The mass of the two lightest quarks

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Standard Model at low energies

- Low energies ($E \ll M_{\rm W}$): weak interaction is frozen
  only generates tiny effects, visible in the finite lifetime of the particles, e.g.

$\Rightarrow$ Standard Model reduces to QCD + QED

Precision theory for cold matter ($T \ll M_{\rm W}$) size and structure of atoms, solids, etc.

- QED is infrared stable

$\Rightarrow$ at low energies, electromagnetic interaction can be treated as a perturbation

- Parameters in Lagrangian: g, θ, e, m_u, m_d, m_s, m_c, m_b, m_t, m_e, m_μ, m_τ

Bohr radius: $$a = \frac{4\pi}{e^2\, m_e}$$

- Pattern of quark and lepton masses looks bizarre . . .
- This talk: $\boxed{m_u\,, m_d}$

Symmetries

- Symmetry plays an essential role in our understanding of nature at low energies
- QCD with N_f massless quarks: Hamiltonian has an exact chiral symmetry, $\mathrm{SU_L}(N_f)\times\mathrm{SU_R}(N_f)$
- Unless N_f is taken too large, $|0\rangle$ is symmetric only under the subgroup $\mathrm{SU_{L+R}}(N_f)$ Symmetry is hidden, "spontaneously broken"

$\Rightarrow$ Spectrum contains N_f^2-1 Goldstone bosons

- m_u, m_d, m_s happen to be small

$\Rightarrow$ $\mathrm{SU_L}(3)\times\mathrm{SU_R}(3)$ is an approximate symmetry of QCD

  - broken spontaneously: $|0\rangle$ not invariant
  - broken explicitly: $\mathcal{L}_{\mathrm{QCD}}$ not invariant
    Symmetry broken by mass term $m_u\,\bar{u}u + m_d\,\bar{d}d + m_s\,\bar{s}s$,
    but since m_u, m_d, m_s are small, the breaking is weak

Light quark masses as perturbations

- Masses of the light quarks enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$
$$H_1 = \int d^3x \,\{m_u\, \bar{u}u + m_d\, \bar{d}d + m_s\, \bar{s}s\}$$

  H_0 describes u, d, s as massless, c, b, t as massive

  H_0 is invariant under SU(3)$_L \times$SU(3)$_R$

Expansion in powers of $m_u\,,m_d\,,m_s$	$\Longleftrightarrow$	Perturbation series in powers of H_1

- H_0 treats π, K, η as massless particles

  H_1 gives them a mass

Gell-Mann-Oakes-Renner formula

- First order perturbation theory yields:

$$M_\pi^2 = (m_u + m_d) \times |\langle 0|\, \bar{u}u\, |0\rangle| \times \frac{1}{F_\pi^2}$$

$\Uparrow$ explicit $\qquad\qquad$ $\Uparrow$ spontaneous

Gell-Mann, Oakes & Renner 1968

Coefficient: decay constant F_π

$$\langle 0|\, \bar{d}\gamma^\mu \gamma_5 u|\pi^+\rangle = i\, p^\mu \sqrt{2}\, F_\pi$$

Value of F_π is known from $\pi^+ \to \mu^+ \nu$

$\Rightarrow$ The main low energy properties of QCD can be understood on the basis of this formula

Pattern of lowest levels

- $M_\pi^2 = (m_u + m_d)\,B + O(m^2)$

$\Rightarrow$ The energy gap of QCD is small because $m_u\,, m_d$ happen to be small

- $M_{K^+}^2 = (m_u + m_s)\,B + O(m^2)$
  $M_{K^0}^2 = (m_d + m_s)\,B + O(m^2)$

$\Rightarrow$ M_K^2 is much larger than M_π^2, because m_s happens to be large compared to $m_u\,, m_d$

- Goldstone boson masses measure the strength of symmetry breaking

$\Rightarrow$ strongly violate SU(3)

- Check: first order perturbation theory also yields
  $M_\eta^2 = \frac{1}{3}\,(m_u + m_d + 4m_s)B + O(m^2)$

$\Rightarrow$ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 $\surd$

Isospin breaking

- The symmetry properties of the vacuum shield the pions from isospin breaking.
  - The difference between m_u and m_d only generates a tiny effect of order $M^2_{\pi^+} - M^2_{\pi^0} \propto (m_u - m_d)^2$.
  - The mass difference between π^0 and π^+ is due almost exclusively to electromagnetism.

  $\Rightarrow$ More easy to determine the mean mass $m_{ud} \equiv \frac{1}{2}(m_u + m_d)$ than the difference $m_u - m_d$.
- Estimate the e.m. self-energies with the Dashen theorem:

$$M^2_{K^+}\big|_{e.m.} = M^2_{\pi^+}\big|_{e.m} \qquad\qquad M^2_{\pi^0}\big|_{e.m.} = M^2_{K^0}\big|_{e.m.} = 0$$

Quark mass ratios

- Solve the tree level mass formulae for the ratios $m_s : m_{ud}$ and $m_u : m_d$:

$$\frac{m_s}{m_{ud}} = \frac{M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}}{M^2_{\pi^0}} = 25.9$$

Weinberg 1977

$$\frac{m_u}{m_d} = \frac{M^2_{K^+} - M^2_{K^0} + 2M^2_{\pi^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.56$$

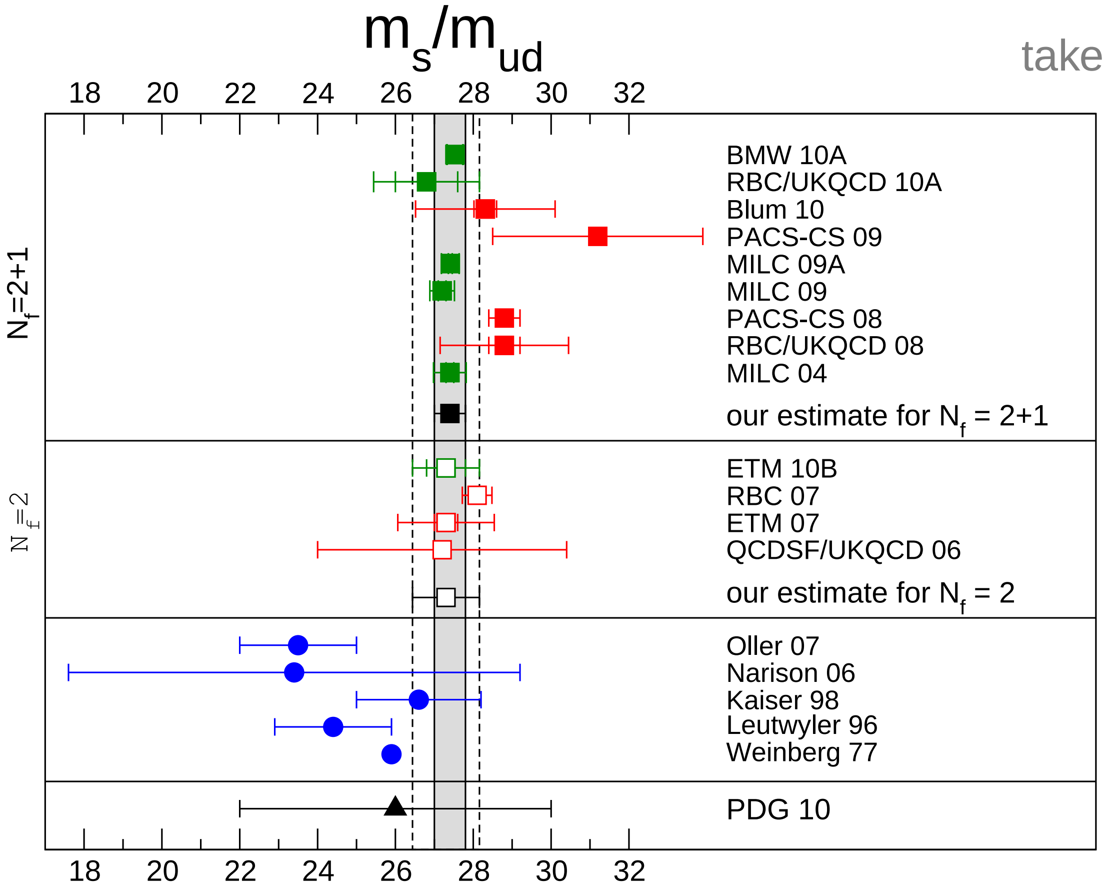
- Low energy theorems, valid to leading order of the chiral expansion.
  Corrections from higher orders ? Could they strongly modify the numerical result ?

  What is the uncertainty to be attached to these predictions ?

Lattice

- Very significant progress achieved in simulation of QCD on a lattice.

taken from FLAG review 2011



current lattice average:

$$\frac{m_s}{m_{ud}} = 27.4 \pm 0.4$$

accuracy reached: 1.5 %

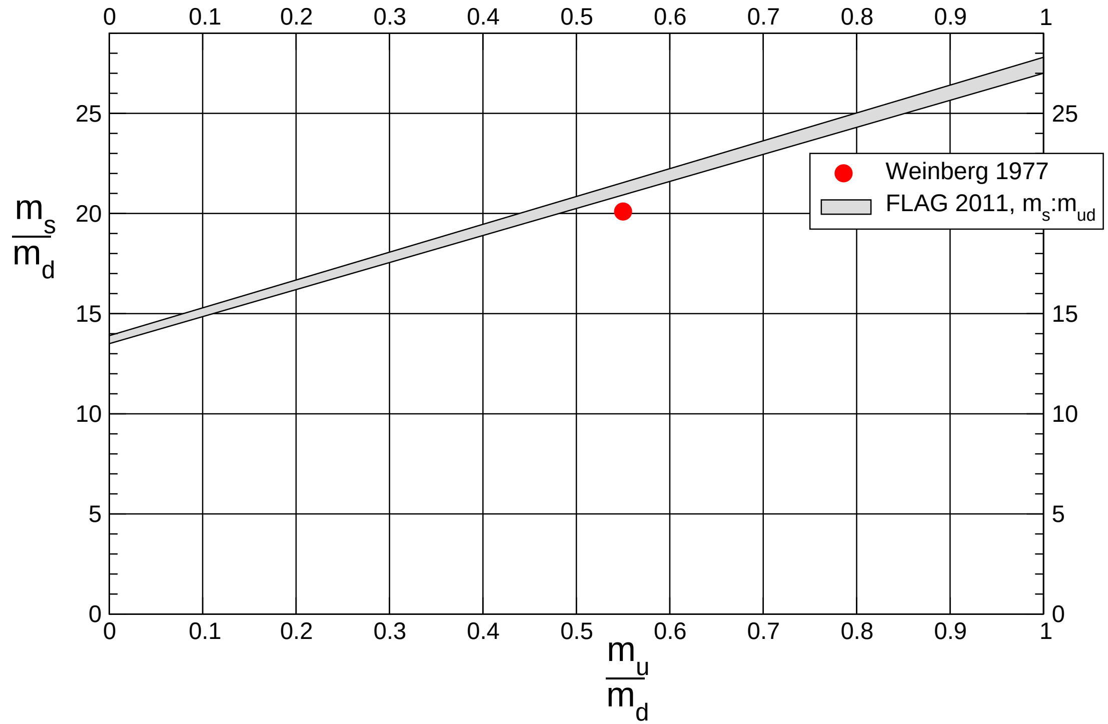
27.4 = 25.9 + 1.5

⇑

higher orders

$\Rightarrow$ correction is small, leading term of chiral perturbation series dominates

