Particle Physics:
Low Energy, High Accuracy

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Pomeranchuk Memorial, June 3, 2013

- During the last 50 years, a remarkable development took place in our understanding of particle physics.
- I plan to first discuss some qualitative features which gradually emerged and then focus on the methods used to analyze the Standard Model at low energies.
- As a student, I first encountered the name of Pomeranchuk in connection with the high energy properties of cross sections.

  My talk concerns the opposite end of the energy scale.

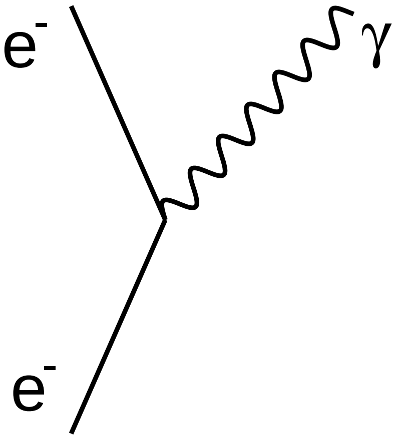
Qualitative aspects of the Standard Model

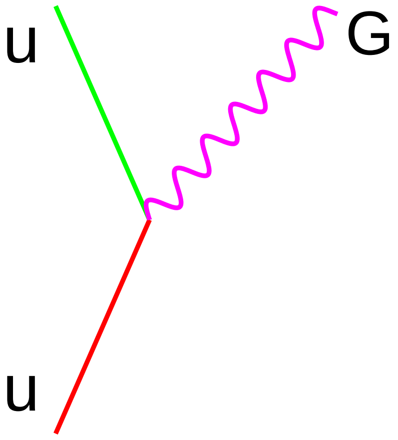
The Standard Model is a miracle:

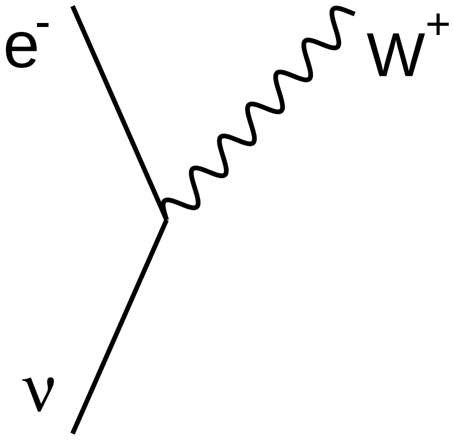
- Since a long time, we know that the microscopic world is governed by three types of interaction:

  strong, electromagnetic, weak
- These have qualitatively very different properties.
- All three generated by gauge fields ?

Gauge field interactions







electromagnetic	strong	weak
QED	QCD	QFD
charge	colour	flavour
photon	gluons	$W^\pm, Z$

Behaviour at short distance

- At short distances (1 TeV $\leftrightarrow 2 \cdot 10^{-19}$ m) all of the forces obey the inverse square law.

$$V = \text{constant} \times \frac{\hbar c}{r} \qquad \text{interaction energy}$$

- The constant is a pure number.

$\Rightarrow$ Interaction strength is fixed by 3 pure numbers.

e.m.	strong	weak
$\frac{e^2}{4\pi}$	$\frac{g_s^2}{4\pi}$	$\frac{g_w^2}{4\pi}$

Why are the three interactions so different ?

- strong $\simeq$ weak ??
- $\frac{1}{r}$ – potential describes an interaction of long range.

  strong and weak interactions are short range !
- Photons can be seen by eye, gluons not,

  etc. etc. etc.

Two features are responsible for the difference:

- Properties of the vacuum.
- Photons do not carry charge, but gluons carry colour.

Properties of the vacuum

$$\boxed{\text{vacuum = condensate of Higgs particles.}}$$

- The Higgs particles do not carry charge.

⇒ Photons do not notice these.

⇒ Vacuum is transparent for photons.

- The Higgs particles do not carry colour.

⇒ Gluons do not notice these.

⇒ Vacuum is transparent for gluons.

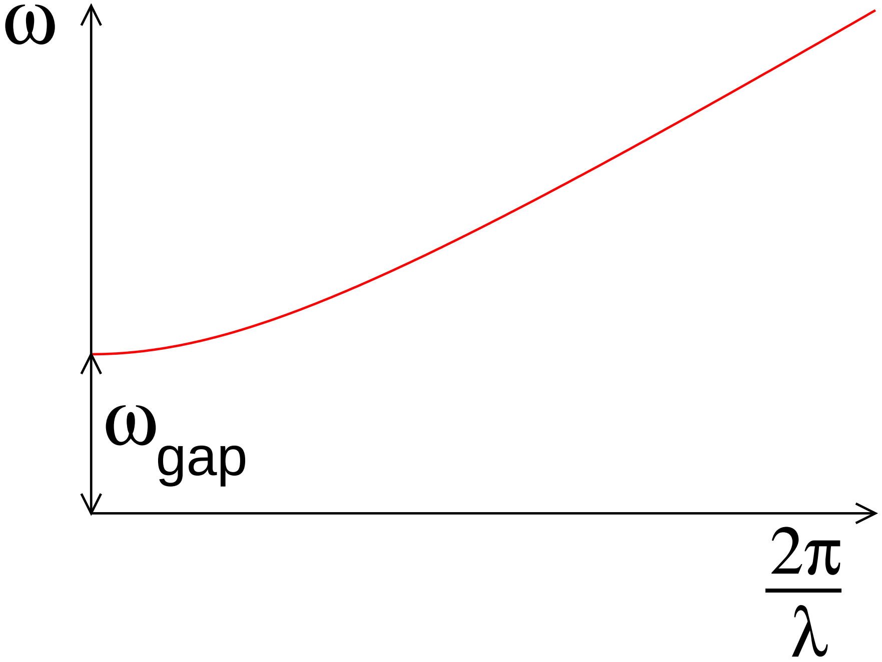
- The Higgs particles do carry flavour.

⇒ W,Z do take notice.

⇒ W,Z-waves of low frequency cannot propagate.

