

Theoretical aspects of Chiral Dynamics

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- Many of the quantities of interest at the precision frontier of particle physics require a good understanding of the strong interaction at low energies.
- In this context, the lightest hadrons are the most important

$$\pi^+ \quad \pi^0 \quad \pi^-$$

- It is essential that we know why the pions are so light. This understanding relies on symmetry.

- Heisenberg 1932: strong interaction is invariant under isospin rotations – this is why $M_p \simeq M_n$.

⇒ Mass difference must be due to the e.m. interaction.

- Puzzle: e.m. field around the proton is stronger, makes the proton heavier than the neutron.
- Numerous unsuccessful attempts at solving this puzzle.
- Gasser, L. 1975:

If QCD describes the strong interaction correctly, then m_u must be very different from m_d .

$m_u/m_d \simeq 0.67$, $m_s/m_d \simeq 22.5$ first crude estimate

- Weinberg 1977: Dashen theorem yields independent result $m_u/m_d \simeq 0.56$, $m_s/m_d \simeq 20.1$
- Current lattice estimates of FLAG:
 $m_u/m_d = 0.46 \pm 0.03$, $m_s/m_d = 20.0 \pm 0.5$

- Since m_u is very different from m_d : how come that isospin is a nearly perfect symmetry of the strong interaction ?
- QCD explains this very neatly: for yet unknown reasons, it so happens that m_u and m_d are very small.
- If m_u and m_d are set equal to zero \Rightarrow QCD becomes invariant under independent flavour rotations of the right- and left-handed u, d -fields.
- Symmetry group: $SU(2)_R \times SU(2)_L$
- This symmetry was discovered before QCD: Nambu 1960.
 - strong interaction has an approximate chiral symmetry
 - chiral symmetry is hidden, spontaneously broken
 - spontaneous symmetry breakdown generates massless bosons
 - the pions are the massless bosons of chiral symmetry
 - are not exactly massless, because the symmetry is not exact

- For $m_u = m_d = 0$ the pions are massless (Nambu-Goldstone bosons of an exact, spontaneously broken symmetry).
- For small values of m_u, m_d : M_π^2 is proportional to $m_u + m_d$:

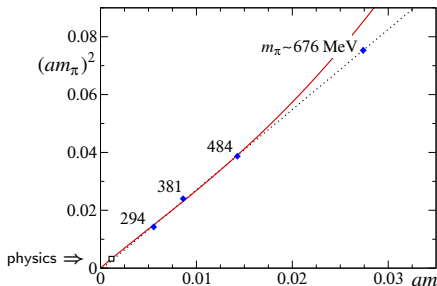
$$M_\pi^2 = (m_u + m_d) \times |\langle 0 | \bar{u}u | 0 \rangle| \times \frac{1}{F_\pi^2}$$

\uparrow explicit \uparrow spontaneous symmetry breaking

Gell-Mann, Oakes, Renner 1968

- Only $m_u + m_d$ counts.
- F_π is known from $\pi^+ \rightarrow \mu^+ \nu$, but $|\langle 0 | \bar{u}u | 0 \rangle| = ?$
Non-perturbative method required to calculate $|\langle 0 | \bar{u}u | 0 \rangle|$.

- GMOR formula is beautifully confirmed on the lattice:
can determine M_π as a function of $m_u = m_d = m$.



Lüscher
Lattice conference 2005

- Proportionality of M_π^2 to m holds out to about
 $m \simeq 10 \times$ physical value of $m_{ud} \equiv \frac{1}{2}(m_u + m_d)$.

Dürr, arXiv:1412.6434

- Switch the electroweak interactions off, consider pure QCD.

$$M_\pi = M_\pi(\Lambda_{\text{QCD}}, m_u, m_d, m_s, m_c, m_b, m_t)$$

- Expand this function in powers of m_u, m_d . The formula of GMOR gives the leading term of the expansion:

$$M^2 \equiv (m_u + m_d) |\langle 0 | \bar{u}u | 0 \rangle| \frac{1}{F^2}$$

$\langle 0 | \bar{u}u | 0 \rangle, F$ independent of m_u, m_d (values in chiral limit)

- χ PT shows that the next term in the expansion is given by

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{2(4\pi F)^2} \bar{\ell}_3 + O(M^4) \right\}$$

$$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M^2} \quad \text{depends logarithmically on } M$$

