strong interactions behave differently at low energies

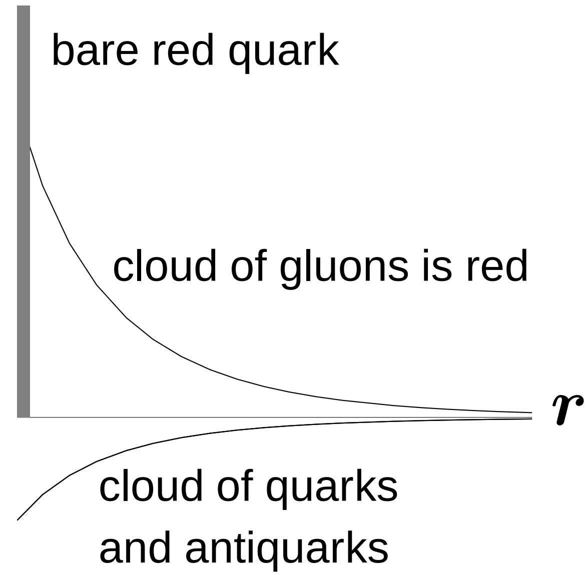
Compare structure of leptons and quarks

QED	QCD
density of charge	density of colour



$$e < e|_{\text{bare}}$$

vacuum shields charge



$$g_{\text{s}} > g_{\text{s}}|_{\text{bare}}$$

vacuum amplifies colour

⇒ The electromagnetic and strong interactions polarize the vacuum very differently

Consequence of shielding/amplification

- Vacuum amplifies gluonic field of a quark
  Vacuum shields electric field of a lepton
- The difference has dramatic consequences:
  although the Lagrangians of QCD and QED
  are very similar, the properties of the strong
  and electromagnetic interactions are totally different

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  are very similar, the properties of the strong
  and electromagnetic interactions are totally different
- Field energy surrounding isolated quark = ∞
  only colour neutral states have finite energy

$\Rightarrow$ colour is confined, "infrared slavery"

- Field energy surrounding a charged particle is finite

$\Rightarrow$ charge is not confined

- nuclear forces = van der Waals forces of QCD

Interaction at large distances, low energies

QED remains weak

$$\frac{e^2}{4\pi} \simeq \frac{1}{137}$$

photons, leptons
nearly decouple

QCD becomes strong

$$\frac{g_s^2}{4\pi} \simeq 1$$

gluons, quarks
confined

$\Rightarrow$ In QED, perturbation theory works at low energies
Spectrum of states can be seen in the Lagrangian

$\Rightarrow$ In QCD, perturbation theory fails at low energies
Spectrum of states cannot be seen in the Lagrangian
Fields in the Lagrangian $\not\Leftrightarrow$ observed particles

This is why it took so long to realize that the strong interactions originate in a gauge field
O. Klein (1938), W. Pauli (1953) had studied nonabelian gauge fields, but dropped the idea because the Fock space contains massless gluons

Summary: Standard Model at low energies

- Vacuum is opaque for $W^{\pm}$, Z

$\Rightarrow$ For $E \ll M_{\mathrm{W}}c^2,\ M_{\mathrm{Z}}c^2$, the weak interaction is frozen

SM reduces to QED + QCD

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- QED is infrared stable, characterized by pure number

The number happens to be small: $\dfrac{e^2}{4\pi} \simeq \dfrac{1}{137}$

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- At low energies, the weak and e.m. interactions can be treated perturbatively

Effects from g_{w} are tiny, those from e are small

$\Rightarrow$ At low energies: SM = QCD + corrections

For e.m. bound states (atoms, solids, . . .): QED needs to be summed up

Pièce de résistance: QCD

- Interaction fully determined by group geometry
Lagrangian contains 2 coupling constants:

$$\boxed{g_{\rm s},\ \theta}$$

- Quark mass matrix, can be brought
to diagonal form, eigenvalues real, positive

$$\boxed{m_{\rm u},\ m_{\rm d},\ m_{\rm s},\ m_{\rm c},\ m_{\rm b},\ m_{\rm t}}$$

- Pattern of quark masses is bizarre, not understood
- High energy side looks like what we are used to:
relevant degrees of freedom are visible in the
Lagrangian, can treat the interaction as a perturbation

QCD at low energies

- QCD gives rise to a rich structure at low energies
- Low energies are out of reach of perturbation theory

$\Rightarrow$ Not a simple matter to work out the consequences of the Standard Model at low energies

- $\exists$ many models that resemble QCD: instantons, monopoles, bags, superconductivity, gluonic strings, linear σ model, hidden gauge, NJL, AdS/CFT, but ...
- Nonperturbative methods needed

$\Rightarrow$ Progress in understanding is slow

- Two model independent methods:
  - Effective field theory, χPT
  - Numerical simulation on a lattice

Low energy expansion

- Cannot treat g_s as small
- If the spectrum has an energy gap

$\Rightarrow$ No singularities in scattering amplitudes or Green functions near $p = 0$

$\Rightarrow$ Low energy behaviour may be analyzed with Taylor series expansion in powers of momentum

$$f(t) = 1 + \frac{1}{6}\langle r^2\rangle\, t + \ldots \quad \text{form factor}$$

$$T(p) = a + b\, p^2 + \ldots \quad \text{scattering amplitude}$$

Cross section dominated by S–wave scattering length

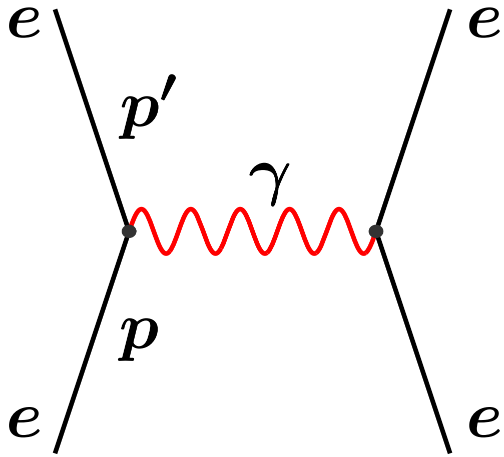
$$\frac{d\sigma}{d\Omega} \simeq |a|^2$$

Energy gap plays crucial role

- Energy gap: difference between the energies of ground state and first excited state
- Particle physics: (mass of the lightest particle)$\times c^2$
- Expansion parameter: $\dfrac{p}{m} = \dfrac{\text{momentum}}{\text{energy gap}} \qquad (c = 1)$
- Taylor series only works if the spectrum has an energy gap, i.e. if there are

  no massless particles

Illustration: Coulomb scattering



- Photon exchange $\Rightarrow$ pole at $t = 0$

$$T = \frac{e^2}{(p' - p)^2}$$

Scattering amplitude does not admit Taylor series expansion in powers of p

Energy gap in QCD

- QCD does have an energy gap, but the gap is very small: $M_\pi \simeq 140$ MeV

$\Rightarrow$ Taylor series has very small radius of convergence, useful only for $p < M_\pi$

- Why is M_π so small ?

- In 1960, Nambu found out why that is so Nobel Prize 2008
  - Has to do with a hidden approximate symmetry
  - Hamiltonian is approximately symmetric, but the state of lowest energy is not

  $\Rightarrow$ Symmetry is "hidden", "spontaneously broken”
  - Nambu realized that the spontaneous breakdown of an approximate symmetry entails approximately massless particles, concluded that in the case of the strong interaction, the pions must play this role

$$SU(2)_L \times SU(2)_R$$

Isospin

- Where is Nambu's approximate symmetry in QCD ?

m_u and m_d happen to be very small

  why ???

- Interactions of u and d are identical. If the masses are set equal, there is no difference at all: QCD Lagrangian becomes symmetric under $u \leftrightarrow d$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$q' = V \cdot q \qquad V \in \mathrm{SU(2)} \qquad \text{isospin symmetry}$$

V acts on quark flavour, mixes u and d

Chiral symmetry

- Massless fermions: right and left do not communicate

⇒ For $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$, the Lagrangian becomes invariant under independent rotations of q_{R} and q_{L}

$$q_{\mathrm{R}} = \tfrac{1}{2}(1+\gamma_5)\, q \;, \qquad q_{\mathrm{L}} = \tfrac{1}{2}(1-\gamma_5)\, q$$

$$q'_{\mathrm{R}} = V_{\mathrm{R}} \cdot q_{\mathrm{R}} \qquad\qquad q'_{\mathrm{L}} = V_{\mathrm{L}} \cdot q_{\mathrm{L}}$$

$$G = \mathrm{SU(2)}_{\mathrm{R}} \times \mathrm{SU(2)}_{\mathrm{L}}$$

- 3 vector charges (R+L), 3 axial charges (R-L)
  Strictly conserved, commute with Hamiltonian

$$[\vec{Q}_{\mathrm{V}}, H_0] = 0 \qquad [\vec{Q}_{\mathrm{A}}, H_0] = 0$$

- QCD with two massless quarks has an exact chiral symmetry

Chiral symmetry is hidden

- For dynamical reasons, the lowest eigenstate of H_0 is not symmetric under chiral rotations Nambu 1960

$$\vec{Q}_{\mathrm{A}}\,|0\rangle \neq 0$$

$\Rightarrow$ Chiral symmetry is "hidden", "spontaneously broken"
Compare spontaneous magnetization

- The spontaneous breakdown of a continuous symmetry entails massless particles: "Goldstone bosons"