⇒ For such waves, the vacuum is opaque.

Frequency versus wave number



$$\hbar\omega_{\text{gap}} = E_{\text{gap}} = Mc^2$$

- $M_\gamma, M_{\text{G}} = 0 \Rightarrow$ γ, G have $v = c$
- $M_{\text{W}}, M_{\text{Z}} \neq 0 \Rightarrow$ W, Z have $v < c$
- Penetration depth for low frequency:

$$d = \frac{\hbar}{Mc} \qquad d_{\text{W}}, d_{\text{Z}} \sim 2 \cdot 10^{-18}\,\text{m}$$

Consequence for strength of weak interaction

- Interaction energy is reduced for $r \gtrsim d$:

$$\frac{g_{\mathrm{W}}^2}{4\pi r} \;\Rightarrow\; \frac{g_{\mathrm{W}}^2}{4\pi r} \cdot e^{-\frac{r}{d}}$$

Penetration depth of the weak interaction is small:

$$d = \frac{\hbar}{M_{\mathrm{W}} c} = 2.4542(9) \times 10^{-18}\ \mathrm{m}$$

⇒ Weak interaction is of short range.

- Effective strength at low energies:

$$\int d^3 r\, \frac{g_{\mathrm{W}}^2}{4\pi r} \cdot e^{-\frac{r}{d}} \;=\; g_{\mathrm{W}}^2 d^2$$

⇒ At low energies, the weak interaction is weak.

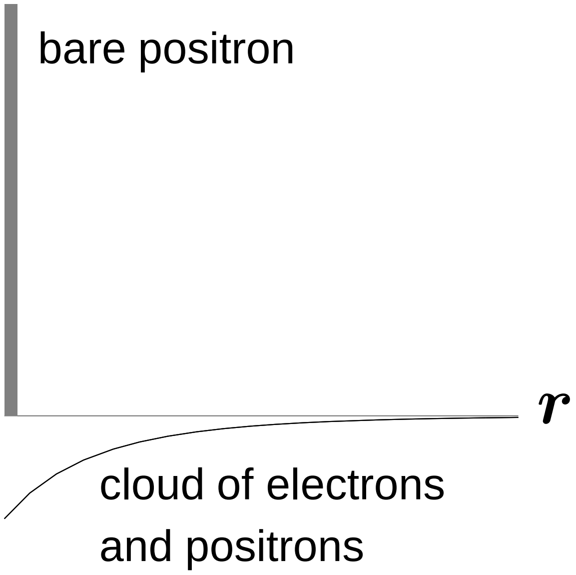
What makes the difference between QED and QCD ?

- Photons do not have charge.
- Gluons do have colour.

⇒ The e.m. and strong interactions behave differently at low energies.

Compare structure of leptons and quarks

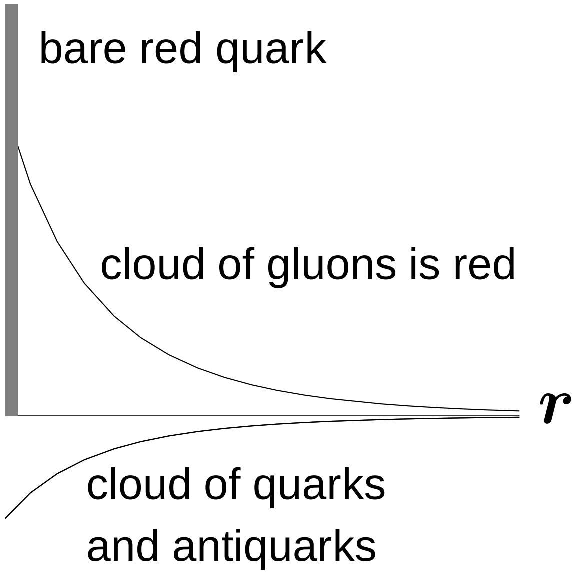
QED
density of charge



$$\boxed{e < e|_{\text{bare}}}$$

vacuum shields charge

QCD
density of colour



$$\boxed{g_{\text{s}} > g_{\text{s}}|_{\text{bare}}}$$

vacuum amplifies colour

⇒ The electromagnetic and strong interactions polarize the vacuum very differently.

Consequence of shielding/amplification

- Vacuum amplifies gluonic field of a coloured source.
  Vacuum reduces electric field of a charged source.
- The difference has dramatic consequences:
  although the Lagrangians of QCD and QED
  are very similar, the properties of the strong
  and electromagnetic interactions are totally different.
- Field energy surrounding isolated quark = ∞
  only colour neutral states have finite energy.

$\Rightarrow$ colour is confined.

- Field energy surrounding a charged particle is finite.

$\Rightarrow$ charge is not confined.

- nuclear forces = van der Waals forces of QCD.

Interaction at large distances, low energies

QED remains weak

$$\alpha_e = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

photons, leptons
nearly decouple

QCD becomes strong

$$\alpha_s = \frac{g_s^2}{4\pi} \simeq 1$$

gluons, quarks
confined

$\Rightarrow$ In QED, perturbation theory works at low energies.
Spectrum of states can be seen in the Lagrangian.

$\Rightarrow$ In QCD, perturbation theory fails at low energies.
Spectrum of states cannot be seen in the Lagrangian.
Fields in the Lagrangian $\not\Leftrightarrow$ observed particles.

This is why it took so long to realize that the strong interactions originate in a gauge field.

Pauli (1953): Kaluza-Klein theory in more than 5 dimensions $\longrightarrow$ nonabelian gauge fields.
He did not pursue this, because he thought that gauge invariance entails massless particles.

Scale dependence of the coupling constants

- Values of e and g_s depend on scale: $e(\mu)$, $g_s(\mu)$.
- If μ is increased: $e(\mu)$ rises, $g_s(\mu)$ falls.

$$g_s(\mu)^2 \propto \frac{1}{\ell n\,(\frac{\mu}{\Lambda_{\text{QCD}}})}$$

- $g_{\text{W}}(\mu)$ also falls.
- Possibly, the e.m., strong and weak interactions have the same strength at $r \sim 10^{-30}$ m (GUT).