Lattice



- Most lattice calculations are done in pure QCD.
- For $m_s : m_{ud}$, this is a good approxiation, because the uncertainties in the violations of the Dashen theorem do not strongly affect this ratio.
- For $m_u : m_d$, the situation is different. Lattice simulations of QCD + QED cannot be done with the same level of confidence as for QCD alone. The e.m. self energies are evaluated in the quenched approximation $\longrightarrow$ not all systematic errors are under control.

Low energy theorem valid to NLO

- The lattice result for $m_s : m_{ud}$ determines the size of the correction in the relation

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + m_{ud}}{m_u + m_d}\left\{1 + \Delta_M\right\}$$

$$m_s : m_{ud} = 27.4 \pm 0.4 \quad \Rightarrow \quad \Delta_M = -0.053 \pm 0.013.$$

- Remarkably, chiral symmetry implies that the correction of NLO in the ratio of mass splittings is the same:

$$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - m_{ud}}\left\{1 + \Delta_M + O(\mathcal{M}^2)\right\}$$

Hence the quark mass ratio

$$Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}$$

is given by a ratio of meson masses, up to corrections of NNLO:

$$Q^2 = \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2} \cdot \frac{M_K^2}{M_\pi^2}\left\{1 + O(\mathcal{M}^2)\right\}$$

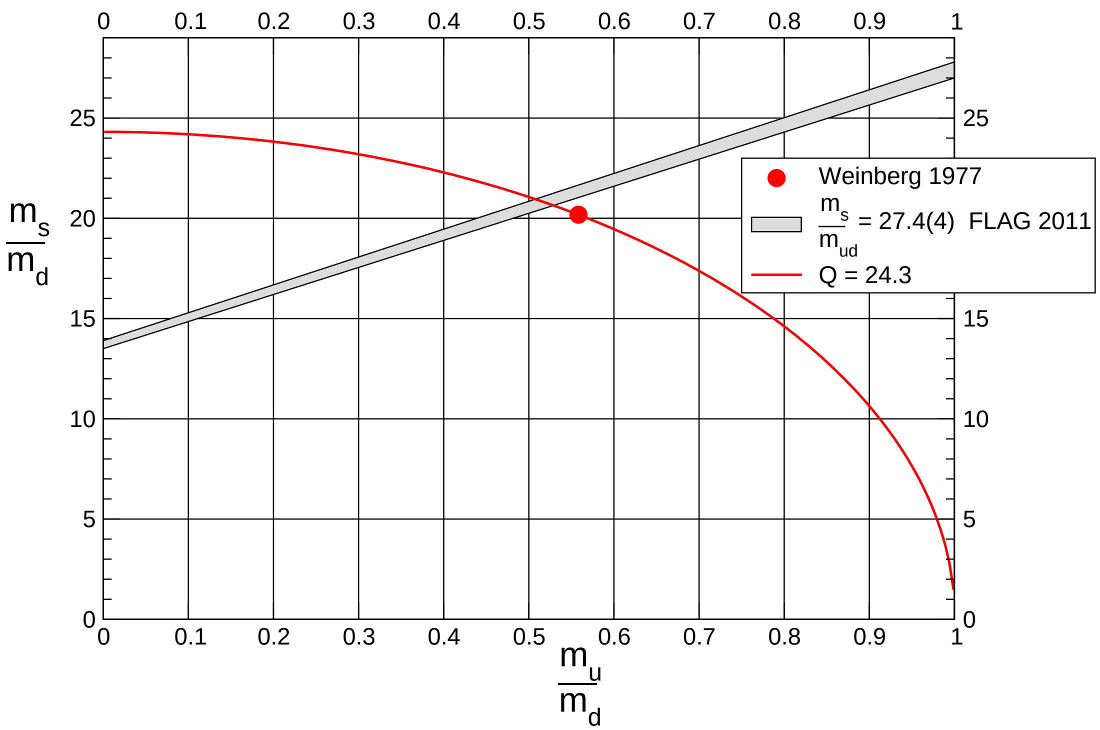
Gasser & L. 1985

Consequences of the low energy theorem for Q

- Insert Weinberg's leading order ratios $\Rightarrow$ Q = 24.3.
- Q^2 is a ratio of quark mass squares

$\Rightarrow$ a given value of Q imposes a homogeneous quadratic constraint on $m_u\,, m_d\,, m_s$

$\Rightarrow$ represents an ellipse in the plane of the quark mass ratios:



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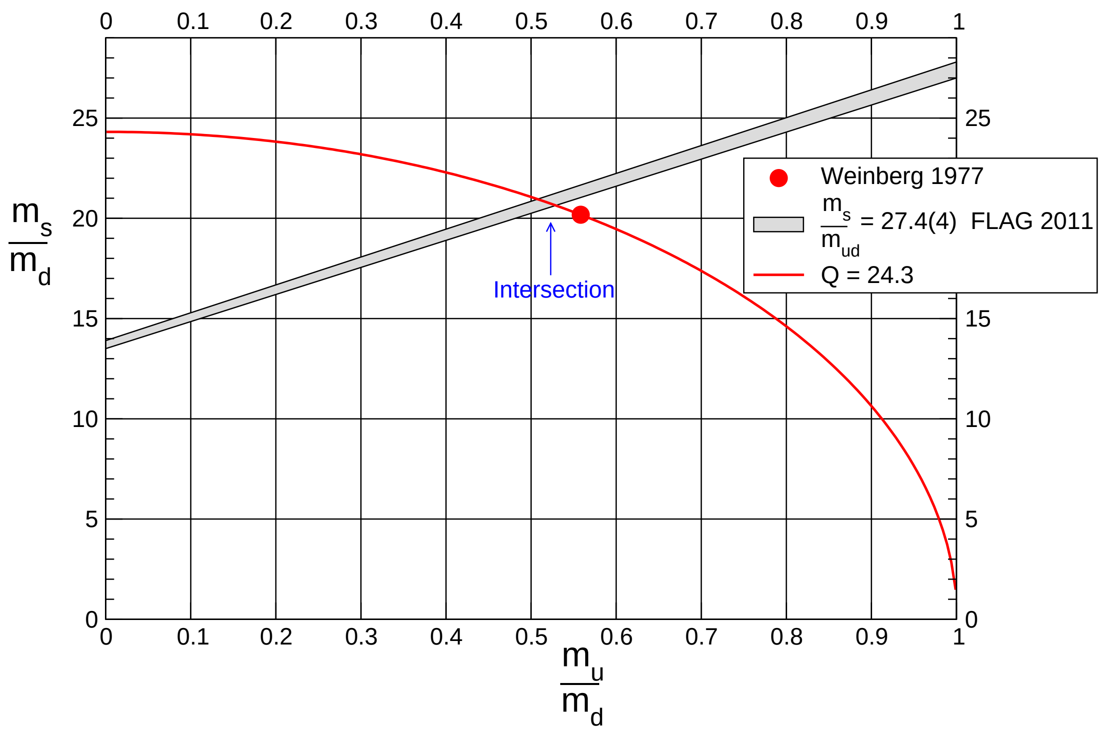


- Critical input here is the "Dashen theorem": Weinberg's estimates for the quark mass ratios account for QED only to LO.

$\eta \to 3\pi$

- The decay $\eta \to 3\pi$ provides a better handle on Q than the mass splitting between K^+ and K^0, because the e.m. interaction is suppressed (Sutherland's theorem).
  - For $e = 0$ and $m_u = m_d$, isospin is conserved, hence G-parity is conserved. In this limit, the η is a stable particle: $G_\eta = 1$, $G_\pi = -1$.
  - $\Rightarrow$ Since the e.m. contributions are tiny, the transition amplitude is to a very good approximation proportional to $(m_u - m_d)$.
  - Parameter free prediction for the leading term of the chiral perturbation series:

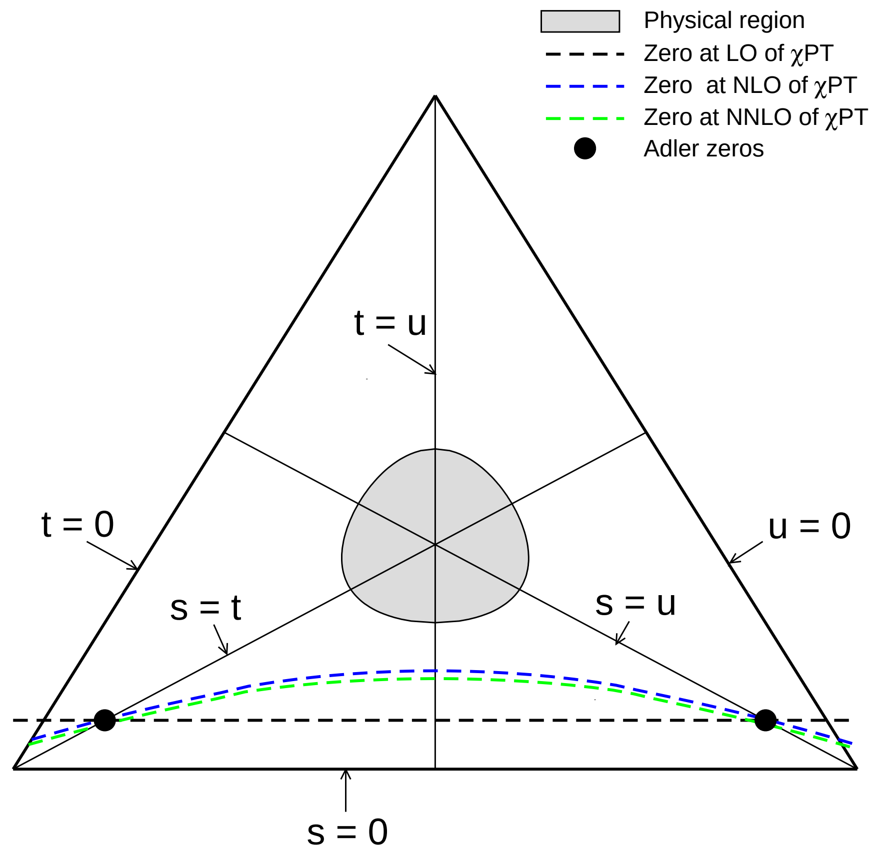
$$A(\eta \to \pi^+\pi^-\pi^0) = -\frac{\sqrt{3}}{4} \cdot \frac{m_d - m_u}{m_s - m_{ud}} \cdot \frac{s - \frac{4}{3}M_\pi^2}{F_\pi^2}$$

  - Compare leading term in the chiral expansion of the $\pi\pi$ scattering amplitude:

$$A(\pi\pi \to \pi\pi) = \frac{s - M_\pi^2}{F_\pi^2}$$

- In both cases, the leading term is linear in s and contains an Adler zero
  $s_A = M_\pi^2$ for $\pi\pi$ scattering $\qquad$ $s_A = \frac{4}{3}M_\pi^2$ for η decay

Mandelstam triangle, Adler zeros



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  $s_A = M_\pi^2$ for $\pi\pi$ scattering $\qquad$ $s_A = \frac{4}{3}M_\pi^2$ for η decay
- The analytic structure of the two amplitudes is very similar.
- In both cases, the higher order contributions of the chiral perturbation series are dominated by the final state interaction among the pions.

One loop

- Most remarkable property of the one loop representation: expressed in terms of F_π, M_π, M_K, M_η, Q, all LECs except L_3 drop out. Gasser & L. 1985

$$A(\eta \to \pi^+\pi^-\pi^0) = -\frac{1}{Q^2} \cdot \frac{M_K^2(M_K^2 - M_\pi^2)}{3\sqrt{3}M_\pi^2 F_\pi^2} \cdot M(s,t,u)$$

- Moreover, L_3 concerns the momentum dependence of the amplitude, can be determined quite well from $\pi\pi$ scattering.

$\Rightarrow$ At one loop, the result for the rate is of the form

$$\Gamma_{\eta \to \pi^+\pi^-\pi^0} = \frac{C}{Q^4} \qquad\qquad Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}$$

where C is a known constant $\Rightarrow$ Q can be determined from the observed rate.

- The main problem is not the uncertainty in L_3, but the contributions from higher orders. In 1985, we estimated the uncertainty in the result for Q at

$$\frac{1}{Q^2} = (1.9 \pm 0.3) \cdot 10^{-3} \qquad \leftrightarrow \qquad Q = 22.9^{+2.1}_{-1.6}$$

Gasser & L. 1985

- The result is consistent with the value $Q = 24.3$ obtained from the kaon mass difference with the Dashen theorem, but the uncertainties are large.

Dispersion theory

- The structure of the decay amplitude is governed by the final state interaction. Standard method for the analysis of this interaction: dispersion theory.
- Up to and including NNLO, the amplitude can be represented in terms of 3 functions of a single variable: Fuchs, Sazdjian and Stern 1993

$$M(s,t,u) = M_0(s)+(s-u)M_1(t)+(s-t)M_1(u)+M_2(t)+M_2(u)-\tfrac{2}{3}M_2(s)$$

(discontinuities from partial waves with $\ell \geq 2$ start contributing only at N^{3}LO).

- The dispersion relations obeyed by the three functions can be brought to the form

$$M_I(s) = \Omega_I(s)\left\{P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^\infty ds'\,\frac{\sin\delta_I(s')\hat{M}_I(s')}{|\Omega_I(s')|s'^n(s'-s)}\right\} \qquad I=0,1,2$$

where $\delta_0(s), \delta_1(s), \delta_2(s)$ are the S- and P-wave phase shifts of $\pi\pi$ scattering and

$$\Omega_I(s) \equiv \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds'\,\frac{\delta_I(s')}{s'(s'-s)}\right\}$$

is the corresponding Omnès factor.

Anisovich & L. 1996

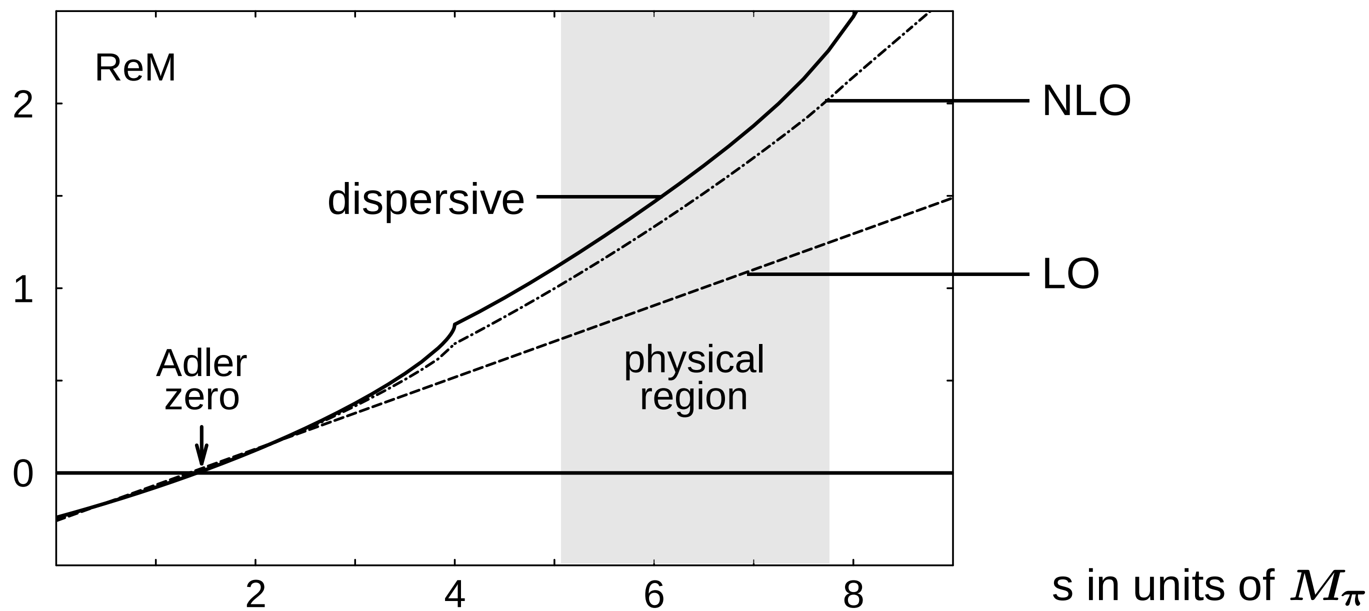
The polynomials $P_0(s), P_1(s), P_2(s)$ collect the subtraction constants.

$\Rightarrow$ S- and P-wave phase shifts of $\pi\pi$ scattering are needed. If these are known, dispersion theory fixes the amplitude up to the subtraction constants.

Dispersive analysis of η decay

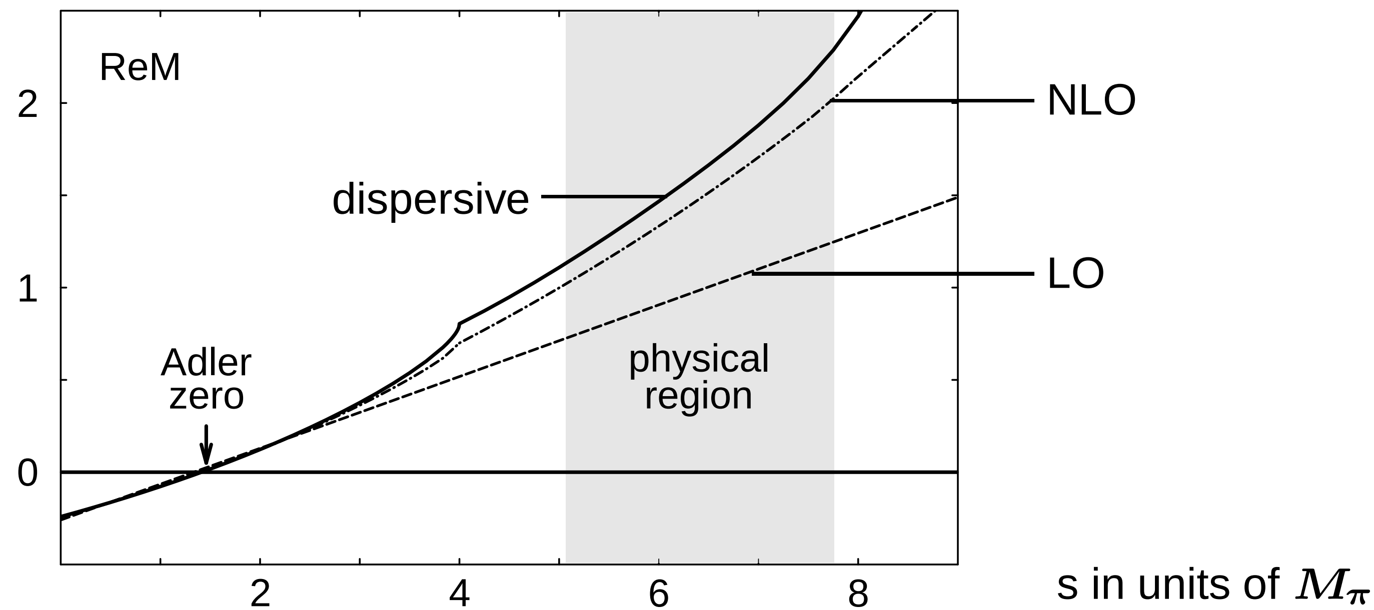
- Main difference to $\pi\pi$ scattering: the subtraction constants relevant for $\eta \to 3\pi$ cannot be predicted to the same precision.
  - Can analyze $\pi\pi$ scattering by treating only m_u and m_d as small: SU(2)×SU(2)
  - In η decay, need to treat m_s as an expansion parameter as well: SU(3)×SU(3)
  - Only the occurrence of an Adler zero follows from SU(2)×SU(2) symmetry alone.
- The subtraction constants can be estimated by comparing the dispersive and chiral representations at small values of s, t or u and requiring the occurrence of an Adler zero at the proper place.