Goldstone, Salam, Weinberg 1962

- Carry the quantum numbers of the states $\vec{Q}_{\mathrm{A}}\,|0\rangle$:

spin zero, negative parity, isospin triplet: $\pi^+,\ \pi^0,\ \pi^-$

Effective theory

- For $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$, pion exchange gives rise to poles and branch points at $p = 0$

$\Rightarrow$ Low energy expansion is not a Taylor series, contains infrared singularities

- Properties of the Goldstone bosons are governed by the hidden symmetry that is responsible for their occurrence

$\Rightarrow$ Goldstone bosons of low momentum interact only weakly: can treat the momenta as well as $m_{\mathrm{u}}, m_{\mathrm{d}}$ as perturbations

Chiral Perturbation Theory

- Formulation in terms of an effective Lagrangian

  Weinberg 1967, Coleman, Wess, Zumino, Callan, Dashen, Weinstein 1969

- Lagrangian $\supset$ massless Goldstone Bosons

$\Rightarrow$ Perturbation series has infrared singularities

Li + Pagels 1971, Langacker + Pagels 1973

Weinberg 1979, Gasser + Zepeda 1980, Gasser 1981

> Singularities due to Goldstone bosons can be worked out with an effective field theory "Chiral Perturbation Theory"

- χPT reproduces the low energy structure of QCD

Meson field theory

- χPT originally formulated as a meson field theory
  - $\langle 0|\, T\pi^i(x)\pi^k(y)\, |0\rangle$ plays central role
  - Depends on choice of variables, but the result for meson masses, S-matrix is unambiguous
  - Studying the Green functions of the pion field amounts to perturbing the system with

    $\mathcal{L}_{\text{eff}} \longrightarrow \mathcal{L}_{\text{eff}} + \vec{f}(x) \cdot \vec{\pi}(x)$

    $\vec{\pi}(x)$ transforms in nonlinear manner

$\Rightarrow$ Ruins the symmetry of the effective Lagrangian

Effective theory for QCD Green functions

- Further shortcoming of original framework: current matrix elements ? Noether currents of $\mathcal{L}_{\text{eff}}$ are correct only at leading order, F_π at NLO ?
- $\vec{\pi}(x) \notin$ QCD, but $\vec{V}^\mu(x),\ \vec{A}^\mu(x), \ldots \in$ QCD
  $\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{QCD}} + \vec{f}(x)\cdot\vec{\pi}(x)$?
  $\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{QCD}} + \vec{f}_\mu(x)\cdot\vec{V}^\mu(x) + \ldots$ $\surd$
- Need effective theory for Green functions of QCD
  $\mathcal{L}_{\text{eff}} \longrightarrow \mathcal{L}_{\text{eff}} + \vec{f}_\mu(x)\cdot\vec{V}^\mu_{\text{eff}}(x) + \ldots$
- Symmetry in terms of Green functions:
  symmetry $\Rightarrow$ current conservation $\Rightarrow$ Ward identities
  WI remain exact even for $m_{\text{u}}, m_{\text{d}} \neq 0$
  anomalies show up in WI, not in QCD Lagrangian

Gasser + L. 1984, 1985

Plethora of effective coupling constants

- χPT merely exploits the symmetries of QCD:
  yields the general solution of the Ward identities
- $\mathcal{L}_{\mathit{eff}}$ contains all functions that can be formed with the pion field and its derivatives, only subject to the condition that the sum is chirally invariant
- Order in number of derivatives (powers of momentum)

$\Rightarrow$ Number of terms in $\mathcal{L}_{\mathit{eff}}$ rapidly grows with the order:
LO: 2, NLO: 7, NNLO: 53, ...

- Symmetries only relate – do not determine
- In principle, the effective theory is exact:
  yields expansion of QCD Green functions in $\boldsymbol{p, m_q}$

Illustration: energy gap of QCD

- Energy gap of QCD: M_π
- Ignore e.m. self energy, $e = 0$, pure QCD

$\Rightarrow$ M_π is a function of $\Lambda_{\text{QCD}}, m_{\text{u}}, m_{\text{d}}, \ldots, m_{\text{t}}$

- How does M_π depend on $m_{\text{u}}, m_{\text{d}}$?

  Chiral symmetry: $M_\pi \longrightarrow 0$ for $m_{\text{u}}, m_{\text{d}} \longrightarrow 0$

- Leading order formula (tree level of χPT):

$$M_\pi^2 = (m_{\text{u}} + m_{\text{d}})B$$

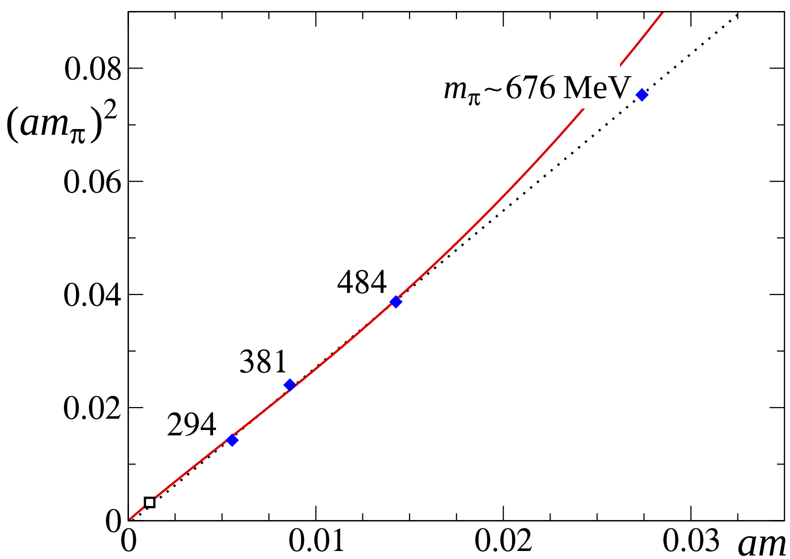
Gell-Mann, Oakes, Renner 1968

- The coefficient is determined by the quark condensate:

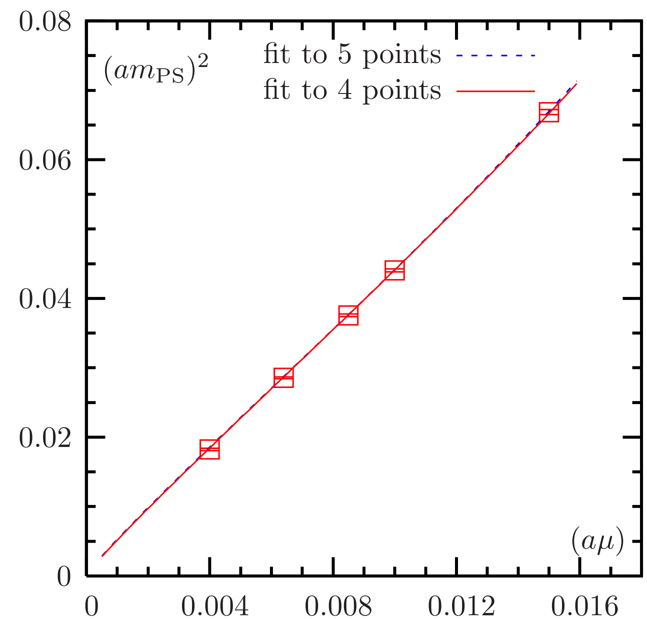
$$B = \frac{|\langle 0|\, \bar{u}u \,|0\rangle|}{F_\pi^2}$$

Lattice results for M_π

- GMOR formula can now be checked on the lattice: determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- The plots shown concern QCD with $N_f = 2$

Lattice

- Quality of data is impressive
- No quenching, quark masses are sufficiently light

$\Rightarrow$ legitimate to use χPT for the extrapolation to the physical values of $m_{\rm u}, m_{\rm d}$

- Proportionality of M_π^2 to

  $$m_{\rm ud} \equiv \frac{1}{2}(m_{\rm u} + m_{\rm d})$$

  holds out to $m_{\rm ud} \simeq 10 \times m_{\rm ud}^{\rm phys}$
- Main limitation: systematic uncertainties from lattice artifacts, continuum extrapolation, finite size effects, etc.