Summary: Standard Model at low energies

- Vacuum is opaque for $W^{\pm}$, Z.

⇒ For $E \ll M_{\mathrm{W}}c^2,\ M_{\mathrm{Z}}c^2$, the weak interaction is frozen.

SM reduces to QED + QCD.

- QED is infrared stable, characterized by pure number.

  The number happens to be small: $\dfrac{e^2}{4\pi} \simeq \dfrac{1}{137}$

- At low energies, the weak and e.m. interactions can be treated perturbatively.

  Effects from g_{w} are tiny, those from e are small.

⇒ At low energies: SM = QCD + corrections.

Pièce de résistance: QCD

- Parameters in the Lagrangian:

$$\boxed{g_s, \theta, m_\mathrm{u},\; m_\mathrm{d},\; m_\mathrm{s},\; m_\mathrm{c},\; m_\mathrm{b},\; m_\mathrm{t}}$$

- Pattern of quark masses is bizarre, not understood.
- At low energies, $g_s \leftrightarrow \Lambda_\mathrm{QCD}$ and $m_\mathrm{u}+m_\mathrm{d}$ play the key role.
- At high energies, quarks and gluons barely interact. QCD is "asymptotically free".

$\Rightarrow$ Fields $\in$ Lagrangian are relevant degrees of freedom. Interaction can be treated as a perturbation.

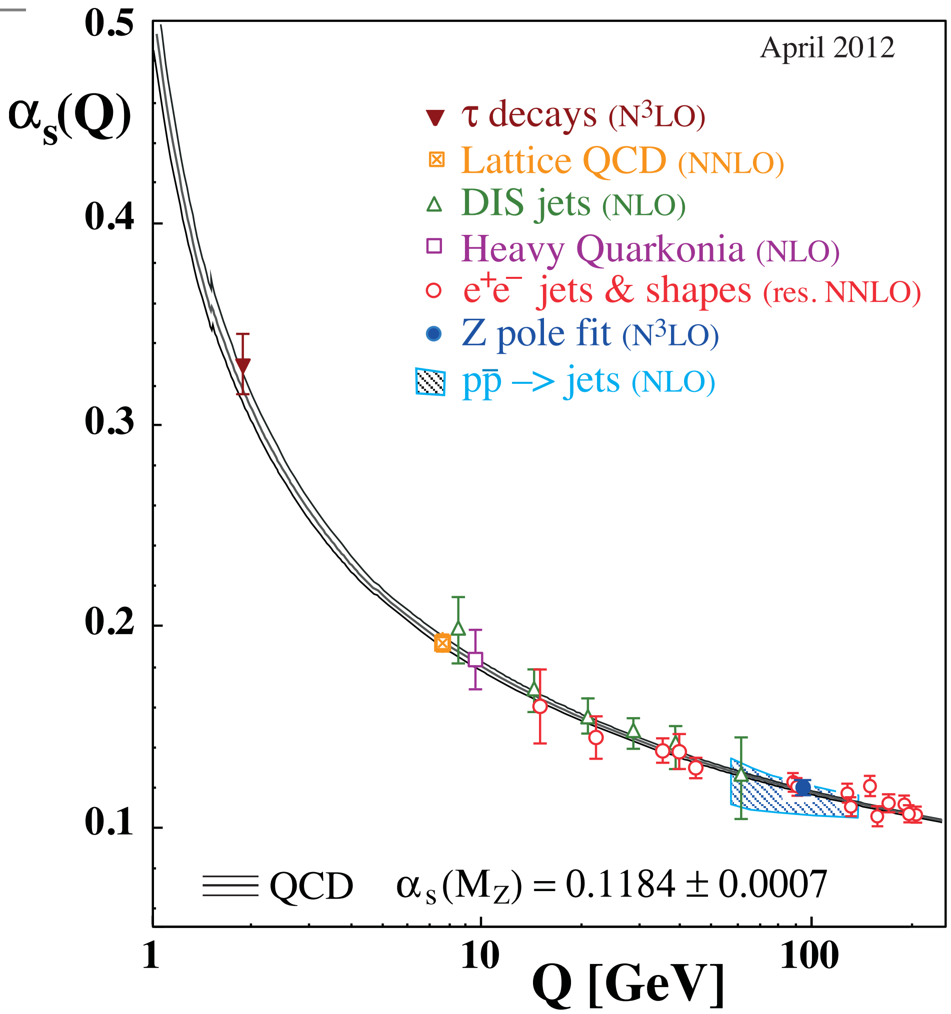
Perturbative QCD

- At short distances, QCD can be analyzed by means of perturbation theory.
- At high energies, inclusive processes, such as

  $e + p \longrightarrow e +$ anything or $e^+ + e^- \longrightarrow$ hadrons

  can be calculated in powers of $\alpha_s(\mu)$, in terms of the running quark masses $m_u(\mu), \ldots, m_t(\mu)$.
- Main characteristic of the jets generated by quarks or gluons with large transverse momenta can be analyzed within perturbation theory.
- Effective theories for the region where some of the loop momenta become soft or collinear.
- Very successful and very active field of research, useful in particular also for processes where QCD merely generates unwanted background.

Running coupling constant of QCD



taken from minireview on QCD by Bethke, Dissertori and Salam (PDG online)

⇐ dominated by lattice results

Sources test very different scales.
Clear evidence for asymptotic freedom.

QCD at low energies

- QCD gives rise to a rich structure at low energies.
- Low energies are out of reach of perturbation theory.

⇒ Not a simple matter to work out the consequences of the Standard Model at low energies.

- Exploring higher energies is one way to try peeking beyond the Standard Model. The alternative is to improve the accuracy in the accessible energy range.

⇒ Low energy properties of QCD then play essential role.

- ∃ many models that resemble QCD: AdS/CFT, NJL, monopoles, skyrmions, bags, superconductivity, gluonic strings, linear σ model, hidden gauge, . . .
- May help developing an intuitive understanding of QCD, but are totally inadequate when accuracy matters.

