On the interface between lattice results and χPT

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QCD: The Modern View of the Strong Interactions

October 5, 2009, Academy of Sciences, Berlin

Standard Model at low energies

- Low energies ($E \ll M_{\rm W}$): weak interaction is frozen

$\Rightarrow$ Standard Model reduces to QCD + QED

- Lagrangian only involves g_s, θ, e, fermion masses

$\Rightarrow$ Precision theory for cold matter ($T \ll M_{\rm W}$), size and structure of atoms, solids, etc.

- QED is infrared stable, characterized by pure number, which happens to be small, $1/137$

$\Rightarrow$ QED can be accounted for with perturbation theory

- Hadrons at low energies: SM = QCD + corrections

Pièce de résistance: QCD

- $\exists$ many models that resemble QCD: instantons, monopoles, bags, superconductivity, gluonic strings, linear σ model, hidden gauge, NJL, AdS/CFT, but ...
- Nonperturbative methods needed

$\Rightarrow$ Progress in understanding is slow

- Model independent methods:
  - Numerical simulation on a lattice
  - Sum rules, dispersion relations
  - Effective field theory (χPT)

Hidden symmetries in particle physics

Already in 1960, Nambu realized that

1. $SU(2)_L \times SU(2)_R$ is an approximate symmetry of the strong interaction
2. The symmetry is "hidden", "spontaneously broken": $|0\rangle$ invariant only under the isospin subgroup $SU(2)_{L+R}$
3. The spontaneous breakdown of an exact symmetry entails massless particles
4. For the strong interaction, the pions play this role
5. The pions are not massless, only light, because the symmetry is only an approximate one

Nobel Prize 2008

Explains why the energy gap of the strong interaction is so small : $M_\pi \simeq$ 135 MeV
When Nambu proposed this idea, the origin of the symmetry was mysterious
Approximate symmetries ? Partially conserved currents ?
For gauge theories like QCD, approximate symmetries do occur naturally

Chiral symmetry

Where is Nambu's hidden approximate symmetry in QCD ?

- QCD with N_f massless quarks: Lagrangian has an exact chiral symmetry: SU$(N_f)_{\rm L}\times$SU$(N_f)_{\rm R}$
- $|0\rangle$ is symmetric only under the subgroup SU$(N_f)_{\rm L+R}$
Symmetry is spontaneously broken

$\Rightarrow$ Spectrum contains N_f^2-1 Nambu-Goldstone bosons

- $m_{\rm u}$ and $m_{\rm d}$ happen to be small

$\Rightarrow$ SU(2)$_{\rm L}\times$SU(2)$_{\rm R}$ is an approximate symmetry of QCD

  - broken spontaneously
  $\Rightarrow$ $|0\rangle$ not invariant
  - broken explicitly by mass term $m_{\rm u}\,\bar{u}u + m_{\rm d}\,\bar{d}d$
  $\Rightarrow$ $\mathcal{L}_{\rm QCD}$ not invariant
  - $m_{\rm u}, m_{\rm d}$ are very small $\longrightarrow$ symmetry is nearly exact

Chiral perturbation series

- For $m_{\rm u} = m_{\rm d} = 0$, pion exchange gives rise to poles and branch points at $p = 0$

$\Rightarrow$ Low energy expansion is not a Taylor series, contains infrared singularities

- Properties of the Nambu-Goldstone bosons are governed by the hidden symmetry that is responsible for their occurrence

- NG bosons of low momentum interact only weakly: can treat the momenta as well as $m_{\rm u}, m_{\rm d}$ as perturbations

$\Rightarrow$ Chiral perturbation series: simultaneous expansion of the matrix elements in powers of p, m_u, m_d

Effective Lagrangian

- Formulation in terms of an effective Lagrangian

Weinberg 1967, Coleman, Wess, Zumino, Callan, Dashen, Weinstein 1969

- Lagrangian $\supset$ massless Nambu-Goldstone Bosons

$\Rightarrow$ Perturbation series has infrared singularities

Li + Pagels 1971, Langacker + Pagels 1973

Weinberg 1979, Gasser + Zepeda 1980, Gasser 1981

Singularities due to NG bosons can be worked out with an effective field theory "Chiral Perturbation Theory"

- χPT reproduces the low energy structure of QCD, order by order in the expansion in powers of p, m_u, m_d

Plethora of low energy constants

- χPT merely exploits the symmetries of QCD: yields the general solution of the Ward-Takahashi identities
- $\mathcal{L}_{eff}$ contains all functions that can be formed with the pion field and its derivatives, only subject to the condition that the sum is chirally invariant
- Order in number of derivatives (powers of momentum)

$\Rightarrow$ Number of terms in $\mathcal{L}_{eff}$ rapidly grows with the order:
LO: 2, NLO: 7, NNLO: 53, ...