Expansion of M_π^2 in powers of m_u, m_d

- GMOR formula represents leading term of χPT
- Correction of first nonleading order:

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{32\pi^2 F_\pi^2}\,\bar{\ell}_3 + O(M^4) \right\}$$

$$M^2 \equiv B(m_u + m_d)$$

$\ell_3 \in \mathcal{L}_{eff}$ depends logarithmically on running scale

- What counts is the running coupling at scale M_π:

$$\bar{\ell}_3 = \ell n \frac{\Lambda_3^2}{M_\pi^2}$$

$\Rightarrow$ Expansion of M_π contains a chiral logarithm

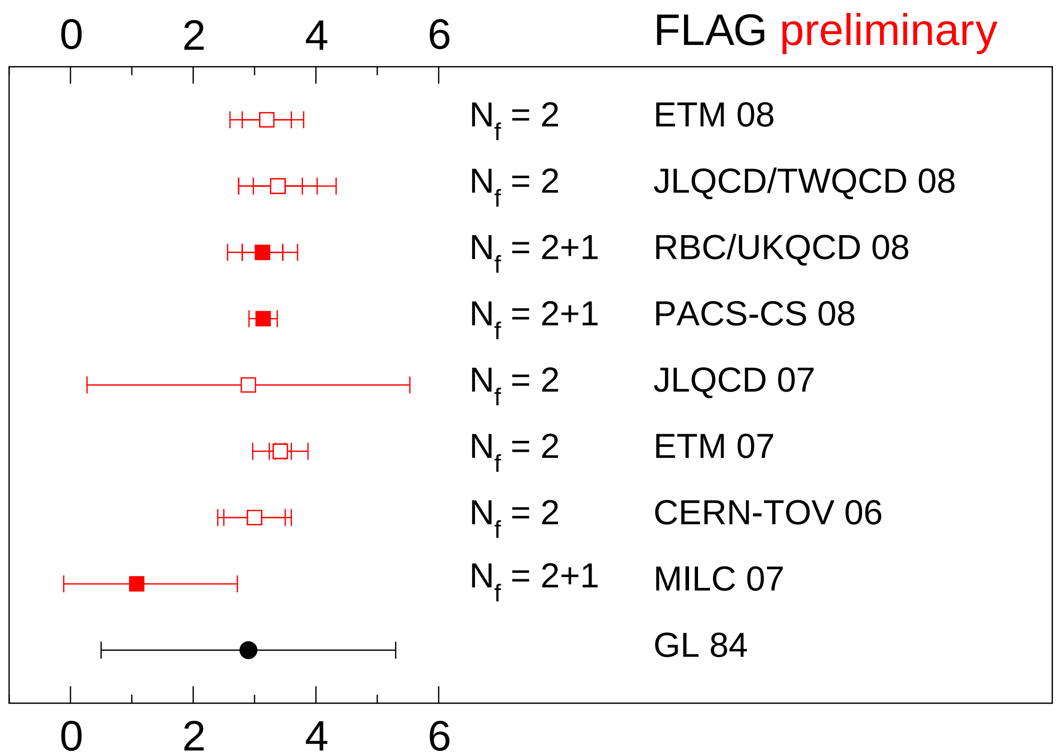
Langacker + Pagels 1973, Gasser + Zepeda 1980, Gasser 1981

Chiral logarithm in M_π

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{32\pi^2 F_\pi^2}\, \ell n \frac{\Lambda_3^2}{M^2} + O(M^4) \right\}$$

- Coefficient is determined by pion decay constant
  Symmetry does not determine the scale Λ_3
- Crude result, based on SU(3)×SU(3):
  $0.2\,\text{GeV} \lesssim \Lambda_3 \lesssim 2\,\text{GeV}$ Gasser + L. 1984

Lattice allows more accurate determination of Λ_3



Horizontal axis shows the value of $\bar{\ell}_3 \equiv \ell n \frac{\Lambda_3^2}{M_\pi^2}$

Range for Λ_3 obtained in 1984 corresponds to $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of RBC/UKQCD 08, for instance, is $\bar{\ell}_3 = 3.13 \underset{stat}{\pm 0.33} \underset{syst}{\pm 0.24}$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

$$F_\pi = F\left\{1+\frac{M^2}{16\pi^2F^2}\,\ell n\,\frac{\Lambda_4^2}{M^2}+O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1-\frac{M^2}{32\pi^2F^2}\,\ell n\,\frac{\Lambda_3^2}{M^2}+O(M^4)\right\}$$

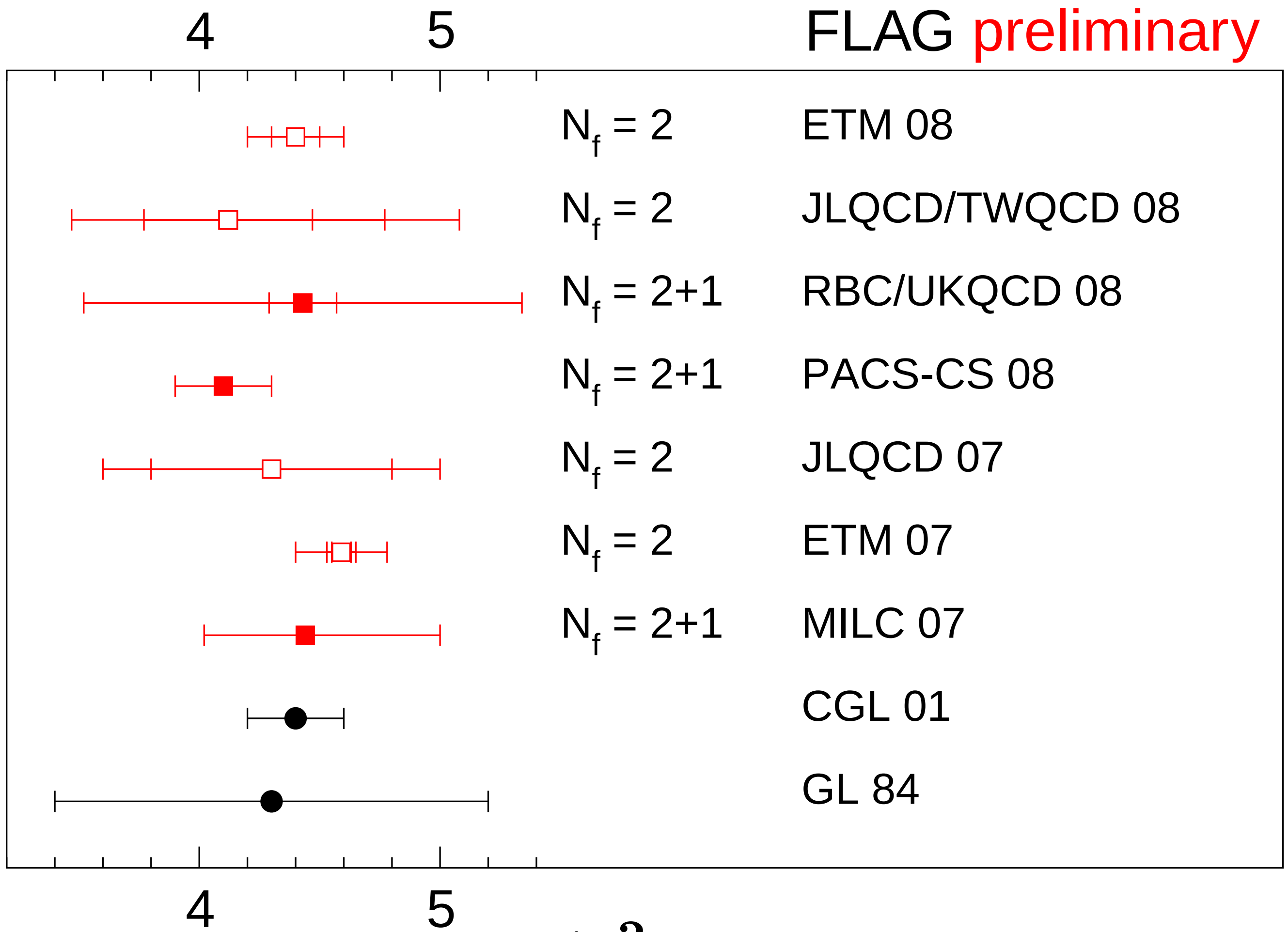
  F is value of pion decay constant in limit $m_\mathrm{u}, m_\mathrm{d} \rightarrow 0$

- Structure is the same, coefficients and scale of logarithm are different

- Quark mass dependence of F_π can also be measured on the lattice $\Rightarrow$ measurement of Λ_4

- Alternative method: determine the scalar form factor of the pion, radius $\langle r^2\rangle_s \Leftrightarrow \bar{\ell}_4 = \ell n(\Lambda_4^2/M_\pi^2)$

Lattice results for Λ_4



$$\bar{\ell}_4 = \ell n \frac{\Lambda_4^2}{M_\pi^2}$$

Lattice determination of scalar radius

- Scalar form factor can be measured on the lattice
- Most recent lattice determination leads to

$$\langle r^2\rangle_s = 0.617 \pm 0.079_{\text{stat}} \pm 0.066_{\text{syst}}\ \text{fm}^2$$

JLQCD/TWQCD collaboration, arXiv:0905.2465

To be compared with the dispersive result:

$$\langle r^2\rangle_s = 0.61 \pm 0.04\ \text{fm}^2$$

CGL 2001

$\Rightarrow$ Central values agree, but the errors are still large

$\pi\pi$ interaction

- Symmetry fixes the interaction among the GB
- LO formulae for the S-wave scattering lengths:

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\,, \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}$$

Weinberg 1966

- NLO corrections are determined by ℓ_3, ℓ_4 Gasser + L. 1983
- $\pi\pi$ scattering amplitude now known to NNLO

Bijnens, Colangelo, Ecker, Gasser + Sainio 1996

Dispersion relations

- Chiral perturbation series for $\pi\pi$ scattering amplitude: uncertainties rapidly grow with energy