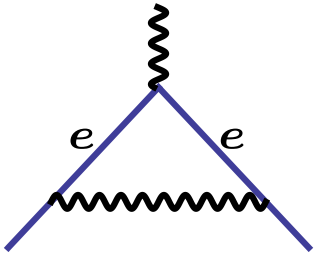
Sensitive low energy probe: leptonic magnetic moments

- $\mu = \dfrac{e\,g}{2m}$ Dirac equation implies $g = 2$.
- For the electron, the experimental uncertainty only shows up in the 13^{th} decimal:

$$\mu_e = \frac{e}{m_e} \times 1.001\,159\,652\,180\,73(28)$$

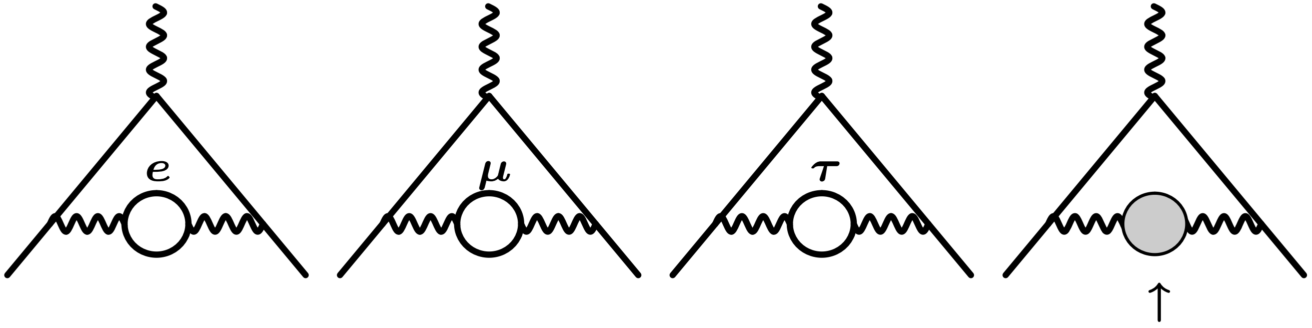
- Leading correction from QED dominates (Schwinger):

$$\mu = \frac{e}{m}\left\{1 + \frac{\alpha_e}{2\pi} + O(\alpha_e^2)\right\}$$



$$\frac{\alpha_e}{2\pi} = 0.00116\ldots$$

Vacuum polarization



Hadronic vacuum polarization
generated by quarks, gluons

- Accuracy of prediction is limited by uncertainty in α_e.
- Contributions from QCD are too small to test our understanding of the strong interaction with μ_e.
- Same applies to contributions from QFD.
- Measurement of μ_e does offer excellent test of QED.

Magnetic moment of the muon

- experiment: $\mu_\mu = \dfrac{e}{m_\mu} \times 1.001\,165\,920\,80(63)$.

  experimental uncertainty shows up in the 10^{th} digit

- theory: $\mu_\mu = \dfrac{e}{m_\mu} \times 1.001\,165\,917\,90(65)$.

  Jegerlehner & Nyffeler

- Contributions from QCD and QFD do matter here:
  - QCD: hadronic vac. pol. $\sim 100\times$exp. error
  - QCD: light-by-light scattering $\sim 2\times$exp. error
  - QFD: $\sim 2\times$exp. error
- Discrepancy amounts to $4.6\times$exp. error

⇒ Standard model prediction is off by more than 3 σ.

Origin of the discrepancy ?

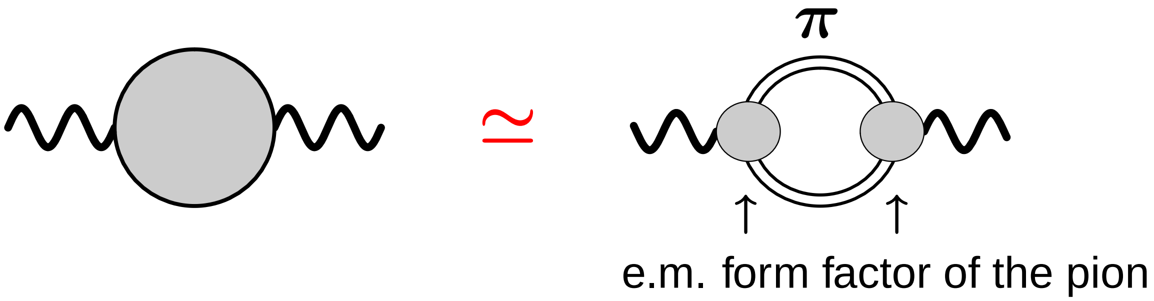
- Three possibilities:
  - Physics beyond the Standard Model ?
  - Error in the evaluation of the SM prediction ?
  - Did something go wrong with the experiment ?
- To match the experimental precision, need the contributions from QCD to an accuracy of 1%.
- More than $\frac{3}{4}$ stem from the region below 1 GeV.

Energy gap of QCD

- Low energy properties of QCD play a central role in our understanding of the laws of nature.
- Essential qualitative feature: QCD has an energy gap.
  No massless particles in spectrum of physical states.
- Lightest particle: π, bound state of quarks and gluons.

$\Rightarrow$ Energy gap is determined by its mass: $E_{\text{gap}} = M_\pi c^2$.

- Pions are the most important degrees of freedom at low energies. In particular, the QCD contribution to μ_μ is dominated by the one from the pions:



Energy gap of QCD is small

- The energy gap is very small: $M_\pi c^2 \simeq$ 140 MeV.

Why ?

- In 1960, Nambu found out why the gap is so small.
  - Has to do with a hidden approximate symmetry.
  - The Hamiltonian is approximately symmetric, but the state of lowest energy is not.

$\Rightarrow$ Symmetry is "hidden", "spontaneously broken”.

  - Nambu realized that the spontaneous breakdown of an approximate symmetry entails approximately massless particles, concluded that in the case of the strong interaction, the pions must play this role.

Chiral symmetry

- Where is Nambu's approximate symmetry in QCD ?