Anisovich & L. 1996



Dispersive analysis of η decay

Anisovich & L. 1996



$\Rightarrow$ Final state interaction amplifies the transition.

- Effect of the higher order contributions on the result for Q is modest:

  $Q = 22.4 \pm 0.9$ Kambor, Wiesendanger & Wyler 1996

  $Q = 22.7 \pm 0.8$ Anisovich & L. 1996

  Confirms the one loop result, $Q = 22.9^{+2.1}_{-1.6}$, uncertainty reduced by a factor of 2.

- KWW also investigated the transition $\eta \to 3\pi^0$, predicted the slope of the corresponding Dalitz plot and showed that the result for the branching ratio $\Gamma_{\eta\to 3\pi^0}/\Gamma_{\eta\to\pi^+\pi^-\pi^0}$ is consistent with experiment.

Recent work on $\eta \longrightarrow 3\pi$

- In the meantime, the experimental situation improved a lot: KLOE, MAMI, WASA.
- At low energies, the $\pi\pi$ phase shifts are now known to remarkable accuray:
  - Low energy precision experiments (E865, NA48, DIRAC).
  - Low energy theorems for scattering lengths.
  - Dispersion theory (Roy equations).
- Simulations of QCD on a lattice now reach sufficiently small quark masses. Powerful source of information, in particular also for the quark masses.
- For η decay, χPT has been worked out to NNLO. Bijnens & Ghorbani 2007
- At the precision reached, isospin breaking needs to be accounted for. Ditsche, Kubis & Meissner 2009
- Nonrelativistic effective theory. Gullström, Kupsc & Rusetsky 2009 Schneider, Kubis & Ditsche 2011
- Improved dispersive analysis, comparison with experiment
  - Diploma work of Manuel Walker 1998, PhD thesis of Stefan Lanz 2011. Thorough investigation in this framework is close to completion. Colangelo, Lanz, L. & Passemar
  - Entirely different approach: Kampf, Knecht, Novotny & Zdrahal 2011

Excursion: $\pi\pi$ scattering

- The interaction among the pions plays a central role at low energies, particularly when looking for physics beyond the Standard Model (precision).
- Dispersion theory of the $\pi\pi$ scattering amplitude: Roy equations. Roy 1971
- In the isospin limit, the Roy equations are exact.
  Inelastic processes such as $\pi\pi \longrightarrow K\bar{K} \longrightarrow \pi\pi$ are explicitly accounted for.
- Dispersion relations involve two subtractions, integrals converge rapidly.
  Subtraction constants can be identified with the S-wave scattering lengths, a_0, a_2.

$\Rightarrow$ If a_0, a_2 are known, the scattering amplitude can be calculated very accurately.

Ananthanarayan, Caprini, Colangelo, Gasser, L.
Descotes, Fuchs, Girlanda, Moussallam, Stern
García-Martín, Kamiński, Nebreda, Peláez, Ríos, Ruiz de Elvira, Ynduráin

Scattering lengths

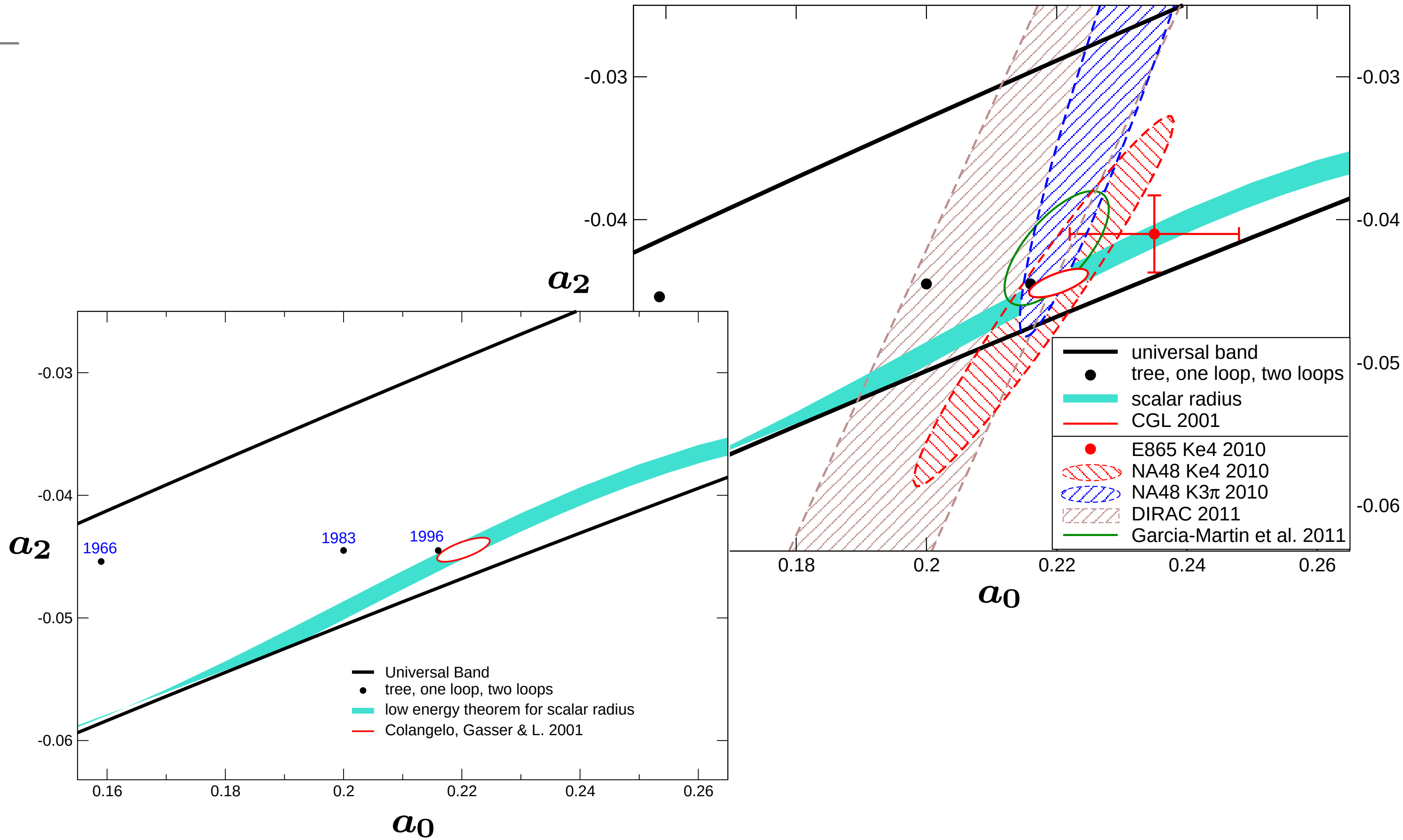
- Prediction at leading order of χPT:

$$a_0 = \frac{7M_\pi^2}{32\pi F_\pi^2} = 0.16, \qquad a_2 = -\frac{M_\pi^2}{16\pi F_\pi^2} = -0.045$$

Weinberg 1966

- χPT allows to analyze the contributions of higher order. Chiral expansion has been worked out to NNLO. Using dispersion theory, this leads to remarkably sharp predictions for a_0, a_2, which triggered new low energy precision experiments:
  - $\pi^+\pi^-$ atoms, DIRAC.
  - $K^\pm \to \pi^0\pi^0\pi^\pm$, $K^0 \to \pi^0\pi^0\pi^0$: cusp near threshold, NA48/2.
  - $K^\pm \to \pi^+\pi^- e^\pm \nu$ data: E865, NA48/2.