$\Rightarrow$ Dispersion theory needed to reach the physical region

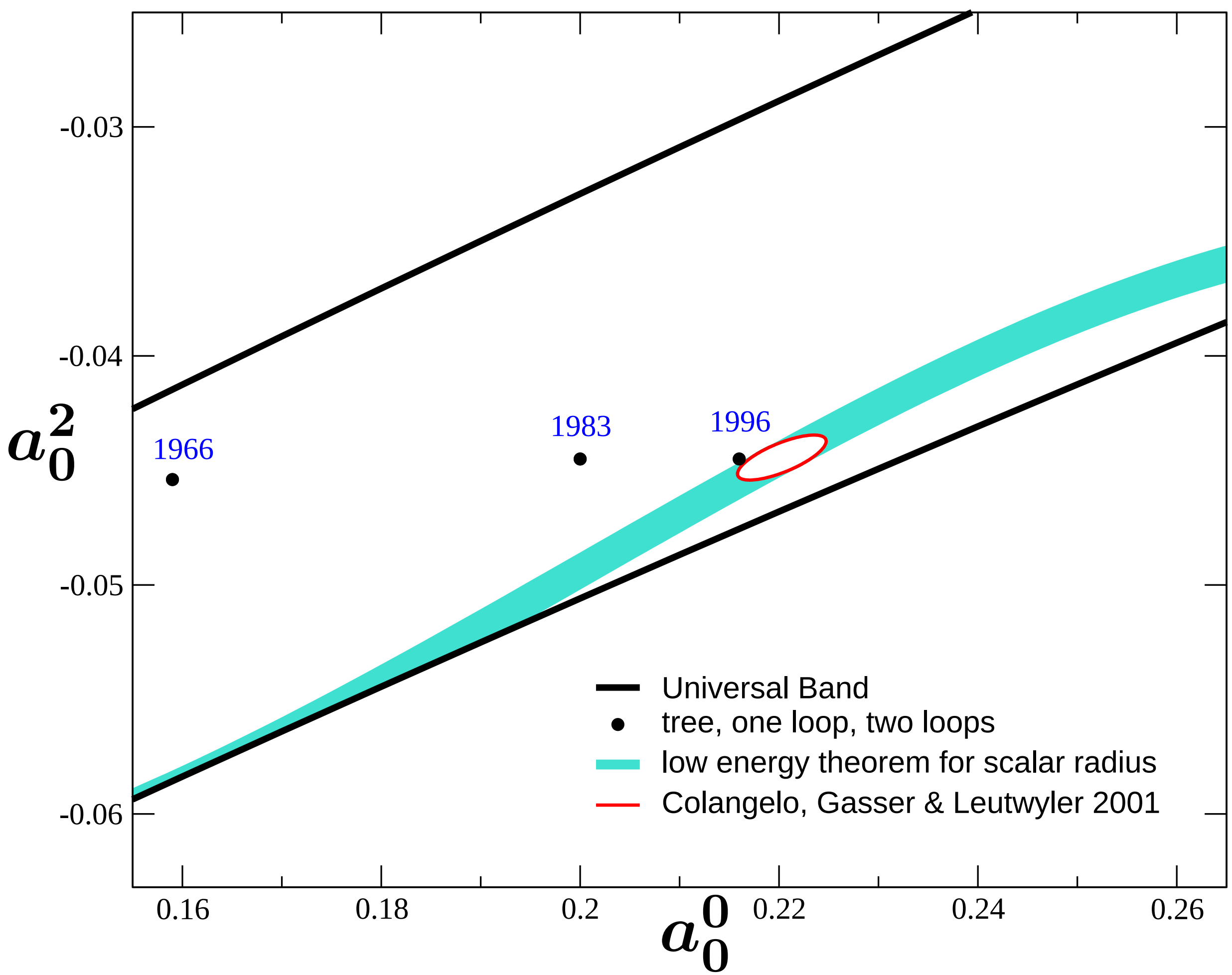
- Dispersive representation of the partial waves: Roy equations

S.M. Roy 1971

- Contain two subtraction constants: a_0^0, a_0^2
- Most accurate results for a_0^0, a_0^2 are obtained by matching the chiral and dispersive representations in the unphysical region below threshold

Colangelo, Gasser + L. 2001

Predictions for the S-wave $\pi\pi$ scattering lengths



Sizable corrections in a_0^0, while a_0^2 nearly stays put

Critical reanalysis of the predictions for a_0^0, a_0^2

- At NNLO, the predictions for a_0^0, a_0^2 involve coupling constants entering the effective Lagrangian at $O(p^6)$
- In 2001, we assumed that the estimates for these quoted in the literature are valid within a factor of two

  Bijnens, Colangelo, Ecker, Gasser, Hannah, Sainio, Talavera 1996-98

- Guo + Sanz-Cillero: thorough investigation of the contributions from the couplings of $O(p^6)$ arXiv:0904.4178

  $a_0^0 = 0.220(5)$ confirmed, $a_0^2 = -0.0444(10)$ replaced by $a_0^2 = -0.0444(11)$

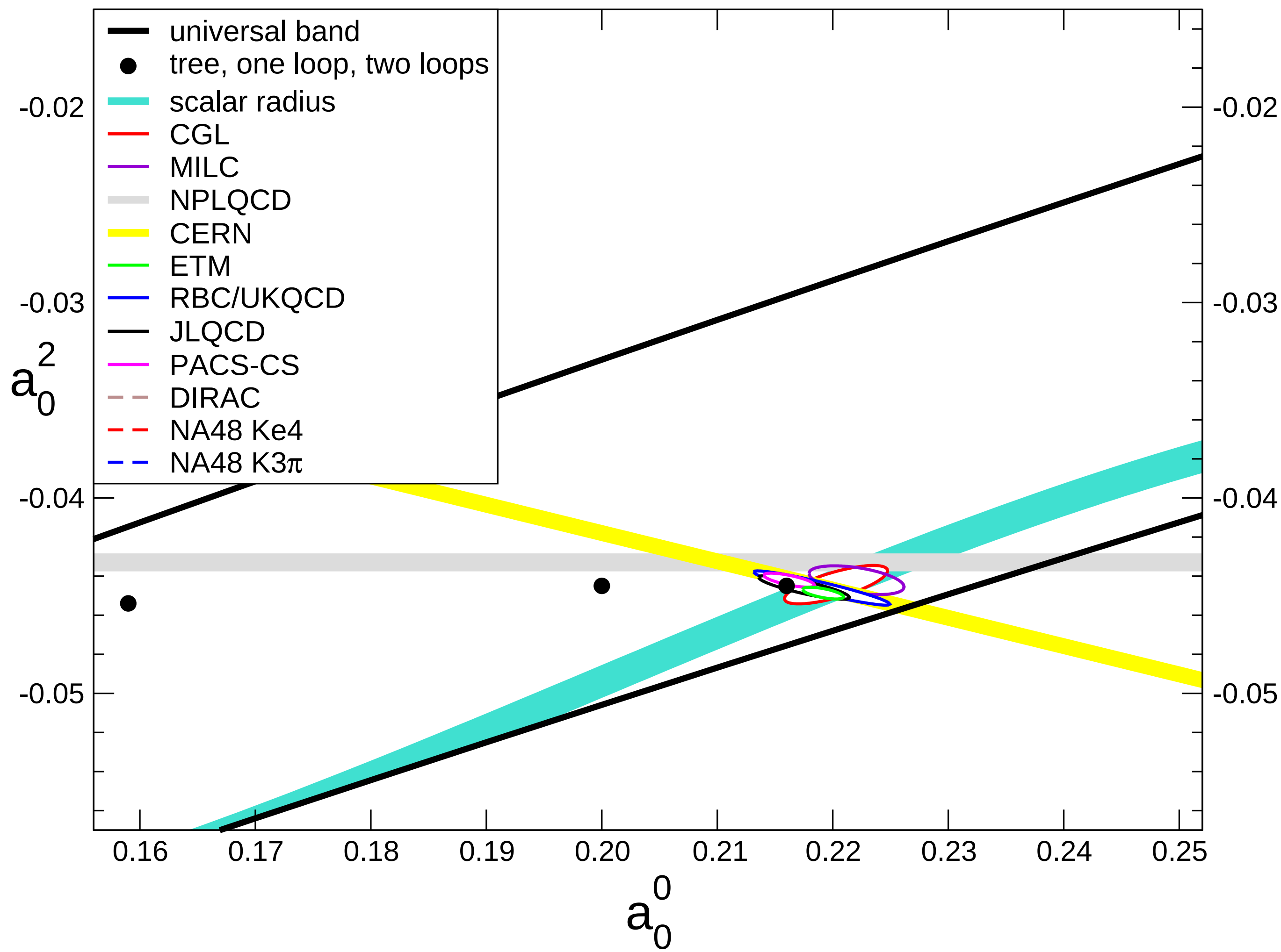
$\Rightarrow$ At the accuracy reached until now, the predictions for a_0^0, a_0^2 are not sensitive to the couplings of $O(p^6)$

- Main source of uncertainty: values of ℓ_3, ℓ_4

Consequence of lattice results for ℓ_3, ℓ_4

- Uncertainty in predictions for a_0^0, a_0^2 is dominated by the uncertainty in the effective coupling constants ℓ_3, ℓ_4

$\Rightarrow$ Can make use of the lattice results for these



Experiments on light flavours at low energy

- Production experiments $\pi N \longrightarrow \pi\pi N$, $\psi \longrightarrow \pi\pi\omega \ldots$