- Massless fermions: right and left do not communicate.

⇒ In the theoretical limit $m_u, m_d \rightarrow 0$, the Hamiltonian of QCD becomes invariant under chiral transformations:

Independent isospin rotations of $\binom{u_{\text{L}}}{d_{\text{L}}}$ and $\binom{u_{\text{R}}}{d_{\text{R}}}$.

- Symmetry group: $\text{SU(2)}_{\text{L}} \times \text{SU(2)}_{\text{R}}$.

⇒ QCD with two massless quarks is indeed invariant under the group introduced by Nambu.

- Currents $\vec{V}^\mu = \bar{q}\gamma^\mu \frac{1}{2}\vec{\tau} q, \ \vec{A}^\mu = \bar{q}\gamma^\mu\gamma_5 \frac{1}{2}\vec{\tau} q$ conserved.

  Charges $\vec{Q}_V = \int d^3x \vec{V}^0, \vec{Q}_A = \int d^3x \vec{A}^0$ commute with the Hamiltonian.

Chiral symmetry is hidden

- For dynamical reasons, the lowest eigenstate is not symmetric under chiral rotations: Nambu 1960

$$\vec{Q}_{\mathsf{A}}\,|0\rangle \neq 0$$

Compare spontaneous magnetization.

- The spontaneous breakdown of a continuous symmetry entails massless particles: "Nambu-Goldstone bosons".
- Carry the quantum numbers of the states $\vec{Q}_{\mathsf{A}}\,|0\rangle$:

spin zero, negative parity, isospin triplet: π^+, π^0, π^-

- In the limit $m_u = m_d = 0$, chiral symmetry is exact, pions are massless.

Approximate chiral symmetry

- Chiral symmetry is exact only if $m_u, m_d \to 0$.
  Quark masses break chiral symmetry.
- Hamiltonian of QCD can be split into two parts:

$$H_{\text{QCD}} = H_0 + H_1\,, \qquad H_1 = \int d^3x\,\{m_{\text{u}}\,\bar{u}u + m_{\text{d}}\,\bar{d}d\}$$

H_0 is invariant, H_1 breaks the symmetry, links L to R:

$$\bar{u}u = \bar{u}_{\text{L}}u_{\text{R}} + \bar{u}_{\text{R}}u_{\text{L}}$$

⇒ Have an approximate symmetry if m_u, m_d are small.

- When they were discovered, the occurrence of approximate symmetries was mysterious.

  In QCD, this is a natural phenomenon:

  $\boxed{\text{It so happens that } m_{\text{u}} \text{ and } m_{\text{d}} \text{ are very small}}$ why ???

Light quark masses as perturbations

$$H_{\text{QCD}} = H_0 + H_1\,, \qquad H_1 = \int d^3x\,\{m_{\text{u}}\,\bar{u}u + m_{\text{d}}\,\bar{d}d\}$$

- H_0 describes u, d as massless, s, c, b, t as massive.
- Spectrum of H_0 contains massless pions.

  H_1 gives them a mass.

Expansion in powers of $m_{\text{u}}, m_{\text{d}}$ $\Longleftrightarrow$ Perturbation series in powers of H_1

- First order perturbation theory: Gell-Mann, Oakes & Renner 1968

$$M_\pi^2 = (m_u \underset{\substack{\Uparrow \\ \text{explicit}}}{+} m_d) \times |\langle 0|\,\underset{\substack{\Uparrow \\ \text{spontaneous}}}{\bar{u}u}\,|0\rangle| \times \frac{1}{F_\pi^2}$$

Coefficient: pion decay constant, known from $\pi \to \mu\nu$.

Gell-Mann-Oakes-Renner formula

- In QCD, M_π^2 is a function of $\Lambda_{\text{QCD}}, m_{\text{u}}, m_{\text{d}}, \ldots, m_{\text{t}}$.
- Chiral symmetry: M_π^2 tends to zero if $m_u, m_d \rightarrow 0$.
- The GMOR formula shows that the expansion of M_π^2 in powers of m_u, m_d starts with a term linear in m_u, m_d:

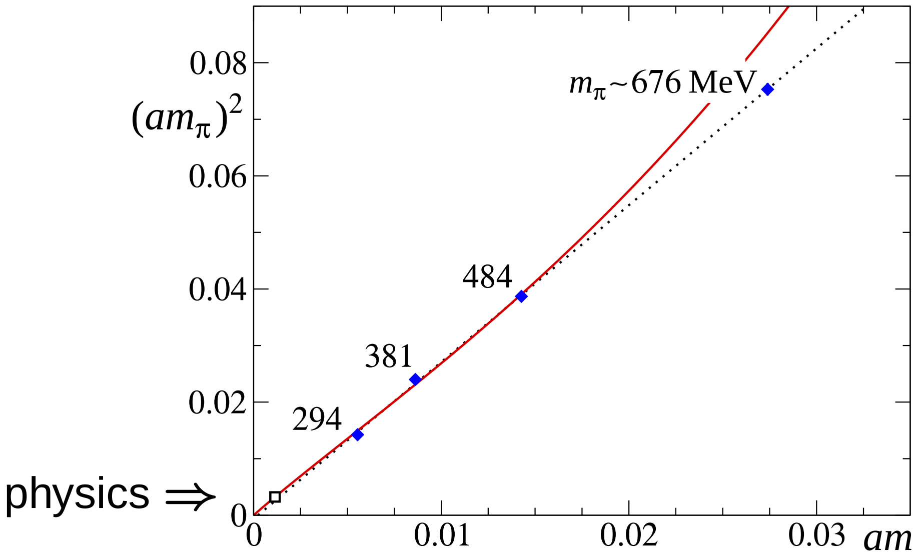
$$\boxed{M_\pi^2 = (m_u + m_d) \times |\langle 0|\bar{u}u\,|0\rangle| \times \frac{1}{F_\pi^2}}$$

⇒ The energy gap of QCD is controlled by m_u, m_d.
The gap is small because m_u, m_d happen to be small.