Experimental tests of the prediction



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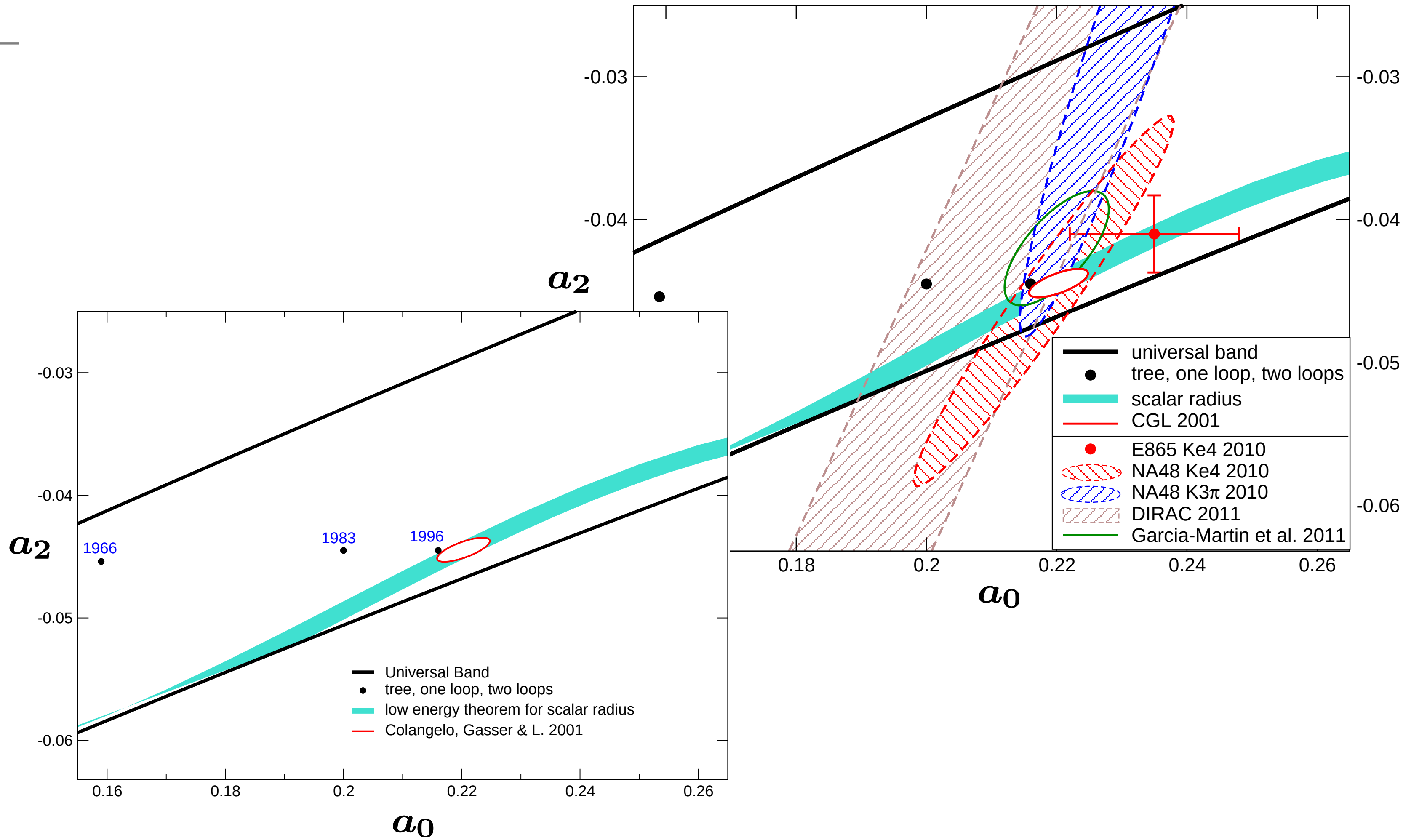
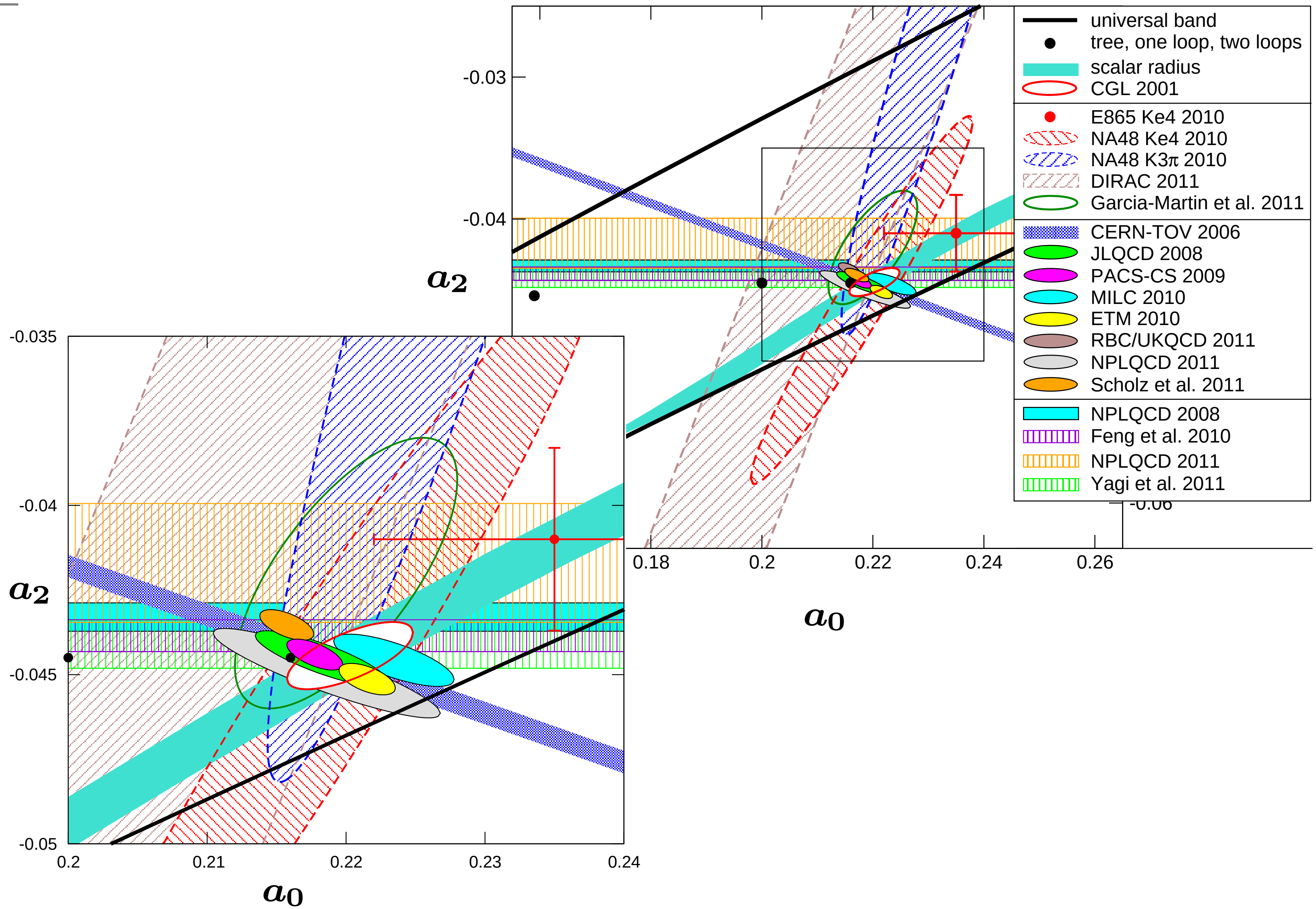


- Uncertainty in χPT prediction for a_0, a_2 is dominated by the uncertainty in the relevant coupling constants of the effective Lagrangian at NLO. These can now reliably be determined on the lattice, from the quark mass dependence of M_π and F_π.
- Direct determination of a_2 via dependence of the energy levels on the size of the box.

Compare the lattice results with prediction and experiment



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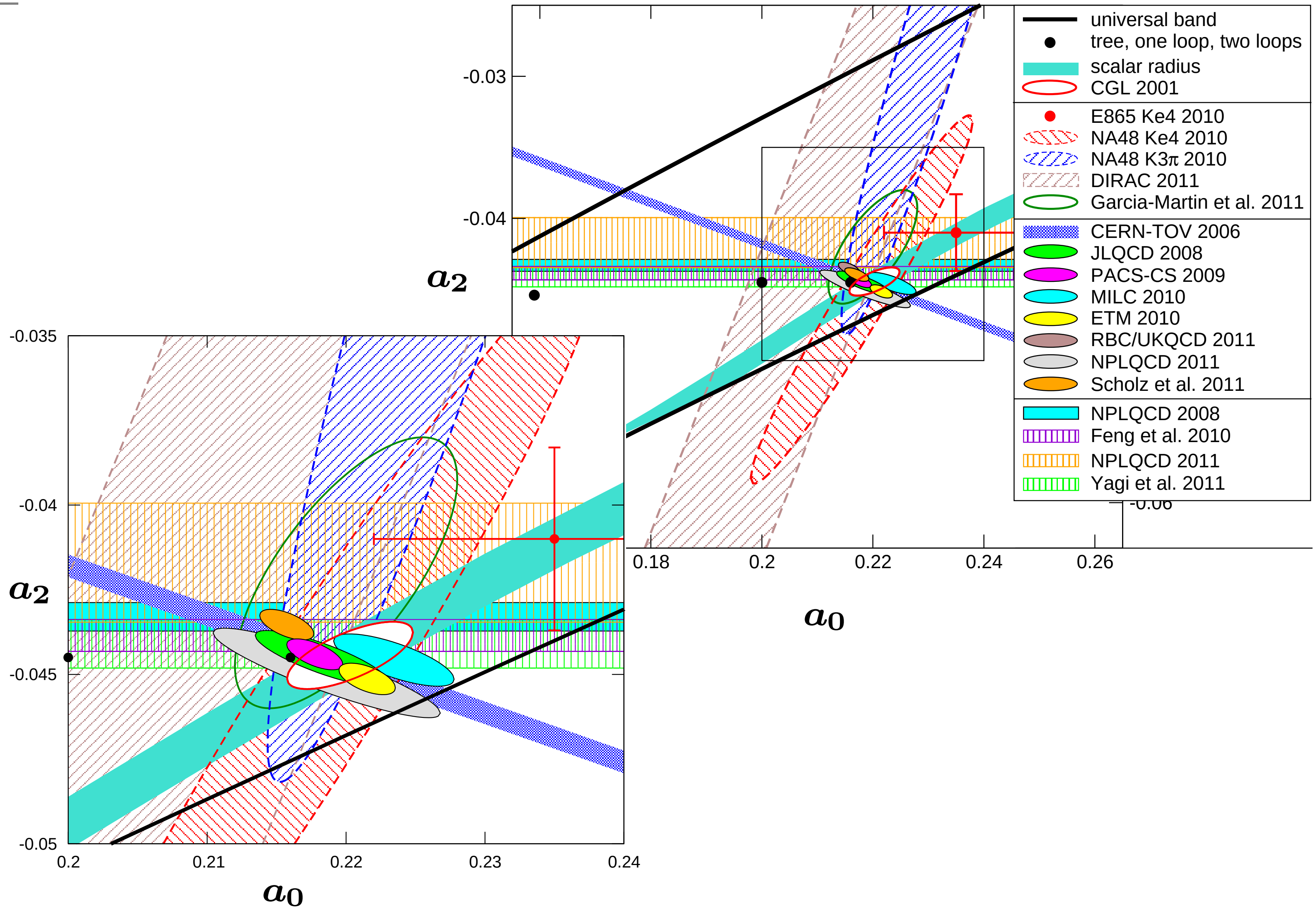


Compared to this, $\eta \longrightarrow 3\pi$ is yet an underdeveloped country.