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{2(4\pi F)^2} \bar{\ell}_3 + O(M^4) \right\}$$

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- Numerical value at $M = 135$ MeV:

$$\bar{\ell}_3 = 3.05 \pm 0.99 \text{ FLAG} \leftrightarrow \Lambda_3 \simeq 600 \text{ MeV.}$$

⇒ Correction in M_π is tiny: $\frac{M_\pi^2}{2(4\pi F_\pi)^2} \bar{\ell}_3 \simeq 0.024$

- Not a surprise: m_u, m_d are small, of order 2 to 5 MeV
SU(2) × SU(2) should be a nearly perfect symmetry !

Why is the strong interaction nearly isospin invariant ?

- m_u, m_d small \Rightarrow $SU(2) \times SU(2)$ a nearly perfect symmetry.
- Isospin is a subgroup of $SU(2) \times SU(2)$.
- \Rightarrow Isospin is a nearly perfect symmetry.
- \Rightarrow The strong interaction is nearly invariant under isospin rotations because m_u, m_d are small.
- But: the fact that $SU(2) \times SU(2)$ symmetry is broken is clearly seen: $M_\pi \neq 0$
Why is the breaking of isospin symmetry so well hidden ?
Why is M_{π^0} nearly equal to M_{π^+} ?
- The Nambu-Goldstone bosons are shielded from isospin breaking: leading term in \mathcal{L}_{eff} only knows about $m_u + m_d$.

Interaction among the pions

- Switch the e.m. interaction off, $\alpha = 0$, set $m_u = m_d$.
- Isospin symmetry then becomes exact \rightarrow the scattering of any of the 6 initial states $\pi^+\pi^+$, $\pi^+\pi^0$, $\pi^+\pi^-$... into any of these final states is described by a single function $\mathbf{A}(s, t)$.
- Expansion in momenta and quark masses starts with

$$\mathbf{A}(s, t) = \frac{1}{F_\pi^2}(s - M_\pi^2) + \dots \quad \text{Weinberg 1966}$$

Parameter free prediction at LO.

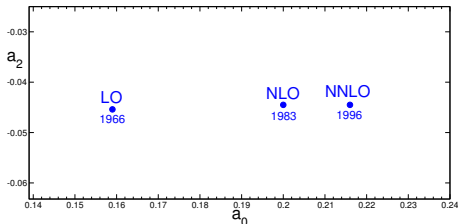
Scattering lengths

- Prediction for the two S-wave scattering lengths:

$$\mathbf{a}_0 = \frac{7M_\pi^2}{32\pi F_\pi^2} = 0.16, \quad \mathbf{a}_2 = -\frac{M_\pi^2}{16\pi F_\pi^2} = -0.045$$

Weinberg 1966

- The chiral perturbation series has been worked out to NNLO.



- \mathbf{a}_2 practically stays put, but the corrections in \mathbf{a}_0 are large !

$$\text{LO} \xrightarrow{26\%} \text{NLO} \xrightarrow{8\%} \text{NNLO}$$

SU(2) × SU(2) a nearly perfect symmetry ??

Why are the corrections in the scattering lengths large ?

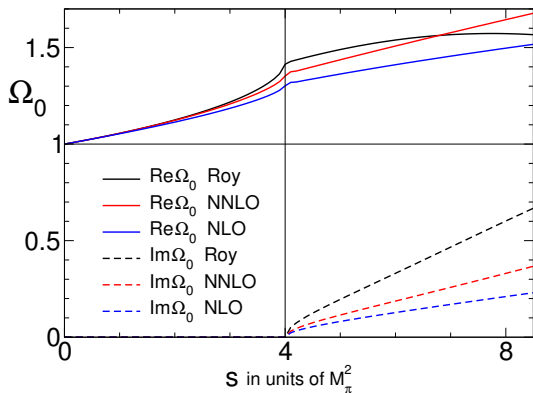
- Expansion of \mathbf{a}_0 in powers of $m_u = m_d$ contains juicy $\chi \log$:

$$\mathbf{a}_0 = \frac{7M_\pi^2}{32\pi F_\pi^2} \left\{ 1 + \frac{9M_\pi^2}{2(4\pi F_\pi)^2} \ln \frac{\Lambda_0^2}{M_\pi^2} + \dots \right\}$$

⇒ Coefficient 9 × larger than the one in the expansion of M_π^2 !