- Symmetries only relate – do not determine
- In principle, the effective theory is exact:
yields expansion of QCD Green functions in p, m_q

Illustration: energy gap of QCD

- Energy gap of QCD: M_π
- Ignore e.m. self energy, $e = 0$, pure QCD

$\Rightarrow$ M_π is a function of $\Lambda_{\text{QCD}}, m_{\text{u}}, m_{\text{d}}, \ldots, m_{\text{t}}$

- How does M_π depend on $m_{\text{u}}, m_{\text{d}}$?

  Chiral symmetry: $M_\pi \longrightarrow 0$ for $m_{\text{u}}, m_{\text{d}} \longrightarrow 0$

- Leading order formula (tree level of χPT):

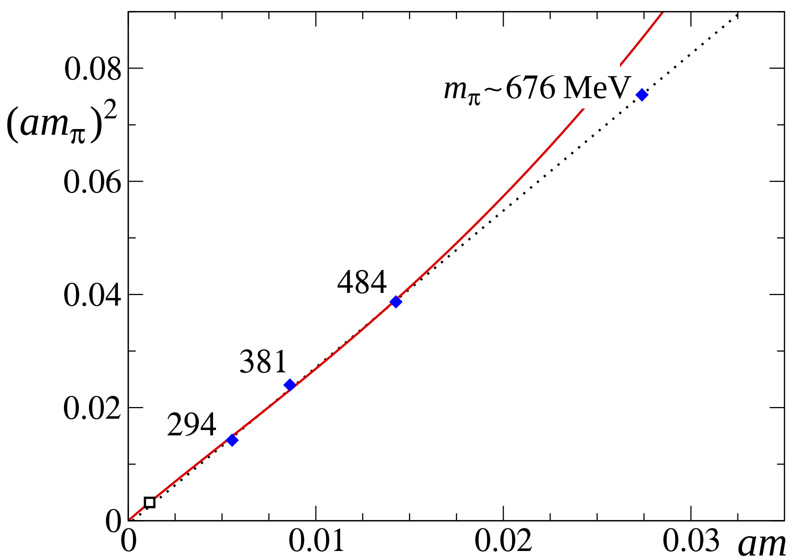
  $M_\pi^2 = (m_{\text{u}} + m_{\text{d}})B$ Gell-Mann, Oakes, Renner 1968

- The coefficient is determined by the quark condensate:

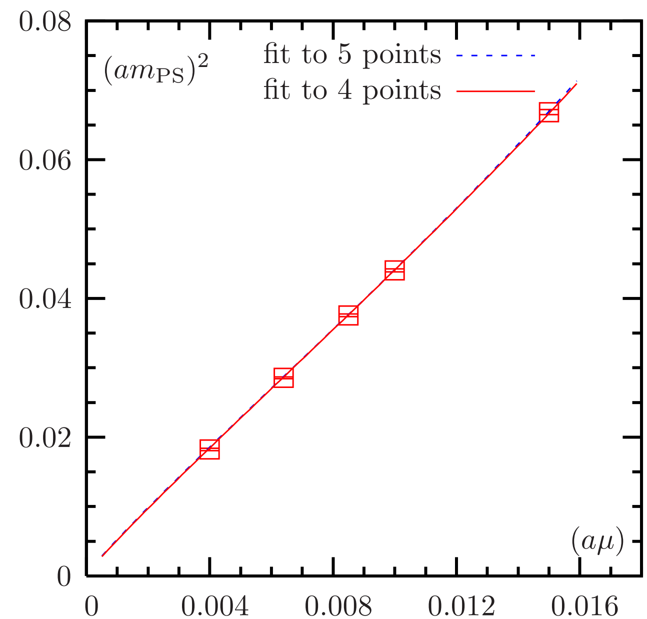
$$B = \frac{|\langle 0| \bar{u}u \,|0\rangle|}{F_\pi^2}$$

Lattice results for M_π

- GMOR formula can now be checked on the lattice: determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

These plots are for QCD with $N_f = 2$. Behaviour in full QCD looks very similar.

Lattice

- Quality of data is impressive
- No quenching, quark masses are sufficiently light

$\Rightarrow$ Legitimate to use χPT for the extrapolation to the physical values of $m_{\rm u}, m_{\rm d}$

- Proportionality of M_π^2 to

  $$m_{\rm ud} \equiv \frac{1}{2}(m_{\rm u} + m_{\rm d})$$

  holds out to $m_{\rm ud} \simeq 10 \times m_{\rm ud}^{\rm phys}$
- Main limitation: systematic uncertainties from lattice artifacts, continuum extrapolation, finite size effects, etc.