Back to $\eta \longrightarrow 3\pi$

- Basic property of the dispersion relations: if the phase shifts are known, the amplitude is uniquely determined by the subtraction constants.
- If $M^{(1)}(s,t,u)$ and $M^{(2)}(s,t,u)$ are solutions, then $\lambda_1 M^{(1)} + \lambda_2 M^{(2)}$ is also a solution: the solutions form a linear space.

$\Rightarrow$ General solution is a linear superposition of basis functions. Number of independent basis functions is determined by the number of subtractions made.

- The subtraction constants are estimated with the following input:
  1. Measured Dalitz plot distributions of the charged and neutral decay modes.
     - Andrzej Kupsc (KLOE), Sergey Prakhov (MAMI) and Patrik Adlarson (WASA) kindly provided us with data tables.
     - In addition, we use the value for the slope of the Z-distribution of the neutral decay mode quoted by the PDG: $\alpha = -0.0317(16)$.
  2. The dispersive representation is glued onto χPT at small values of s, t or u, where the higher orders of the chiral perturbation series are smallest.

Input from χPT

- The isospin components of the amplitude are expanded in a Taylor series:

$$M_0(s) = a_0 + b_0 s + c_0 s^2 + \ldots$$
$$M_1(s) = a_1 + b_1 s + \ldots$$
$$M_2(s) = a_2 + b_2 s + c_2 s^2 + \ldots$$

- χPT provides estimates for the Taylor coefficients, which rely on SU(3)$\times$SU(3). Standard rule for the uncertainties therein: 20 to 30% at LO, 4 to 10% at NLO.

$\Rightarrow$ $a_0, b_0, c_0, a_1, b_1, a_2, b_2, c_2$ are known to an accuracy of 20 to 30%.

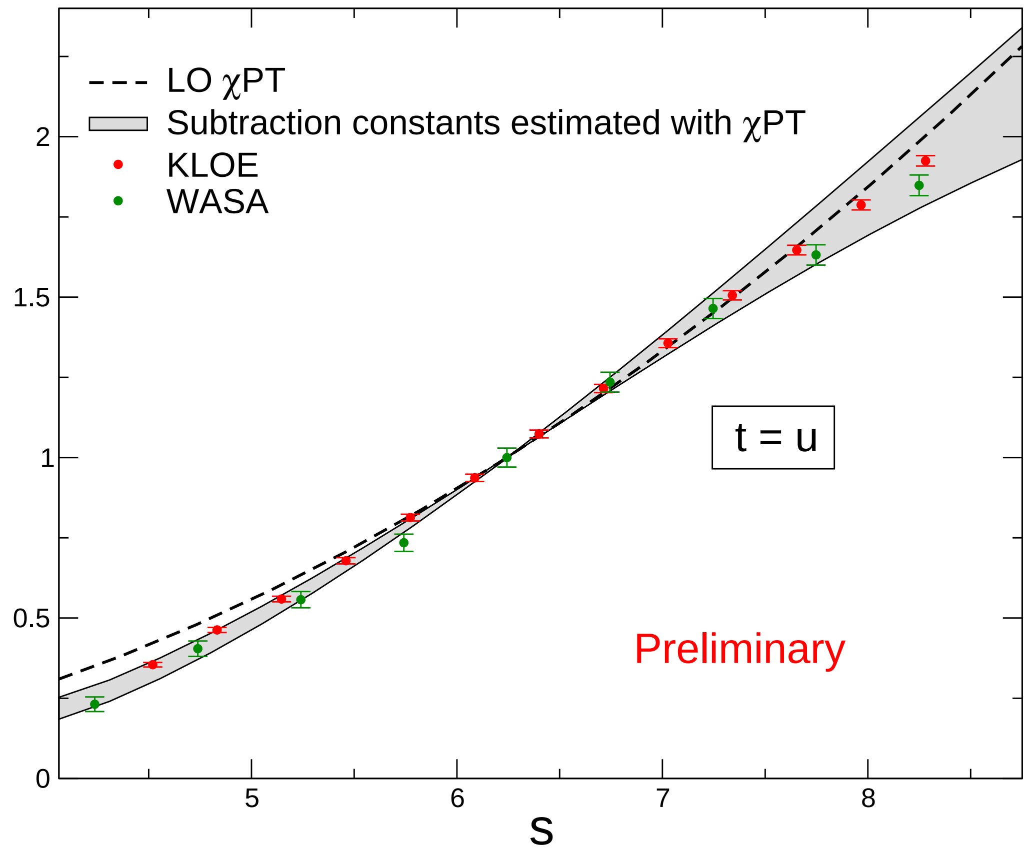
- Unitarity suppresses the imaginary parts of the Taylor coefficients. For these, the NNLO representation yields an estimate which does not involve any unknown LECs. Result is too small to matter $\Rightarrow$ the Taylor coefficients are approximately real.

- Adler zero along the line $s = u$: Re $M(s,t,u) = 0 \;\rightarrow\; s = s_A$

  Slope there $D_A = \partial_s \Big\{ \text{Re}\, M(s,t,u)\big|_{s=u} \Big\}$ at $s = s_A$

  The NLO corrections to the tree level predictions for s_A and D_A are known and are remarkably small. Since we do not know why that is so, we nevertheless attach an uncertainty of 10% to the χPT estimates for s_A and D_A.

Dalitz plot distribution of $\eta \to \pi^+\pi^-\pi^0$

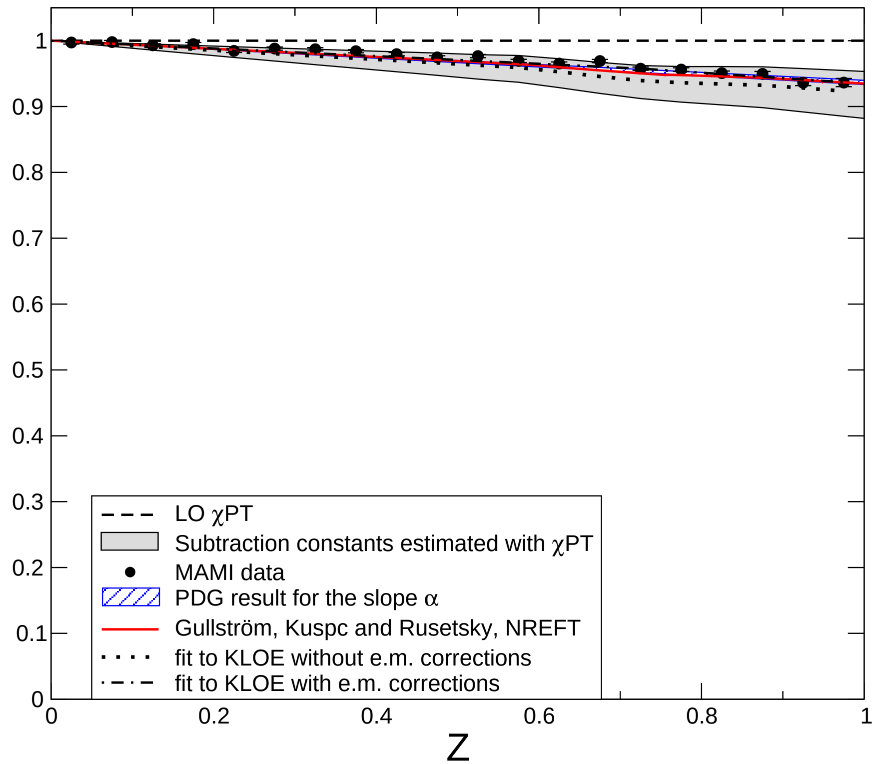


- The current algebra prediction is confirmed.
- Data are more precise than the theoretical estimates.
- Despite the small errors, the tension between KLOE and WASA is modest.
- The dispersive analysis of the final state interaction is consistent with the data.

$\Rightarrow$ Data can be used to reduce the uncertainties in the subtraction constants.

Z-distribution for $\eta \longrightarrow 3\pi^0$

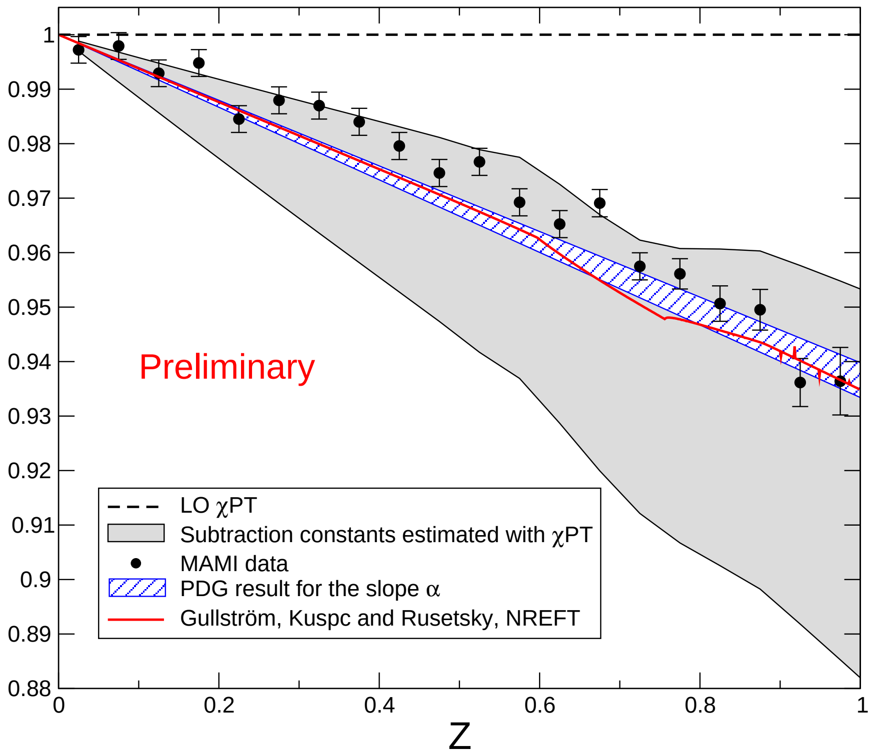
- At first sight, the situation in the neutral channel looks similar:



- Current algebra prediction is confirmed also in this channel.