  Problem: pions are not produced in vacuo

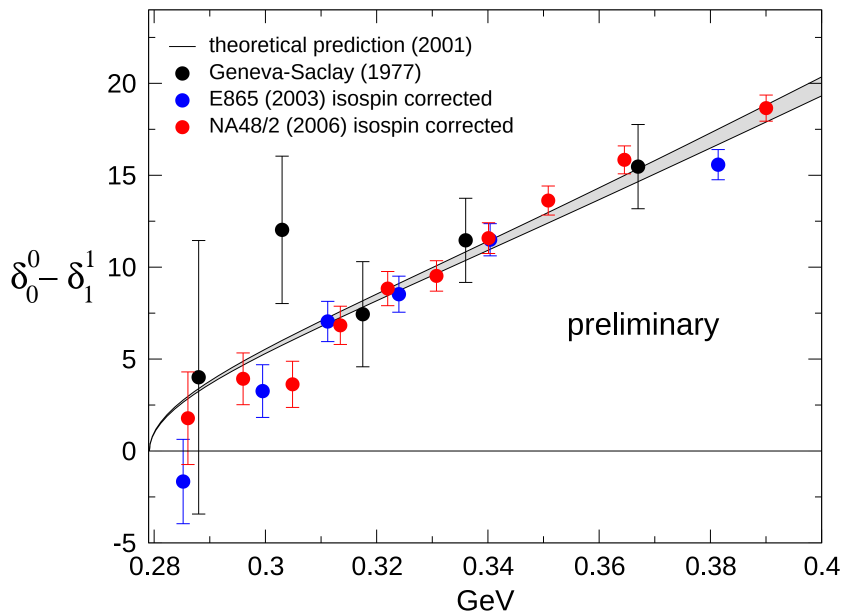
$\Rightarrow$ Extraction of $\pi\pi$ scattering amplitude not simple

  Accuracy rather limited

- $\pi^+\pi^-$ atoms, DIRAC
- $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ cusp near threshold: NA48/2
- $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$ precision data from E865, NA48/2

K_{e4} decay

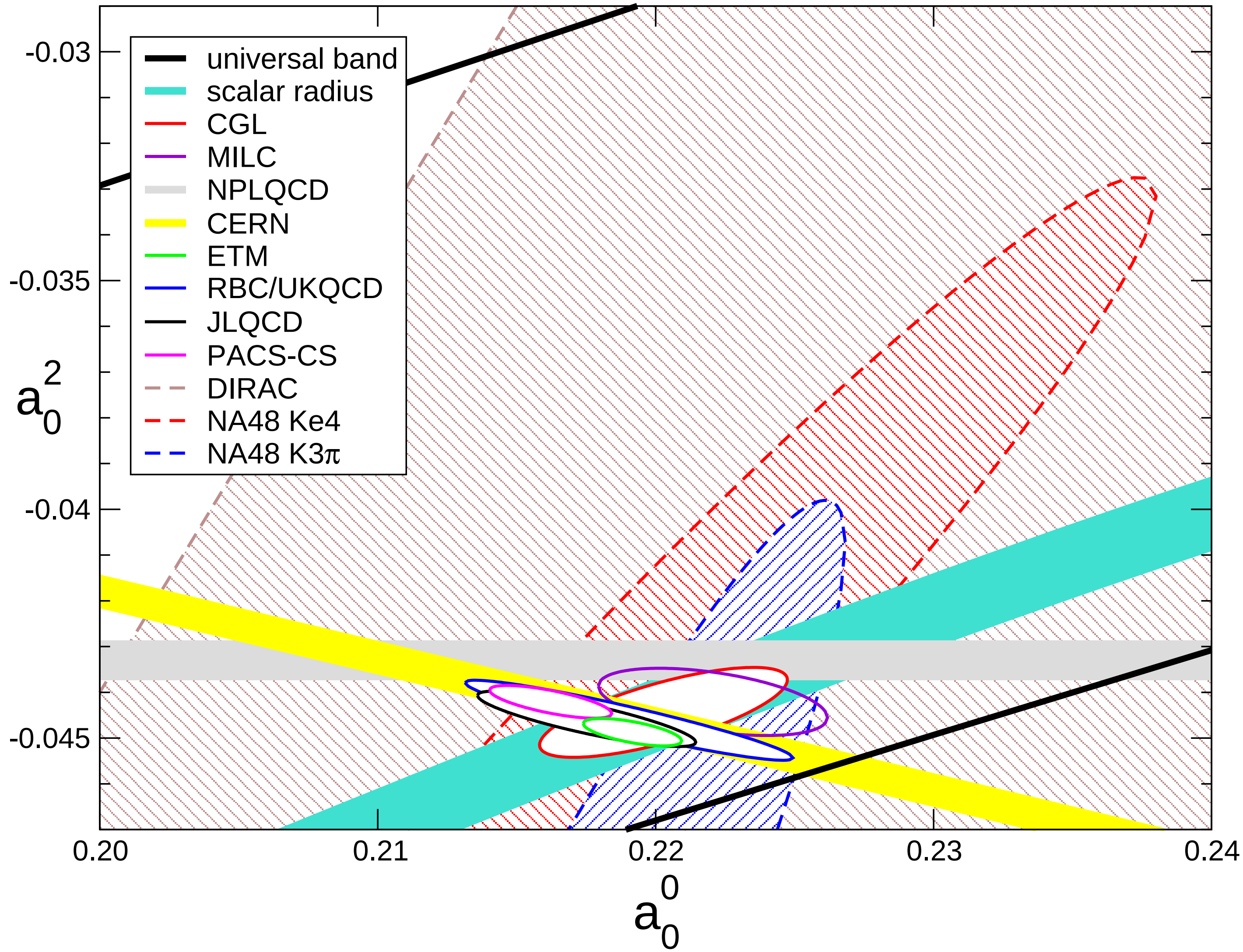
- $K \to \pi\pi e\nu$ allows clean measurement of $\delta_0^0 - \delta_1^1$
- Theory predicts $\delta_0^0 - \delta_1^1$ as function of energy



- There was a discrepancy here, because a pronounced isospin breaking effect from $K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ had not been accounted for in the data analysis

Colangelo, Gasser, Rusetsky 2007, Bloch-Devaux 2007

a_0^0, a_0^2 : predictions, lattice + experiment



$$SU(3)_L \times SU(3)_R$$

Extension to SU(3)×SU(3)

- In the theoretical limiting case $m_{\mathrm{u}} = m_{\mathrm{d}} = m_{\mathrm{s}} = 0$ QCD acquires an exact $\mathrm{SU(3)_L \times SU(3)_R}$ symmetry
- Is m_{s} small enough for this to represent a useful approximate symmetry of full QCD ?
- Theoretical reasoning
  - $\mathrm{SU(3)_{L+R}}$ (eightfold way) is an approximate symmetry
  - Typical size of $\mathrm{SU(3)_{L+R}}$ breaking: $\dfrac{F_K}{F_\pi} = 1.19 \pm 0.01$
  - Only coherent way to understand this in QCD:

    The mass differences $m_{\mathrm{s}} - m_{\mathrm{d}}$, $m_{\mathrm{d}} - m_{\mathrm{u}}$ must be small, can be treated as perturbations
  - Since $m_{\mathrm{u}}, m_{\mathrm{d}} \ll m_{\mathrm{s}}$

$\Rightarrow$ m_{s} is small, $\mathrm{SU(3)_L \times SU(3)_R}$ must be an approximate symmetry, breaking not larger than for $\mathrm{SU(3)_{L+R}}$

Light quark masses as perturbations

- Masses of the light quarks enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$

$$H_1 = \int d^3x \, \{m_{\text{u}} \, \bar{u}u + m_{\text{d}} \, \bar{d}d + m_{\text{s}} \, \bar{s}s\}$$

H_0 describes u, d, s as massless, c, b, t as massive

H_0 is invariant under $\text{SU}(3)_L \times \text{SU}(3)_R$

Expansion in powers of $m_{\text{u}}, m_{\text{d}}, m_{\text{s}}$	$\Longleftrightarrow$	Perturbation series in powers of H_1

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Expansion in powers of $m_\text{u}, m_\text{d}, m_\text{s}$	$\Longleftrightarrow$	Perturbation series in powers of H_1

- Ground state of H_0 is invariant only under SU(3)$_{\text{L+R}}$

$\Rightarrow$ Spectrum of H_0 contains 8 Goldstone bosons

$$\pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, \eta$$

- H_1 gives them a mass

Pattern of lowest levels

- $M_\pi^2 = (m_\mathrm{u} + m_\mathrm{d})\, B + O(m^2)$

$\Rightarrow$ The energy gap of QCD is small because $m_\mathrm{u}, m_\mathrm{d}$ happen to be small

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  $M_{K^0}^2 = (m_\mathrm{d} + m_\mathrm{s})\, B + O(m^2)$

$\Rightarrow$ M_K^2 is much larger than M_π^2, because m_s happens to be large compared to $m_\mathrm{u}, m_\mathrm{d}$

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- Goldstone boson masses measure the strength of symmetry breaking $\Rightarrow$ strongly violate $\mathrm{SU(3)}_{\mathrm{L+R}}$