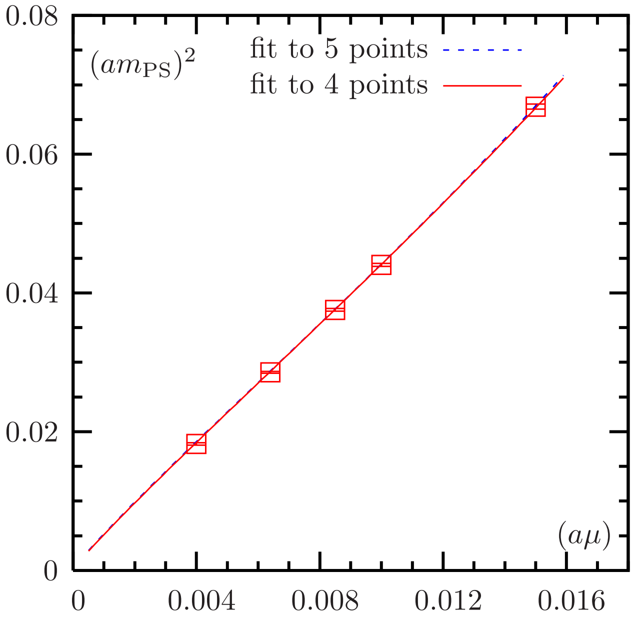
- Formula receives corrections from higher orders.

Lattice results for M_π

- GMOR formula can now be checked on the lattice: determine M_π as a function of $m_{\text{u}} = m_{\text{d}} = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- Proportionality of M_π^2 to m holds out to $m \simeq 10 \times m_{\text{phys}}$

Lattice

- Quality of data is impressive.
- No quenching, can reach small quark masses.
  - $m_{ud} = \frac{1}{2}(m_u + m_d)$ and m_s now known to ∼ 3%.
  - Ratio can be determined even more precisely:

    $$\frac{m_s}{m_{ud}} = 27.5 \pm 0.4.$$

  - $\frac{m_u}{m_d}$ is still subject to significant uncertainties, because the e.m. interaction plays an important role.

    Lattice simulations of QCD + QED cannot be done with the same level of confidence as for QCD alone.

Effective theory

- In the limit $m_u, m_d \to 0$, the spectrum of QCD contains massless particles, Nambu-Goldstone bosons.

$\Rightarrow$ Green functions contain infrared singularities: poles and branch points at $p = 0$.

- Properties of the Nambu-Goldstone bosons are governed by the hidden symmetry.

$\Rightarrow$ Nambu-Goldstone bosons of low momentum interact only weakly: can treat the momenta as perturbations.

- Quark masses also generate perturbations, push the infrared singularities away from $p = 0$.

$\Rightarrow$ Low energy structure of QCD can be analyzed with a simultaneous expansion in powers of p, m_u, m_d.

Effective Lagrangian

- Formulation in terms of an effective Lagrangian:

Weinberg 1967, Coleman, Wess, Zumino, Callan, Dashen, Weinstein 1969

- Variables of the effective theory: pion fields.

⇒ Perturbation series has infrared singularities:

Li & Pagels 1971, Langacker & Pagels 1973
Weinberg 1979, Gasser & Zepeda 1980, Gasser 1981

Singularities due to Nambu-Goldstone bosons can be worked out with an effective field theory

Chiral Perturbation Theory, χPT

- χPT reproduces the low energy structure of QCD, order by order in the expansion in powers of p, m_u, m_d.

Meson field theory

- χPT originally formulated as a meson field theory.
  - $\langle 0|\, T\pi^i(x)\pi^k(y)\,|0\rangle$ plays central role.
  - Drawback: $\mathcal{L}_{\text{eff}}$ depends on choice of variables.

    $\pi^{i\,\prime} = F^i(\pi)$ change of variables.

$\Rightarrow$ Green functions of the pion field change, but physics (masses, S-matrix) remains the same.

- Studying the Green functions of the pion field amounts to perturbing the system with

  $\mathcal{L}_{\text{eff}} \longrightarrow \mathcal{L}_{\text{eff}} + \vec{f}(x)\cdot\vec{\pi}(x) \quad \Big| \vec{f}(x)$ is an external field

$\Rightarrow$ Since $\vec{\pi}(x)$ transforms in a nonlinear manner, this ruins the symmetry of the effective Lagrangian.

Effective theory for QCD Green functions

- Further shortcoming of original framework: current matrix elements ? Noether currents of $\mathcal{L}_{\text{eff}}$ are correct only at leading order, F_π at NLO ?
- $\vec{\pi}(x) \notin$ QCD, but $\vec{V}^\mu(x),\ \vec{A}^\mu(x), \ldots \in$ QCD

  $\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{QCD}} + \vec{f}(x) \cdot \vec{\pi}(x)$?

  $\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{QCD}} + \vec{v}_\mu(x) \cdot \vec{V}^\mu(x) + \vec{a}_\mu(x) \cdot \vec{A}^\mu(x)$ $\surd$
- Need effective theory for Green functions of QCD:

  $$\langle 0|\, T V_i^\mu(x) V_k^\nu(y)\, |0\rangle,\ \langle 0|\, T A_i^\mu(x) A_k^\nu(y)\, |0\rangle,$$
  $$\langle 0|\, T V_i^\mu(x) V_k^\nu(y) A_l^\lambda(z)\, |0\rangle, \ldots$$
- Symmetry in terms of Green functions:
  symmetry $\Rightarrow$ current conservation $\Rightarrow$ Ward identities.
  Ward identities are exact even for $m_{\text{u}}, m_{\text{d}} \neq 0$.
  Anomalies show up in W.i., not in Lagrangian of QCD.

Gasser & L. 1984,1985

Plethora of effective coupling constants

- χPT merely exploits the symmetries of QCD:
  yields the general solution of the Ward identities.
- $\mathcal{L}_{\mathit{eff}}$ contains all functions that can be formed with the pion field and its derivatives, only subject to the condition that the sum is chirally invariant.
- Order in number of derivatives (powers of momentum).