- S-wave scattering length is the value of the partial wave $t_0(s)$ at threshold, $\mathbf{a}_0 = t_0(4M_\pi^2)$
 - At LO, the S-wave has an Adler zero at $s = M_\pi^2$.
 - Slope is large ⇒ $t_0(s)$ very rapidly grows with s .
 - Final state interaction generates strong curvature at threshold.
 - Unitarity ⇒ scattering amplitude is singular there.
 - Scattering length sits at the threshold.

⇒ The large $\chi \log$ stems from the threshold singularity.



$$\Omega_0(s) = e^{\frac{s}{\pi} \int \frac{ds'}{s'-s-i\epsilon} \delta_0(s')}$$

- Roy equations

Roy 1971

- Numerical analysis

Ananthanarayan, Colangelo, Gasser, L. 2001

⇒ Omnès factor known reliably and quite accurately.

- χ PT expands in powers of quark masses and momenta.
Would have to be taken to high order to match this.

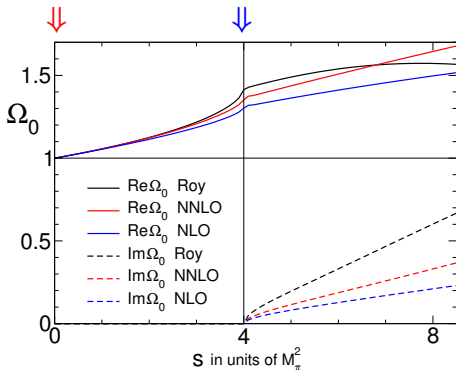
⇒ χ PT is not needed for the momentum dependence.
Dispersion theory provides a better tool for that.

Roy equations + χ PT

- Match Roy equations with χ PT at $s = 0$, not at threshold.

rapid convergence

slow convergence of the chiral series

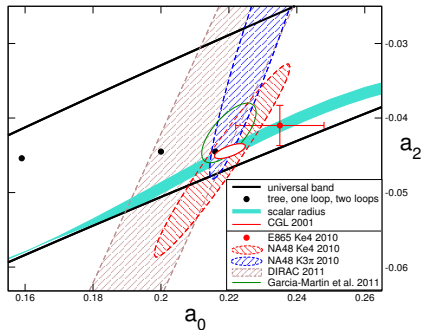
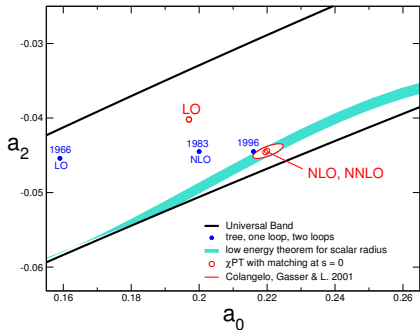


In determination of a_0 with Roy + χ PT:

$$\begin{array}{ccc}
 \text{LO} & \Rightarrow & \text{NLO} & \Rightarrow & \text{NNLO} & & \text{matching at threshold} \\
 26\% & & 8\% & & & & \\
 11\% & & 0.2\% & & & & \text{matching at } s = 0
 \end{array}$$

- Leads to remarkably sharp predictions for $\pi\pi$ scattering
- Triggered new low energy precision experiments:
 - $\pi^+\pi^-$ atoms, DIRAC.
 - $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$, $K^0 \rightarrow \pi^0\pi^0\pi^0$: cusp near threshold, NA48/2.
 - $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ data: E865, NA48/2.

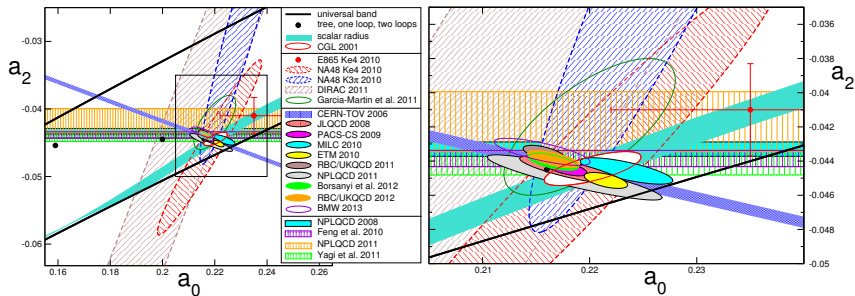
Experimental tests of the prediction



Determination of the scattering lengths on the lattice

- a. Direct determination of \mathbf{a}_2 with Lüscher's method, via dependence of the energy levels on the size of the box.
- b. Uncertainty in χ PT prediction for $\mathbf{a}_0, \mathbf{a}_2$ is dominated by the uncertainty in the coupling constants l_3, l_4 of the effective Lagrangian. These can now reliably be determined on the lattice, from the quark mass dependence of M_π and F_π .