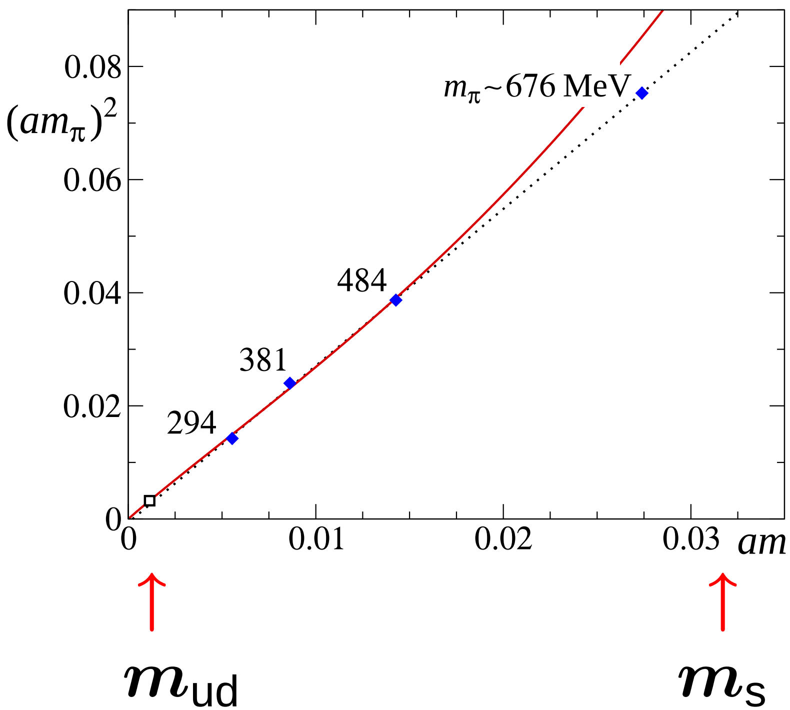
- Check: first order perturbation theory also yields
  $M_\eta^2 = \frac{1}{3}\,(m_\mathrm{u} + m_\mathrm{d} + 4m_\mathrm{s})B + O(m^2)$

$\Rightarrow$ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 $\surd$

Expansion in powers of m_u, m_d, m_s

- Expansion in powers of m_u, m_d, m_s should work, but expect convergence to be comparatively slow
- Recall that $M_\pi^2 \propto m_{ud}$ holds out to $10 \times m_{ud}^{phys}$
- m_s is larger than that: $m_s \simeq 27 \times m_{ud}$



Compare

$$\frac{F_K}{F_\pi} \simeq 1.19$$

Expansion in powers of m_u, m_d, m_s

- Terms of nonleading order in m_s are sizable

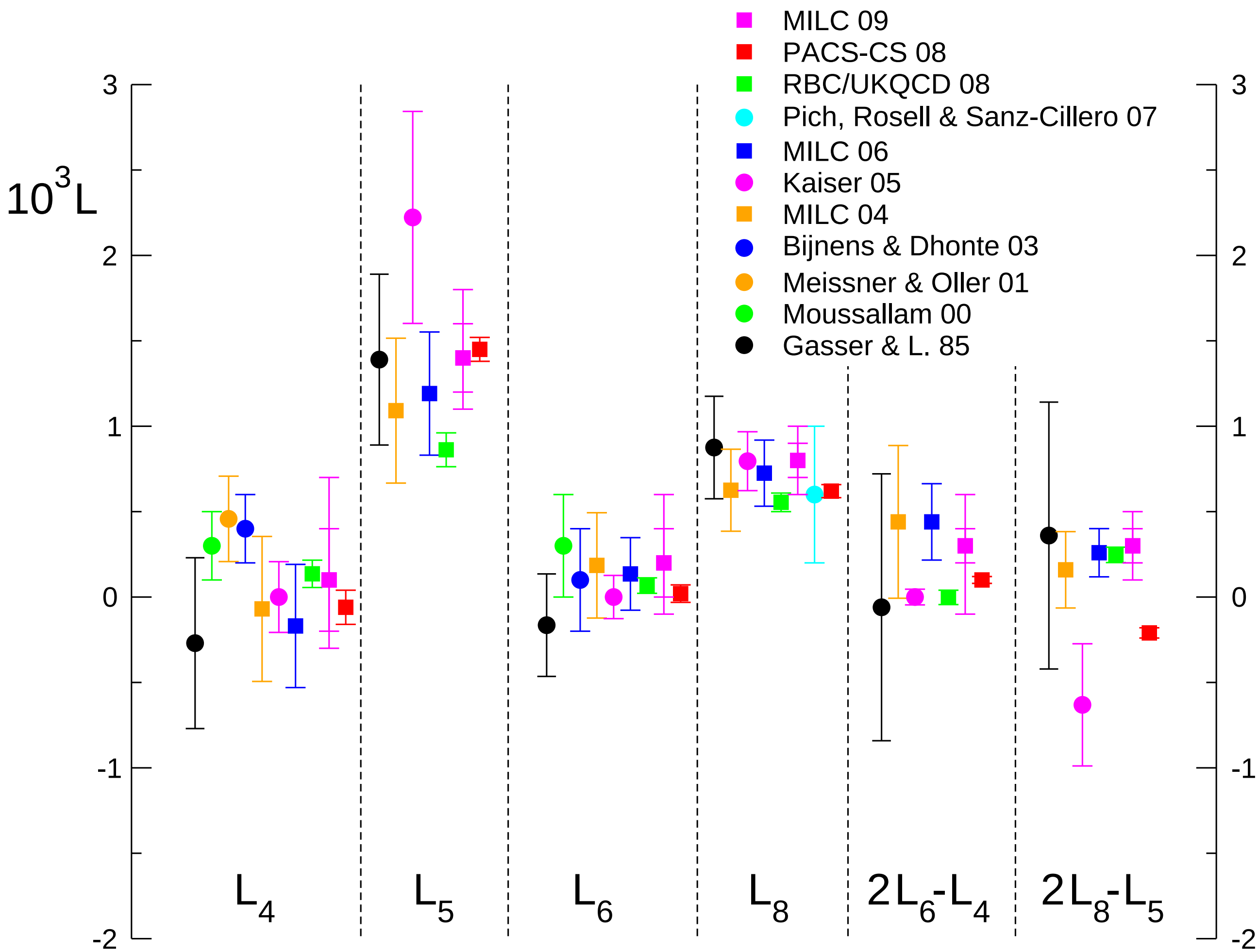
$\Rightarrow$ Need to know the effective coupling constants

- Apart from L_4, L_6, all of the SU(3)$_L \times$SU(3)$_R$ couplings of NLO can be determined from experiment Gasser + L. 1985

- L_4, L_6 are suppressed in large N_c limit, violate OZI rule

$\Rightarrow$ Expect L_4, L_6 to be small (at a running scale where large logarithms do not occur, such as M_η or M_ρ,)

NLO couplings: L_4, L_5, L_6, L_8



Numerical values shown refer to running scale $\mu = M_\rho$

Lattice results for L_4, L_5, L_6, L_8 agree with phenomenology within errors

The large N_c suppression of L_4, L_6 is confirmed

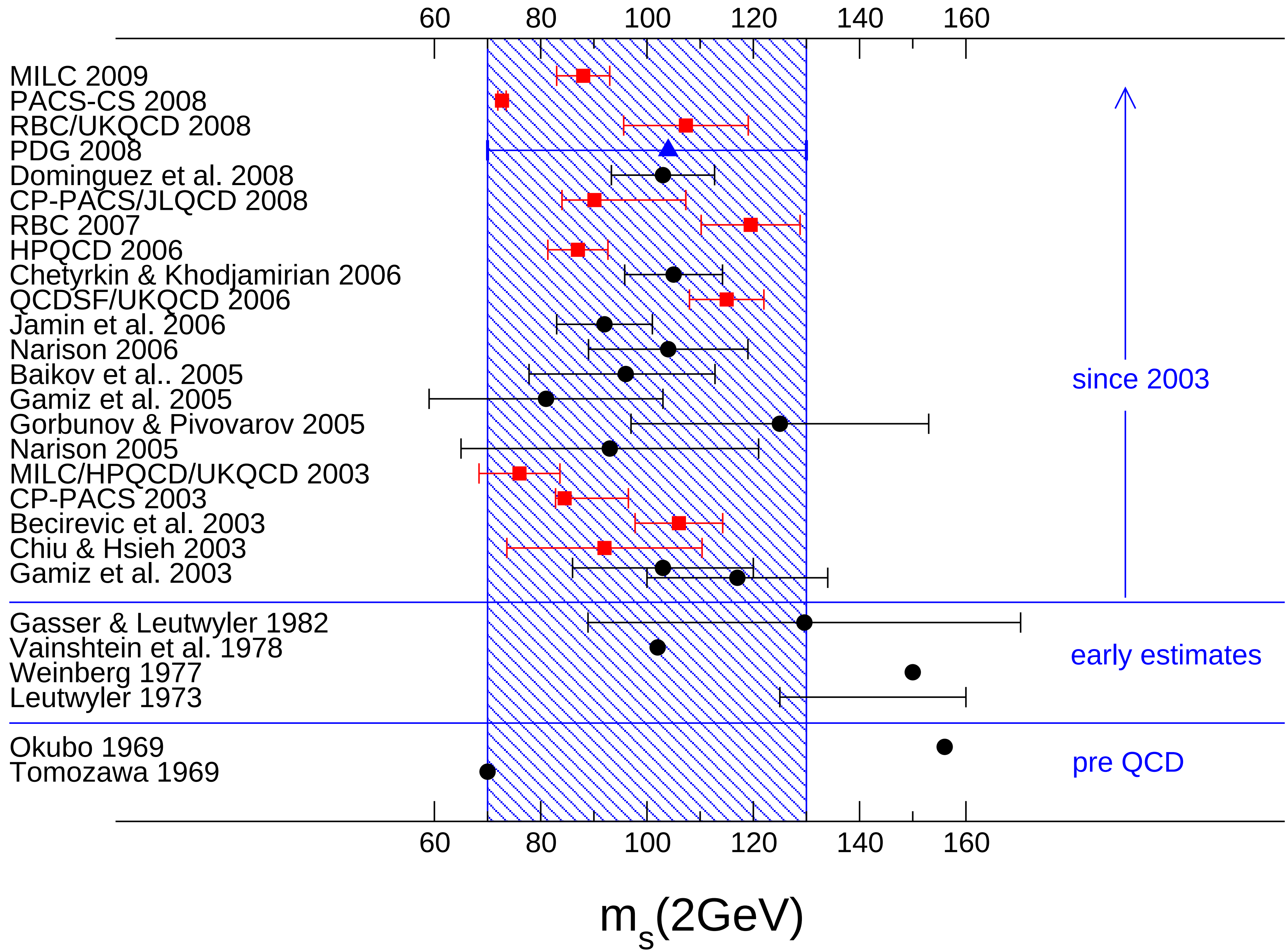
Three light quarks: interface between lattice and χPT