Z-distribution for $\eta \to 3\pi^0$

- Same picture, focused on the vicinity of LO prediction:



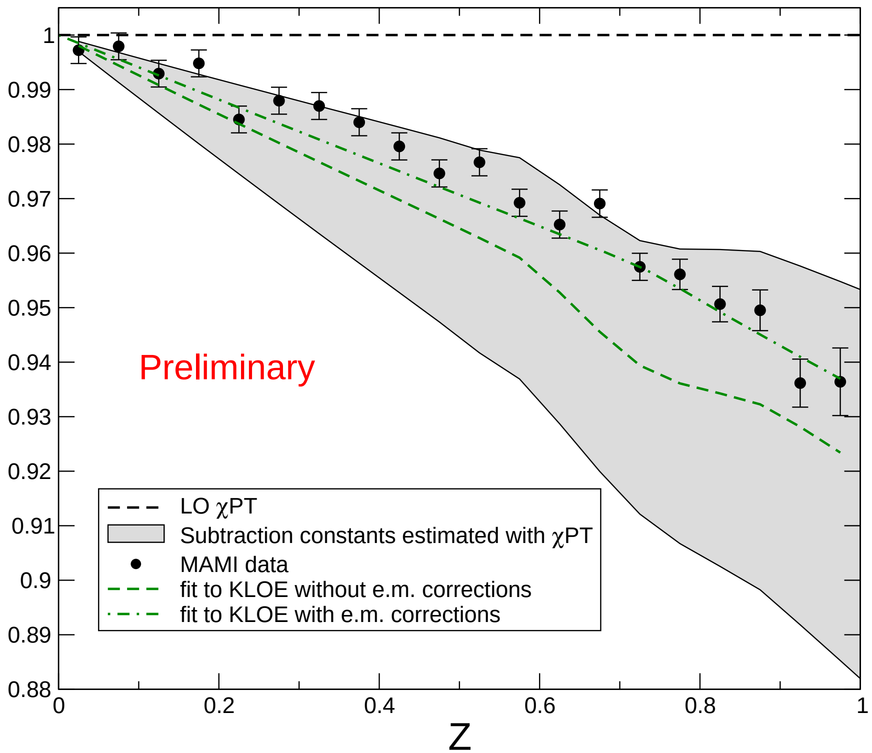
- Isospin symmetry fixes $A(\eta \to 3\pi^0)$ in terms of $A(\eta \to \pi^+\pi^-\pi^0)$.

  Gullström, Kupsc & Rusetsky 2009, Schneider, Kubis & Ditsche 2011

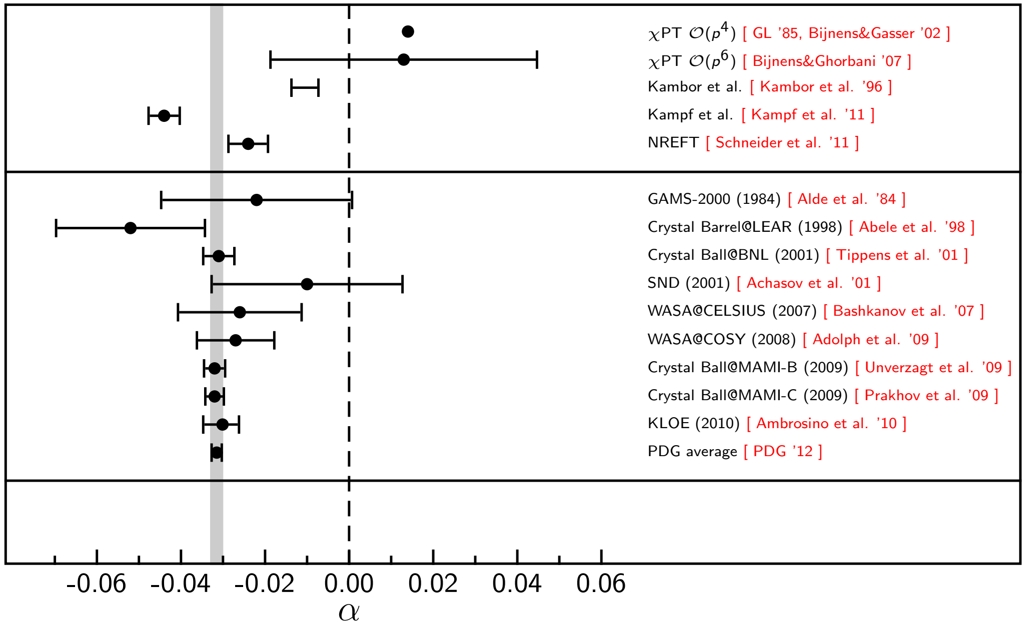
- Dispersion theory + χPT estimates for subtraction constants leave a lot of room.

Z-distribution for $\eta \longrightarrow 3\pi^0$

- At this accuracy, isospin breaking effects cannot be ignored:



Slope α of $\eta \to 3\pi^0$



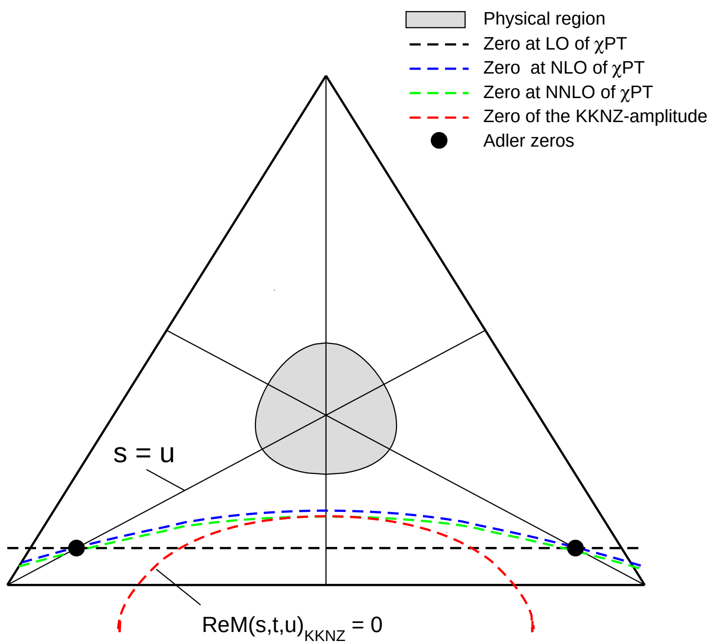
I thank Stefan Lanz for this table

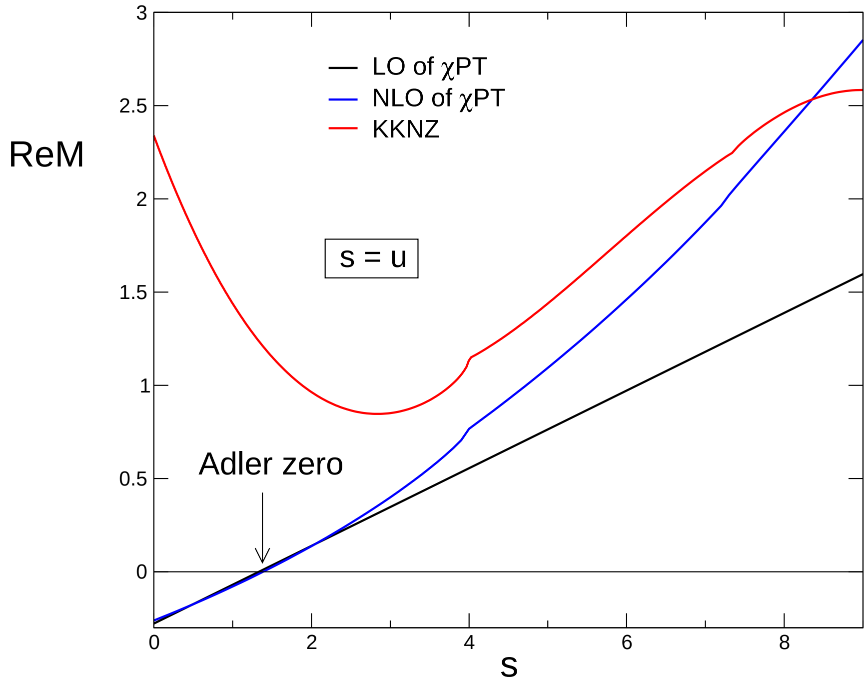
- At LO of χPT, α vanishes. The NLO correction is too small and has the wrong sign.
- The dispersive analysis of Kambor, Wiesendanger and Wyler did get the sign right, but the uncertainties were underestimated.
- Experimental situation is now clear, α is known to good accuracy.

$\Rightarrow$ The slope obtained recently by Kampf, Knecht, Novotny & Zdrahal is not correct. Origin of the problem: their representation is in conflict with SU(2)xSU(2).

Low energy theorem of SU(2)xSU(2)

- Adler zero on the line $s = u$: place where ReM(s,t,u) vanishes. In the limit $m_u = m_d = 0$, this zero sits at the lower left hand corner. If the quark masses are turned on, the zero moves by $\Delta s = O(M_\pi^2)$.
- The amplitude constructed by Kampf et al. does not have an Adler zero.