⇒ Number of terms in $\mathcal{L}_{\mathit{eff}}$ rapidly grows with the order:
LO: 2, NLO: 7, NNLO: 53, . . .

- Symmetries only relate – do not determine.
- In principle, the effective theory is exact:
  yields expansion of QCD Green functions in $\boldsymbol{p, m_u, m_d}$.

Illustration: Gell-Mann-Oakes-Renner formula at NLO

- GMOR formula represents leading term of χPT .
- Correction of first nonleading order:

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{32\pi^2 F_\pi^2}\, \bar{\ell}_3 + O(M^4) \right\}$$

  with $M^2 \equiv (m_u + m_d) |\langle 0| \bar{u}u\, |0\rangle| F_\pi^{-2}$

  $\ell_3 \in \mathcal{L}_{\text{eff}}$ depends logarithmically on running scale.

- What counts is the running coupling at scale M:

$$\bar{\ell}_3 = \ell n \, \frac{\Lambda_3^2}{M^2}$$

⇒ Expansion of M_π^2 contains a chiral logarithm.

Langacker & Pagels 1973, Gasser & Zepeda 1980, Gasser 1981

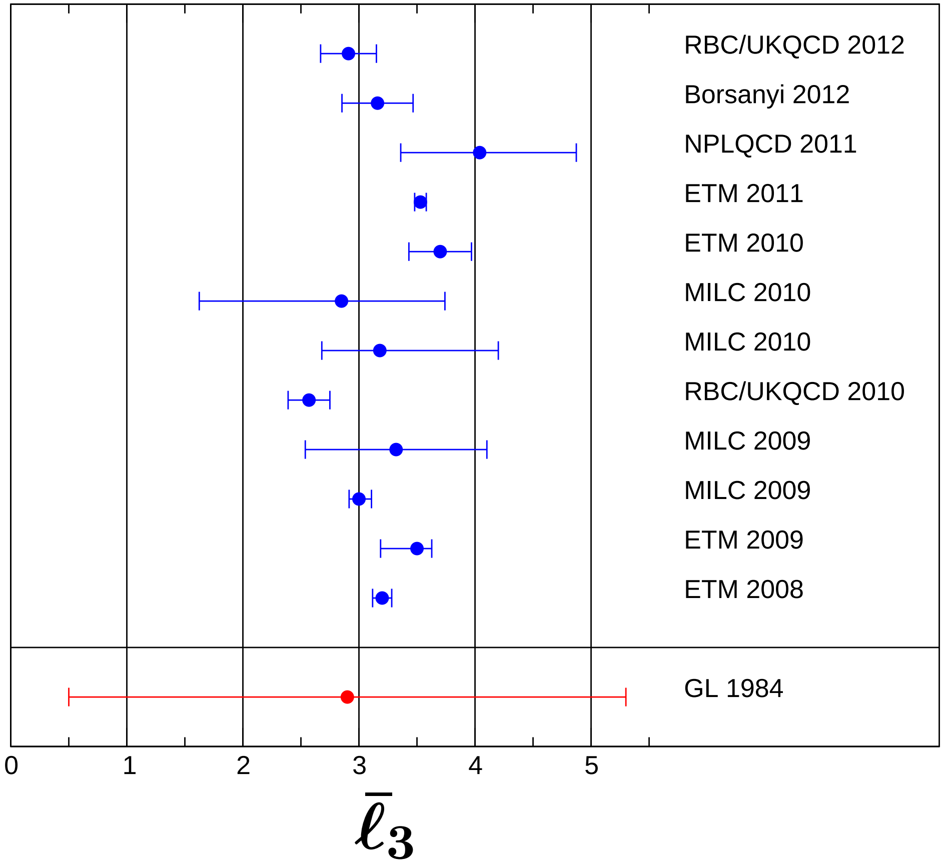
Chiral logarithm in M_π

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{32\pi^2 F_\pi^2}\, \ell n \frac{\Lambda_3^2}{M^2} + O(M^4) \right\}$$

- Coefficient is determined by pion decay constant. Symmetry does not determine the scale Λ_3.
- Crude estimate, based on SU(3)×SU(3): 0.2 GeV $\lesssim \Lambda_3 \lesssim 2$ GeV

Gasser & L. 1984

Lattice allows more accurate determination of Λ_3



Not all of the lattice data are of the same quality. Some of the entries do not show the full systematic error.

⇒ FLAG review

Edition 2013 is in preparation.

Horizontal axis shows the value of $\bar{\ell}_3 \equiv \ell n \frac{\Lambda_3^2}{M_\pi^2}$.

Range for Λ_3 obtained in 1984 corresponds to $\bar{\ell}_3 = 2.9 \pm 2.4$

Result of Borsanyi 2012, for instance, is $\bar{\ell}_3 = 3.16 \underset{\text{stat}}{\pm 0.10} \underset{\text{syst}}{\pm 0.29}$

Interaction among the pions

Dispersion theory

- The interaction among the pions plays an important role in the low energy analysis of the Standard Model.
- $\pi\pi$ scattering has been worked out to NNLO of χPT. Strength of $\pi\pi$ interaction rapidly grows with energy.

$\Rightarrow$ χPT useful only at very low energies.

- $\pi\pi$ scattering is special: crossed channels are identical.

$\Rightarrow$ Re $T(s,t)$ can be represented as a twice subtracted dispersion integral over Im $T(s,t)$ in physical region.

S.M. Roy 1971

- The two subtraction constants can be identified with the two S-wave scattering lengths: a_0, a_2 (isospin 0 or 2)
- Representation leads to dispersion relations for the individual partial waves: *Roy equations*.

Early work on Roy equations

- Pioneering work on the physics of the Roy equations:

Basdevant, Froggatt & Petersen 1974

- Dispersion integrals converge rapidly (2 subtractions).

⇒ Crude phenomenological information on Im $T(s,t)$ for energies above 800 MeV suffices.

- Main problem in early work: a_0, a_2 poorly known. Experimental information near threshold was meagre.