Compare lattice results with prediction and experiment



Prediction is consistent with the lattice results.

Some of the collaborations underestimate the uncertainties.

Low energy constants from the lattice

- No experimental info for dependence on m_u, m_d, \dots
Lattice is the ideal tool for that.
Please do not be content with reaching physical quark masses.
Extract the dependence on them, determine the LECs !
- In $SU(3)_R \times SU(3)_L$ χ PT, the quantities of interest are also expanded in $m_s \Rightarrow$ NNLO contributions sizeable, important.
- Numerical representations for these are available for many quantities of interest. Bijnens and collaborators
- Analysis of lattice data: algebraic expressions preferable.

Kaiser, Schweizer

Ecker, Masjuan, Neufeld

Ananthanarayan and collaborators

Developments in dispersion theory

- Roy equation analysis confirmed

Garcia-Martin, Kaminski, Londergan, Nebreda, Pelaez, Szczepaniak, Yndurain

- σ meson, $f_0(450)$ firmly established by the Roy analysis
Position of pole on second sheet known quite accurately

⇒ Analysis Tools for Next-Generation Hadron Spectroscopy Experiments

arXiv:1412.6393

- Bounds on form factors Abbas, Ananthanarayan, Caprini, Fischer
- Dispersion theory for $K_{\ell 4}$ decay Colangelo, Passemar, Stoffer
- Light-by-light contribution to muon $g-2$
Colangelo, Hoferichter, Kubis, Procura, Stoffer

- Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, arXiv:1506.04142

Result for σ -term:

$$\sigma_{\pi N} \equiv \frac{m_{ud}}{2M_N} \langle p | \bar{u}u + \bar{d}d | p \rangle = 59.1(3.5) \text{ MeV}$$

- I find this result very puzzling because of two prejudices:

(1) SU(3) is a decent approximate symmetry, also for the matrix elements of the operator $\bar{q}\lambda^a q$ in the baryon octet.

(2) Rule of Okubo, Iizuka and Zweig is approximately valid.

⇒ A value around 60 MeV implies that (1) and/or (2) are wrong.

Why is $\sigma_{\pi N} = 60$ MeV puzzling ?

- Mass formula valid to first order in SU(3) breaking:

$$M_{\Sigma} + M_{\Xi} - 2M_N = \frac{m_s - m_{ud}}{2M_N} \langle \mathbf{p} | \bar{u}u + \bar{d}d - 2\bar{s}s | \mathbf{p} \rangle$$

- The ratio $m_s : m_{ud}$ is by now firmly known.

⇒ Experimental values of the baryon masses yield

$$\frac{m_{ud}}{2M_N} \langle \mathbf{p} | \bar{u}u + \bar{d}d - 2\bar{s}s | \mathbf{p} \rangle \simeq 25 \text{ MeV.}$$

- Zweig rule: $\langle \mathbf{p} | \bar{s}s | \mathbf{p} \rangle$ small, confirmed by lattice results.

⇒
$$\frac{m_{ud}}{2M_N} \langle \mathbf{p} | \bar{u}u + \bar{d}d | \mathbf{p} \rangle \simeq 25 \text{ MeV.}$$

- Clash between two independent experimental results:

$$\text{Baryon masses} \begin{array}{c} \longleftrightarrow \\ 25 \end{array} \pi N \text{ scattering} \begin{array}{c} \\ 60 \end{array}$$

- Clash is not new, but recent developments accentuate it:

Zweig rule violations appear to be very small.

Recent work on πN scattering yields higher values at the Cheng-Dashen point than what was obtained with the Karlsruhe-Helsinki analysis.

Left hand side concerns expansion in $m_s - m_{ud}$, not expansion in m_u, m_d .