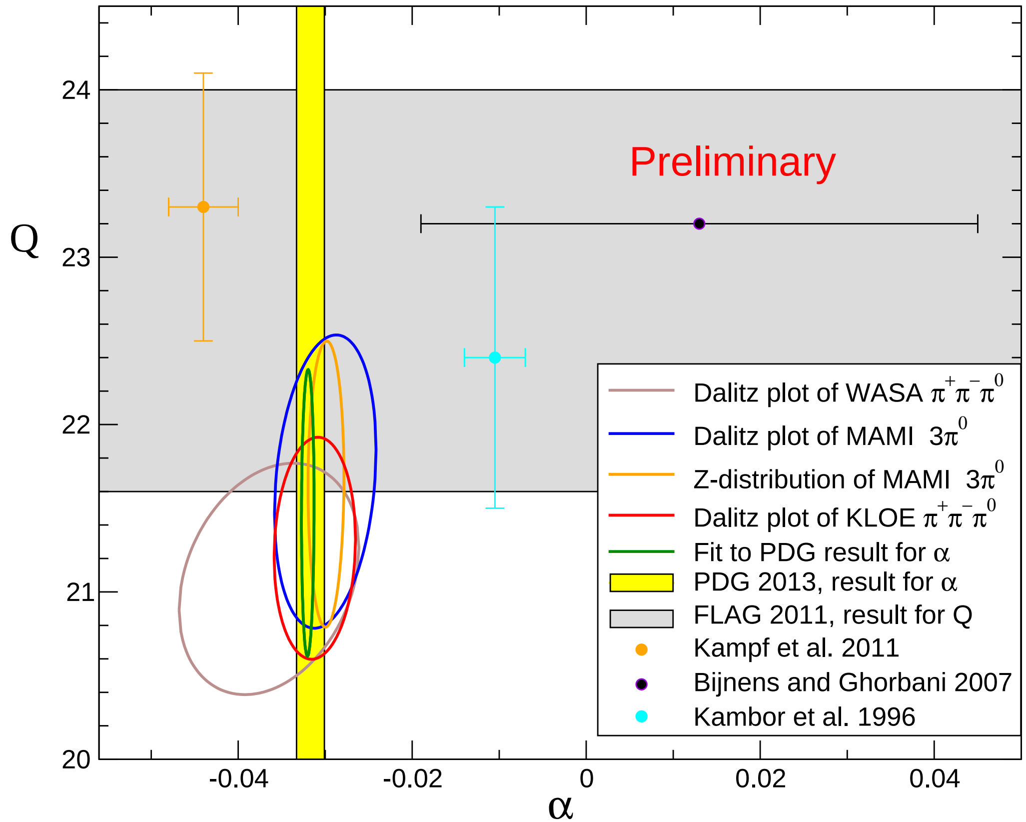




To be done

- The dependence of the result on the uncertainties in the input (phase shifts, subtraction constants, experimental errors) can be worked out explicitly, but we yet need to do this.
- Concerning the input used for the phase shifts, the Roy equations provide a reliable handle on the uncertainties – in their domain of validity: $\sqrt{s} \leq 1.15$ GeV.
- The dispersion integrals extend to ∞, but with the number of subtractions we are using, the contributions from the region above $K\bar{K}$ threshold are tiny.
- We approximate the isospin breaking effects by means of χPT, using the NLO representation of Ditsche, Kubis and Meissner.
- The value of Q can be determined either from the rate of the transition $\eta \rightarrow \pi^+\pi^-\pi^0$ or from $\eta \rightarrow 3\pi^0$. This offers a good test: evaluating the isospin breaking effects in this way, we indeed find that the two results agree.
- I refrain from offering quantitative results for Q and for m_u/m_d at this stage. A detailed report on our work, including a thorough error analysis is forthcoming.

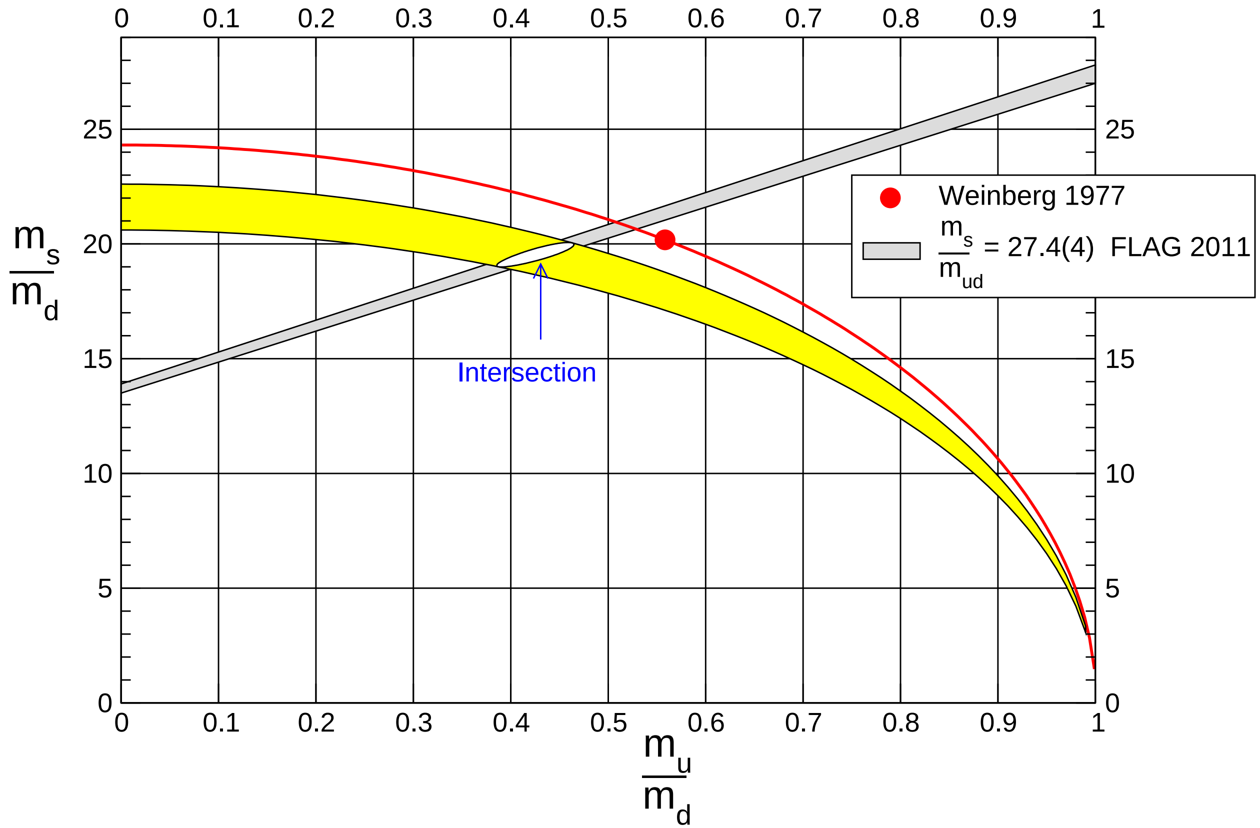
Some qualitative results of our dispersive analysis



I thank Emilie Passemar for some of the curves

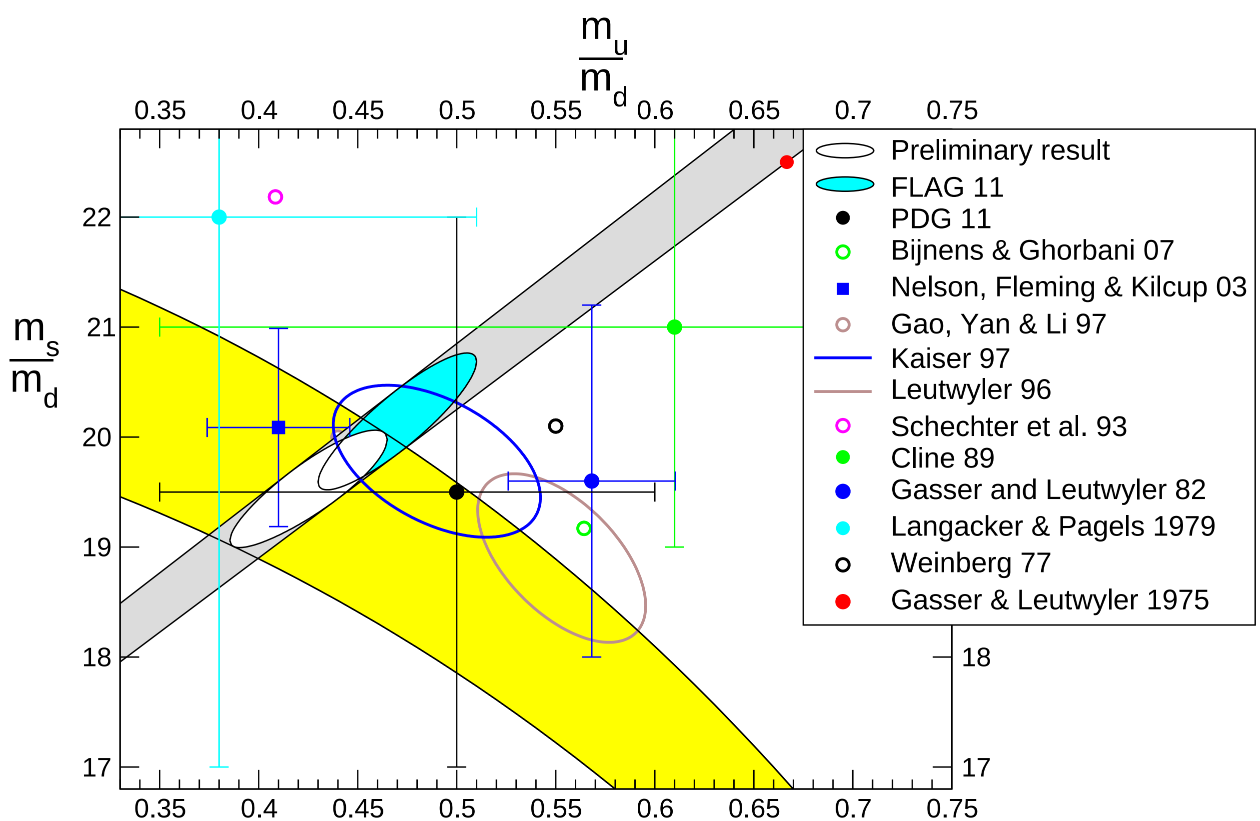
- The plot shows results for α and Q obtained from fits to the various data sets.
- It indicates that the various data sets are compatible.
- In particular, the value of α 'predicted' from the dispersive fit to the Dalitz plot of the charged mode is consistent with experiment.
- Note that in this plot, isospin breaking is not accounted for.

Consequence for the quark mass ratios



- Intersection moves to values of $\frac{m_u}{m_d}$ and $\frac{m_s}{m_d}$ that are somewhat smaller than those obtained with the LO mass formulae of Weinberg.

Comparison with lattice results



- The preliminary results for the ratios of quark masses are consistent with the lattice averages given in FLAG 2011, but tend to be somewhat smaller.