Low energy theorems for the scattering lengths

- Chiral perturbation theory provides the missing piece: accurate theoretical prediction for a_0, a_2.

  Weinberg 1966, Gasser & L. 1983, Bijnens et al. 1996, Colangelo et al. 2001

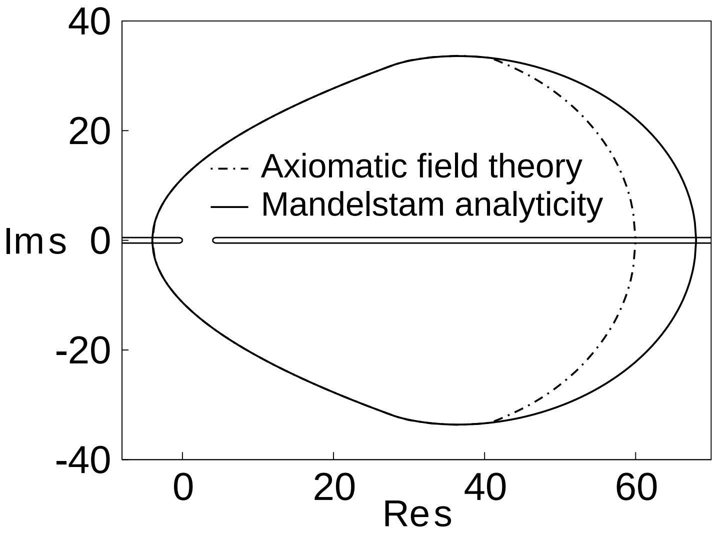
- Triggered new low energy precision experiments:
  - $\pi^+\pi^-$ atoms DIRAC
  - $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$ E865, NA48/2
  - $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$, $K^0 \rightarrow \pi^0\pi^0\pi^0$ NA48/2
- Beautiful data, prediction passes the test.
- In combination with the low energy theorems for a_0, a_2, the dispersion relations for the partial waves fix the $\pi\pi$ scattering amplitude to remarkable accuracy.

Resonances

- Resonances are poles in partial waves, on second sheet. Precise notion, even for broad resonances.
- Unitarity: a pole on the second sheet implies that the S-matrix element has a zero on the physical sheet.
- In the domain of validity of the Roy equations, the S-matrix elements can reliably be calculated on the physical sheet. Are there any zeros there ?

Domain of validity of the Roy equations

- Roy derived his equations for real energies in the interval $-4M_\pi^2 < s < 60M_\pi^2$.
- Equations are valid for complex s in a limited region of the first sheet. Caprini, Colangelo & L. 2006



s in units of M_π^2

- Proof is based on first principles, general quantum field theory.

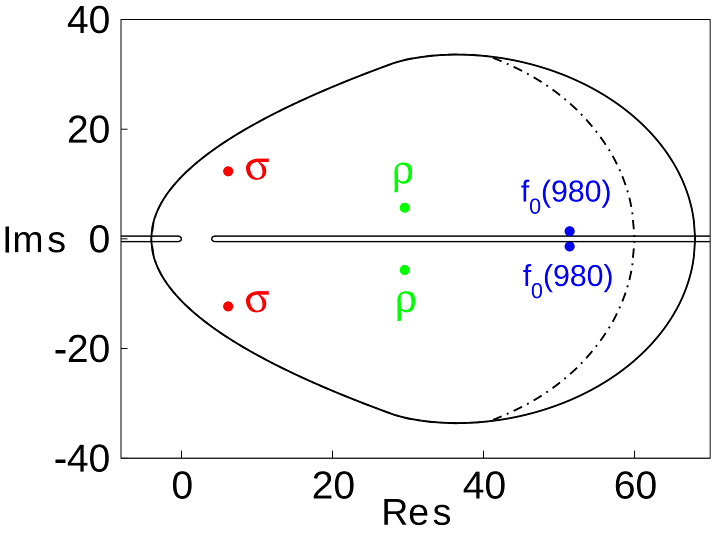
A. Martin, *Scattering Theory: Unitarity, Analyticity and Crossing*, Lecture Notes in Physics, vol. 3, 1969.

Mahoux, Roy & Wanders 1974

⇒ Exact representation for the partial waves in this region. Do not need to parametrize the amplitude.

Resonances in domain of validity of Roy equations

- Result: for $\ell = I = 0$ there are two pairs of zeros:



the central values are at

$$s = (6.2 \pm i\,12.3)\, M_\pi^2 \quad \sigma$$
$$s = (51.4 \pm i\,1.4)\, M_\pi^2 \quad f_0(980)$$

σ occurs far from real axis.

- Error analysis yields

$$M_\sigma - \tfrac{i}{2}\Gamma_\sigma = \sqrt{s_\sigma} = 441\,{}^{+16}_{-8} \;- i\; 272\,{}^{+9}_{-13}\ \text{MeV}$$

Colangelo, Caprini & L. 2006

⇒ Lowest resonance has vacuum quantum numbers.

Lowest resonance

- In the meantime, the analysis has been confirmed by several groups. Average value quoted by the PDG for "most advanced dispersive analyses":

  $$M_\sigma - \tfrac{i}{2}\Gamma_\sigma = 446 \pm 6 - i\,276 \pm 5 \text{ MeV} .$$

- Accuracy not of interest per se, but shows that $\pi\pi$ scattering below 1 GeV is quantitatively understood.
- Low energy flavour physics is a precision laboratory.
- Theoretical tools: χPT, lattice, dispersion theory.
- Limitations:
  - Low energies.
  - Calculations cannot be done on back of an envelope.
  - e.m. interaction must properly be accounted for.

Conclusion

- The Standard Model is a precision theory at low energy.
- To work out the quantitative consequences of this insight is a fascinating challenge.
- Models (AdS/CFT, NJL, hidden gauge, . . .) may help developing an intuitive understanding of QCD . . .

  . . . but are a meagre replacement for the real thing.
- Much work remains to be done to confront low energy precision experiments with our present understanding of the laws of nature: lattice simulations, effective field theory methods, dispersion theory . . .

. . . and to find out what hides behind the Standard Model.