- Steady progress in simulating QCD with light quarks
- Still, the quark masses used are too large for the NLO formulae of χPT to work ($m_{\rm ud}$ too large, $m_{\rm s}$ OK)
- Three options
  - Use smaller quark masses
  - Extrapolate only in $m_{\rm u}, m_{\rm d}$ keep $m_{\rm s}$ fixed
  - Account for NNLO contributions
- Some lattice analyses do allow for NNLO contributions, but the chiral logarithms are accounted for only to NLO
- Discrepancies between different lattice results persist

  In part, these may arise from nonperturbative renormalization effects
  Some of the collaborations still use perturbative renormalization

$\Rightarrow$ Illustrate this with the results for $m_{\rm s}$

Mass of the strange quark



Lattice determination of V_{us}, V_{ud}

- Rely on Standard Model, where

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Precision data on K-decays imply

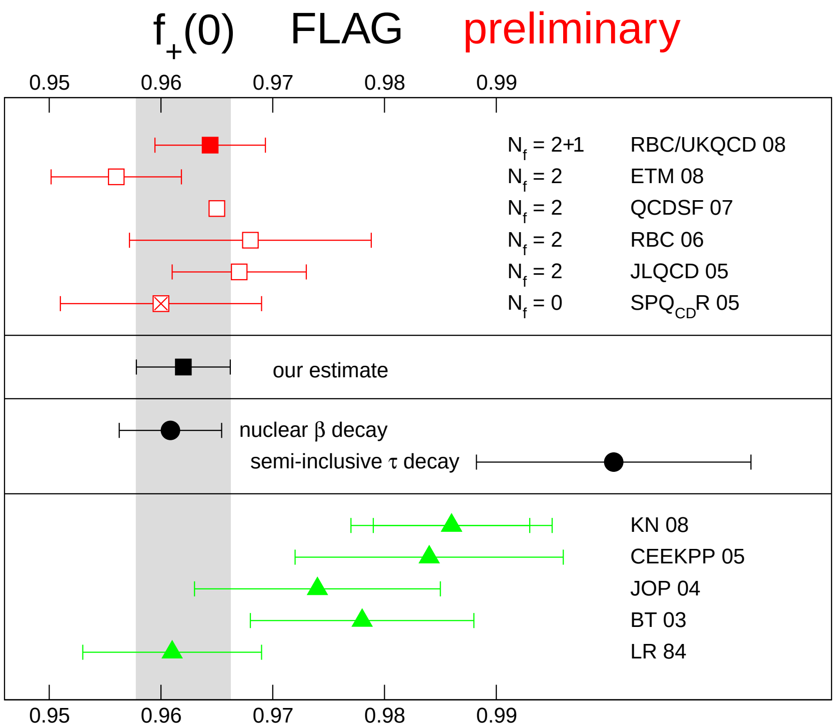
$$|V_{us}|f_+(0) = 0.21661(47)$$

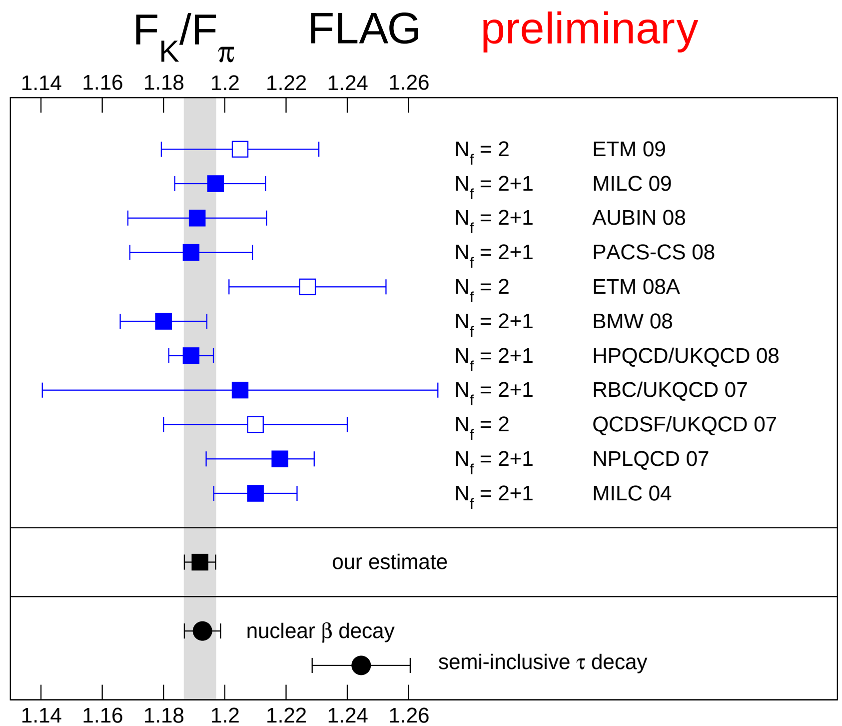
$$\left|\frac{V_{us}F_K}{V_{ud}F_\pi}\right| = 0.27599(59)$$

$\Rightarrow$ Since V_{ub} is tiny and known to good accuracy, $V_{ud}, f_+(0), F_K/F_\pi$ are all determined by V_{us}

- Lattice allows two independent ways to measure V_{us}: calculate $f_+(0)$ or calculate F_K/F_π

Lattice determinations of $f_+(0)$ and F_K/F_π

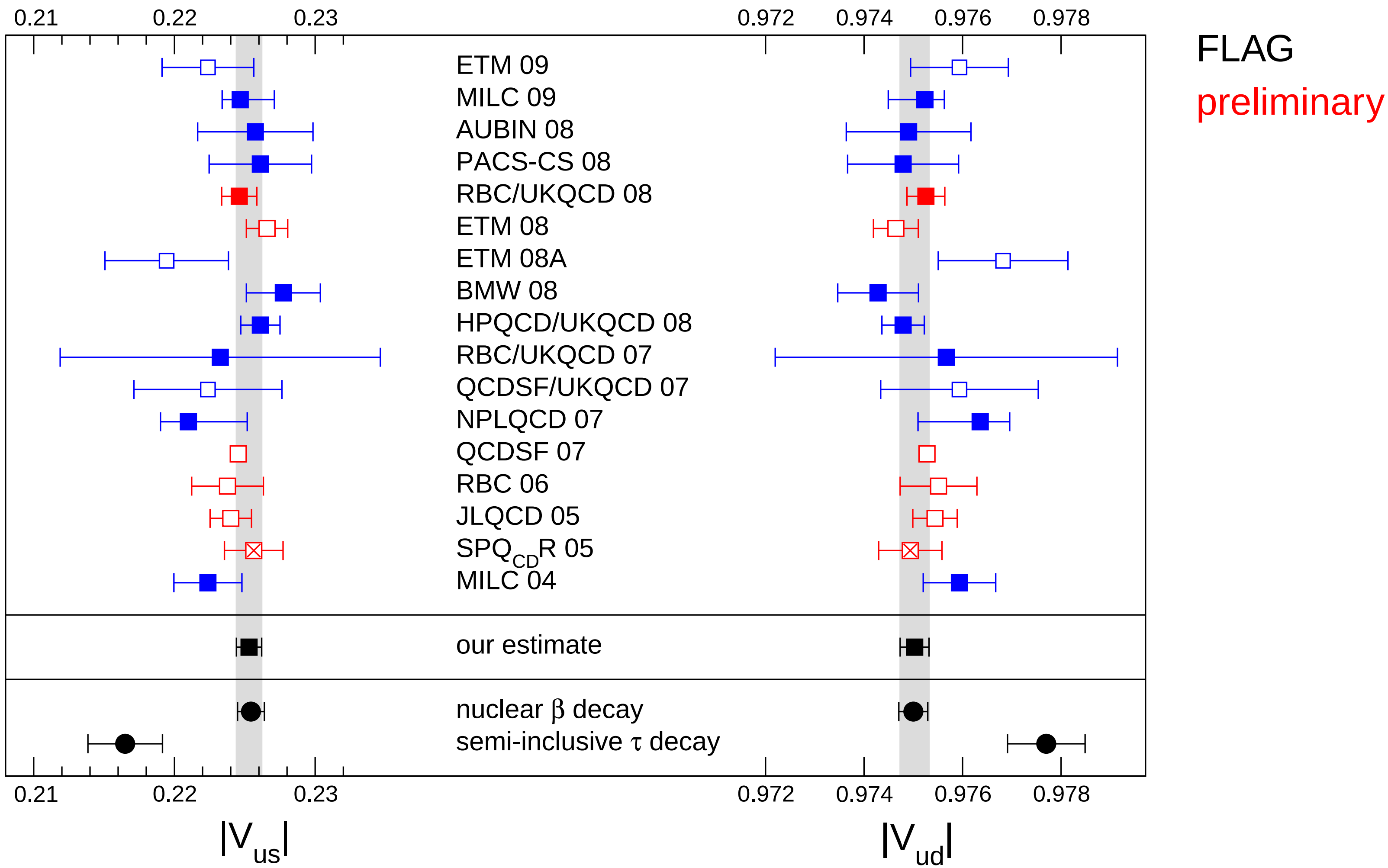




- FLAG estimate combines the lattice data for $f_+(0)$ with those for F_K/F_π

$\Rightarrow$ Determines V_{us} as well as V_{ud}

Lattice determinations of V_{us} and V_{ud}



- Nuclear β decay value for V_{ud} confirmed within errors
- τ decay: physics beyond the Standard Model ?