SU(3)-breaking in $\langle p | \bar{q} \lambda^a q | p \rangle$ extraordinarily large ?

Why then does the Gell-Mann-Okubo formula work so well ?

If a 'correction' is $> 100\%$ \Rightarrow not easy to justify the calculation.

Contribution from the $\Delta(1232)$? Alarcon, Geng, Martin Camalich, Oller

\Rightarrow Look forward to a resolution of this puzzle ...

Started the talk with isospin symmetry, return to this theme now.

Mass difference between proton and neutron

- Prehistoric work: Cottingham 1963
- Ancient: Gasser, L. 1975
- More recent:
 - Walker-Loud, Carlson, Miller 2012 WCM
 - Erben, Shanahan, Thomas, Young 2014 ESTY
 - Thomas, Wang, Young 2015
- Oven fresh:
 - Gasser, Hoferichter, L., Rusetsky arXiv:1506.06747

- Electromagnetic self-energy of a particle: M_γ

$$M_\gamma = \frac{e^2}{4M} \int d^4x D_{\mu\nu}(x) \langle p | T j^\mu(x) j^\nu(0) | p \rangle$$

photon propagator Compton scattering

- QCD is asymptotically free \Rightarrow quarks free at short distances

$$D_{\mu\nu}(x) \propto \frac{1}{x^2} \quad \langle p | T j^\mu(x) j^\nu(0) | p \rangle \propto \frac{1}{x^2}$$

\Rightarrow Integral diverges logarithmically at $x = 0$, like for e in QED.

- Divergence absorbed in renormalization of g, m_u, m_d, \dots
Only operators belonging to m_u, m_d have $l \neq 0$.

\Rightarrow Only the renormalization of m_u, m_d matters for $M_\gamma^p - M_\gamma^n$.

- Renormalization of m_u proportional to $e^2 m_u$, likewise for m_d .

\Rightarrow Coefficient of logarithmic divergence $\propto e^2 m_u, e^2 m_d$ tiny.

In chiral limit: $M_\gamma^p - M_\gamma^n$ finite.

In reality there is a divergence, but with a tiny coefficient.

Self-energy in terms of invariant amplitudes

$$\frac{i}{2} \int d^4x e^{i q \cdot x} \langle p | T j^\mu(x) j^\nu(0) | p \rangle = \\ (q^\mu q^\nu - g^{\mu\nu} q^2) T_1 + \left\{ -p^\mu p^\nu \frac{q^2}{M^2} + \dots \right\} T_2$$

- The invariant amplitudes depend on two variables:

$$T_1 = T_1(\nu, q^2) \quad T_2 = T_2(\nu, q^2) \quad \nu = p \cdot q / M$$

- Explicit formula for the self-energy in terms of T_1, T_2 :

$$M_\gamma = \frac{-i e^2}{2M(2\pi)^4} \int \frac{d^4q}{q^2 + i\epsilon} \{ 3q^2 T_1(\nu, q^2) + (2\nu^2 + q^2) T_2(\nu, q^2) \}$$

Dispersion theory of T_1, T_2

- $T_1(\nu, q^2)$ and $T_2(\nu, q^2)$ are analytic in q^0 .
- Suffices to know these functions for space-like momenta. Values in the time-like region: analytic continuation.
- An analytic function is determined by its singularities and its asymptotic behaviour.
- Singularities: residues of poles, discontinuities across cuts.

$$\text{Im } T_1, \text{Im } T_2 \longleftrightarrow F_1, F_2 \longleftrightarrow \sigma_T, \sigma_L$$

\uparrow structure functions \uparrow cross sections for
 $e + N \rightarrow e + \text{anything}$

- Asymptotics: If $T_1, T_2 \rightarrow 0$ for $\nu \rightarrow \infty$

\Rightarrow unsubtracted dispersion relations

$$T_1(\nu, q^2) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{d\nu'}{\nu' - \nu} \text{Im } T_1(\nu', q^2) \quad \text{likewise for } T_2$$