Trying to understand the size of the effective couplings

- $SU(2)_L \times SU(2)_R$: can understand the size of all NLO couplings in terms of resonance exchange Gasser + L. 1984
- Also true for $SU(3)_L \times SU(3)_R$ Ecker, Gasser, Pich, de Rafael 1989
- χPT formulae have been worked out to NNLO for many quantities of physical interest Bijnens and collaborators
- Formulae involve new unknown couplings
- Chiral resonance theory, couplings of higher order, effective Lagrangian for e.m. + weak interactions . . .

  Gonzalez-Alonso, Guo, Pich, Portoles, Prades, Rosell, Ruiz-Femenia, Sanz-Cillero . . .
- Excellent review of current state of the art:

  Bijnens, arXiv:0904.3713 (Valencia 2009)

Problem with RχPT in case of $f_+(0)$

- Form factor known to NNLO

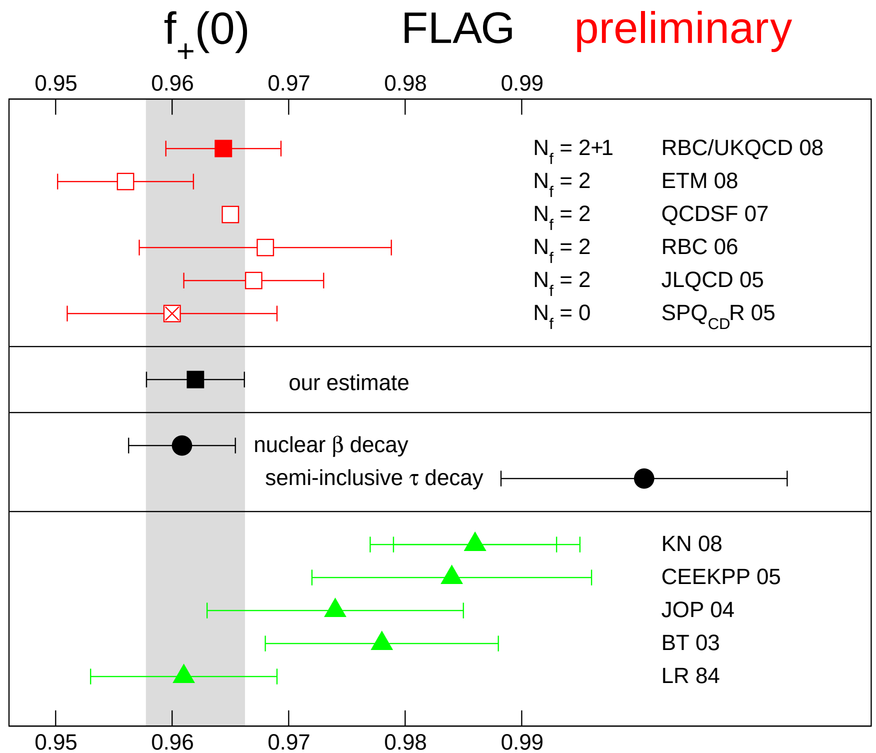
  Post + Schilcher 2002, Bijnens + Talavera 2003

- Account for isospin breaking, use RχPT estimates for the effective couplings

$\Rightarrow$ $f_+(0) = 0.986(7)$ Kastner + Neufeld 2008

Compare $0.961(5)$ (β decay) or $0.962(5)$ (lattice)
Discrepancy amounts to more than 3 σ

Compare with lattice results



- Difference between $f_+(0)$ and 1 is a symmetry breaking effect
- No problem at NLO

$\Rightarrow$ RχPT does not appear to account properly for the symmetry breaking effects at NNLO

Problems with scalar meson dominance ?

- Quark mass term in $\mathcal{L}_{\text{QCD}}$ is a scalar operator
- Matrix elements dominated by scalar resonances ?

  Can the *dependence on the quark masses* be accounted for with scalar meson dominance ?
- Rapidly rising $\pi\pi$ continuum (large chiral logs), σ makes a broad bump, narrow peak from $f_0(980)$, glueballs, etc.
- Failure of scalar meson dominance may be the origin of the problem   more detailed discussion in Erice lectures, arXiv:0808.2825

Second example

- $K \to \pi\pi$ decay: value of $\delta_0^0 - \delta_0^2$ at $s = M_K^2$
  - $\pi\pi$ phase shifts accurately known from dispersion theory $\delta_0^0 - \delta_0^2 = 47.5^\circ \pm 1.5^\circ$ Colangelo, Gasser, L. 2001
  - In the determination from $K \to \pi\pi$ via Watson theorem, isospin breaking is enhanced because of the $\Delta I = \frac{1}{2}$ rule
  - Complete analysis to NLO Cirigliano, Ecker, Neufeld + Pich 2004
  - Recent update of the numerics yields

    $$\delta_0^0 - \delta_0^2 = 57.5^\circ \pm 3.4^\circ$$ FlaviaNet Kaon Working Group 2008
- Results differ by 2.7 σ
- In both examples, resonance estimates are used
  In my opinion, this is the only problematic point

Things on the working bench

- Systematic error in lattice simulations ?
  The discrepancies between results obtained with different lattice formulations need to be understood
- Validity of the OZI rule, for example: Σ/Σ_0

  $\Sigma = |\langle 0|\,\bar{u}u\,|0\rangle|_{m_u,m_d\to 0}$ at physical value of m_s

  $\Sigma_0 = |\langle 0|\,\bar{u}u\,|0\rangle|_{m_u,m_d,m_s\to 0}$
- Zweig rule violations in the decay constants: F/F_0
  F is value of F_π for $m_u, m_d \longrightarrow 0$ at physical value of m_s
  F_0 is value of F_π for $m_u, m_d, m_s \longrightarrow 0$

$\Rightarrow$ Values obtained for Σ/Σ_0 and F/F_0 still scatter strongly

- Determine the NNLO couplings on the lattice
  Which of these cannot be understood with RχPT ?
- Many other open questions can now be tackled . . .
  For quite a few, the answer will soon be known

Conclusions

Conclusions for SU(2)$\times$SU(2)

- Expansion in powers of $m_{\rm u}, m_{\rm d}$ yields a very accurate low energy representation of QCD

Low energy pion physics is a precision laboratory Theoretical tools: χPT, lattice, dispersion theory

- Limitations:
  - Low energies
  - e.m. interaction must properly be accounted for
  - Calculations cannot be done on back of an envelope

Conclusions for SU(2)$\times$SU(2) ctd.

- Lattice results confirm the GMOR relation

$\Rightarrow$ M_π is dominated by the leading order term, related to the quark condensate

$\Rightarrow$ Energy gap of QCD is understood very well

- Lattice approach allows an accurate measurement of some of the effective coupling constants

Conclusions for SU(3)$\times$SU(3)

- Pattern of low levels of QCD and properties of Green functions at low energies can be understood with χPT
- Do get a coherent picture at NLO, couplings reasonably well known, lattice confirms phenomenology
- Expansion in powers of m_s converges only slowly, NNLO couplings cannot be neglected
- Knowledge of these couplings leaves much to be desired
- RχPT appears to work for those couplings that account for the momentum dependence of the amplitudes, but does not properly account for those that specify the dependence on the quark masses
- Lattice determinations of NNLO couplings would help sorting things out, but are not yet available

Conclusions for SU(3)×SU(3) ctd.

- Some of the lattice data call for large violations of the Zweig rule, others do not
- The central value $F/F_0 = 1.23$ of RBC/UKQCD, for instance, leads to a qualitative puzzle:
  - F_π is the pion wave function at the origin
  - One of the valence quarks of the kaon is heavier
    $\longrightarrow$ moves more slowly
    $\longrightarrow$ wave function more narrow
    $\longrightarrow$ higher at the origin: $F_K/F_\pi \simeq 1.19$
  - $F/F_0 = 1.23$ indicates that the wave function is more sensitive to the mass of the sea quarks than to the mass of the valence quarks . . . very strange
    $\longrightarrow$ most interesting if true

$\Rightarrow$ Expect significant progress at the interface between lattice and χPT in the near future

Spares

Quark mass ratios