\uparrow
 σ_T, σ_L

\Rightarrow Cottingham formula: M_γ given by integral over σ_T, σ_L

- Reggeon exchange $\Rightarrow T_1 \propto \nu^\alpha$, $T_2 \propto \nu^{\alpha-2}$
 α is value of the Reggeon trajectory $\alpha(t)$ at $t = 0$
- All Reggeons have $\alpha < 2 \Rightarrow T_2(\nu, q^2) \rightarrow 0$ for $\nu \rightarrow \infty$
 \Rightarrow d.r. for T_2 unsubtracted, fully determined by its singularities.
- Reggeon exchange does generate nontrivial asymptotics in T_1 :

$$T_1^R(\nu, q^2) = - \sum_{\alpha} \frac{\pi \beta_{\alpha}(q^2)}{\sin \pi \alpha} \{ \nu^{\alpha} + (-\nu)^{\alpha} \}$$

- $\Rightarrow T_1$ does not obey an unsubtracted d.r.

$$T_1(\nu, q^2) = S_1(q^2) + \frac{\nu}{\pi} \int_{-\infty}^{\infty} \frac{d\nu'}{\nu'(\nu' - \nu)} \text{Im} T_1(\nu', q^2)$$

- Harari 1966: Could it be that the subtraction term dominates over the remainder and changes the sign of M_{γ}^{p-n} ?

Reggeon dominance hypothesis

- The field theoretic origin of the Reggeons is understood.

Reggeon field theory, Gribov, Balitsky, Fadin, Kuraev, Lipatov, ...

- Basic hypothesis in GL: nothing but Reggeons in asymptotics

$$T_1(\nu, q^2) - T_1^R(\nu, q^2) \rightarrow 0 \quad \text{for } \nu \rightarrow \infty \text{ at fixed } q^2$$

- If the asymptotic behaviour of QCD could not be understood on this basis, that would be most interesting !
- Alternative: $T_1(\nu, q^2) - T_1^R(\nu, q^2) \rightarrow \beta(q^2) \neq 0$

In Regge language: $T_1 \sim \beta(q^2)\nu^\alpha$ fixed pole at $\alpha = 0$

Reggeon dominance hypothesis \iff ~~fixed pole~~

- Theoretical understanding of Pomeron underdeveloped.
Branch point at $\alpha = 1$. \exists daughter at $\alpha = 0$?

Reggeon dominance hypothesis

- Consequence of the hypothesis that the Reggeons dominate:
 $T_1 - T_1^R$ obeys an unsubtracted dispersion relation.
- ⇒ Subtraction $S_1(q^2)$ is determined by σ_T, σ_L , like the rest.
- ⇒ Electromagnetic self-energy can be calculated.
- Numerical evaluation in GL: $M_\gamma^{p-n} = 0.76(30)$ MeV
- Shortcoming of this calculation: in 1975 there was no sign of scaling violations in the data. Deep inelastic contributions were estimated with Bjorken's scaling laws, found to be very small, in the noise of the calculation.

- WCM claim 'technical oversight' in GL.

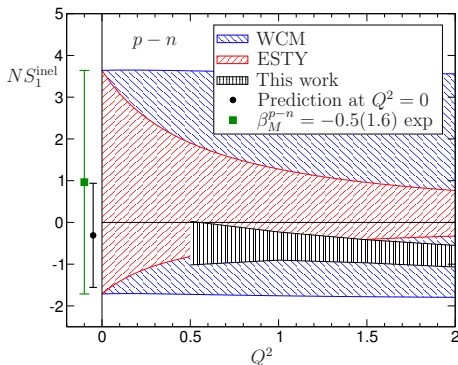
Instead treat $\mathbf{S}_1(\mathbf{q}^2)$ as physically independent of σ_T, σ_L .

$$T_1(\nu, \mathbf{q}^2) = \underset{\substack{\uparrow \\ \text{subtraction}}}{\mathbf{S}_1(\mathbf{q}^2)} + \frac{\nu}{\pi} \int_{-\infty}^{\infty} \frac{d\nu'}{\nu'(\nu' - \nu)} \text{Im} T_1(\nu', \mathbf{q}^2) \underset{\substack{\uparrow \\ \sigma_T, \sigma_L}}{\phantom{T_1(\nu', \mathbf{q}^2)}}$$

- Low energy theorem: $T_1(\mathbf{0}, \mathbf{0}) \Leftrightarrow \beta_M$ magnetic polarizability.
- \Rightarrow Can fix $\mathbf{S}_1(\mathbf{0})$ with experimental value of β_M .
- \nexists exp. info about dependence on $\mathbf{q}^2 \Rightarrow$ invent a model.
- \Rightarrow Result for self-energy: $M_\gamma^{p-n} = 1.30(03)(47)$ MeV.
- ESTY make a different ansatz for $\mathbf{S}_1(\mathbf{q}^2)$.
- \Rightarrow Result for self-energy: $M_\gamma^{p-n} = 1.04(35)$ MeV.
- In either case, the systematic error due to the model-dependence is difficult to assess.

Comparison of the subtraction functions

- Side remark: Born terms are ambiguous, elastic part is unique. Analytic functions determined by asymptotics + singularities. $T_1^{\text{el}}, T_2^{\text{el}}$ contributions to T_1, T_2 from the nucleon pole.
- Decompose $S_1 = S_1^{\text{el}} + S_1^{\text{inel}}$, $S_1^{\text{el}} \Leftrightarrow$ nucleon form factors.



taken from
Gasser, Hoferichter, Leutwyler
and Rusetsky, arXiv:1506.06747

$Q^2 = -q^2$, GeV units

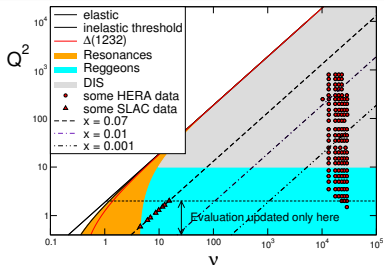
y-axis stretched with
 $N = (1 + Q^2 / 0.71)^2$
for better visibility

- Models yield higher central values for the subtraction function.

Self-energy difference, polarizabilities

- Difference in M_γ^{p-n} is exclusively due to change in S_1 :
Can replace the model made in WCM for S_1 by our prediction, leave everything else as it is \Rightarrow 1.30 MeV drops to 0.63 MeV.
Repeat exercise with ESTY \Rightarrow 1.04 MeV drops to 0.67 MeV.
- \Rightarrow Ancient result, $M_\gamma^{p-n} = 0.76(30)$ MeV, is confirmed.
- Update of estimate for deep inelastic contributions needed, would reduce the uncertainty in result for self-energy.
- As a bonus, we get a prediction: $S_1(0) \iff \beta_M$
- \Rightarrow Using known Baldin sum rule results for $\alpha_E + \beta_M$, can calculate the p-n difference for α_E as well as β_M :
 $\alpha_E^{p-n} = -1.7(4)$, $\beta_M^{p-n} = 0.3(7)$ [in units of 10^{-4} fm^3]
- α_E^p, β_M^p accurately known from experiment.
- \Rightarrow Prediction for neutron: $\alpha_E^n = 12.3(7)$ $\beta_M^n = 2.9(9)$.
Experiment: $\alpha_E^n = 11.55(1.5)$ $\beta_M^n = 3.65(1.50)$.

To be done



- To sharpen the result for the self-energy difference
 - Exp. as well as theor. info about DIS much better now.
 - ⇒ Reanalysis of contributions from that region still missing.
 - Regge region, low Q^2 : Alwall, Ingelman
 - Regge region, modest Q^2 : Capella et al., Sibirtsev et al.
 - Amalgamate the two and make contact with the available parameterizations of DIS, for instance: Alekhin, Blümlein, Moch
- To sharpen the predictions for the polarizabilities
 - Region of the $\Delta(1232)$: MAID, DMT, chiralMAID, SAID
Available information is of excellent quality.
 - Intermediate energies, low Q^2 : Bosted, Christy
Improved information urgently needed, MAMI, JLAB ?

- Main problem: all of the well-established properties of the cross sections drop out in the difference between p and n.
 - Leading term in χ_{PT} is the same for p and n.
 - Isospin \Rightarrow couplings to the $\Delta(1232)$ are the same.
 - Dominating term at high energies (Pomeron) is the same.
 - Even the logarithmic divergences in M_{γ}^p and M_{γ}^n nearly cancel.

What remains is small and not easy to determine accurately.

- It would take a fixed pole with a juicy residue to get a sizeable difference between the subtraction functions relevant for p and n and hence a sizeable difference between β_M^p and β_M^n .
- The data do not rule out the presence of a fixed pole, but show that its residue is small, consistent with zero.
- Lattice is gradually making progress with e.m. self-energies ...

Merci vielmal !