Expansion of M_π^2 in powers of m_u, m_d

- GMOR formula represents leading term of χPT
- Correction of first nonleading order:

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{32\pi^2 F_\pi^2}\, \bar{\ell}_3 + O(M^4) \right\}$$

$$M^2 \equiv B(m_u + m_d)$$

$\ell_3 \in \mathcal{L}_{\mathit{eff}}$ depends logarithmically on running scale

- What counts is the running coupling at scale M_π:

$$\bar{\ell}_3 = \ell n \, \frac{\Lambda_3^2}{M_\pi^2}$$

$\Rightarrow$ Expansion of M_π contains a chiral logarithm

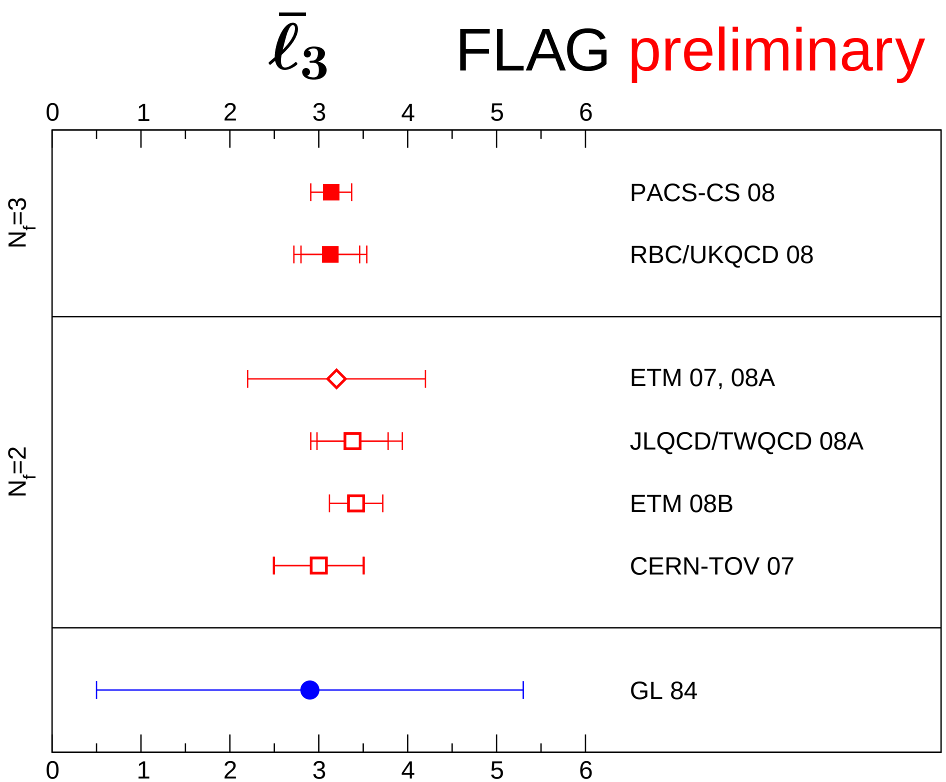
Langacker + Pagels 1973, Gasser + Zepeda 1980, Gasser 1981

Size of the low energy constant ℓ_3

Crude estimate, based on $\mathrm{SU(3)_L \times SU(3)_R}$: $\bar{\ell}_3 = 2.9 \pm 2.4$

Gasser & L. 1984

Lattice allows more accurate determination:



$$\bar{\ell}_3 = \ell n \frac{\Lambda_3^2}{M_\pi^2}$$

Result of RBC/UKQCD 08, for instance: $\bar{\ell}_3 = 3.13 \underset{stat}{\pm 0.33} \underset{syst}{\pm 0.24}$

Expansion of F_π in powers of the quark mass

- Also contains a logarithm at NLO:

$$F_\pi = F\left\{1+\frac{M^2}{16\pi^2F^2}\,\ell n\,\frac{\Lambda_4^2}{M^2}+O(M^4)\right\}$$

$$M_\pi^2 = M^2\left\{1-\frac{M^2}{32\pi^2F^2}\,\ell n\,\frac{\Lambda_3^2}{M^2}+O(M^4)\right\}$$

F is value of pion decay constant in limit $m_\mathrm{u}, m_\mathrm{d} \to 0$

- Structure is the same, coefficients and scale of logarithm are different
- Quark mass dependence of F_π can also be measured on the lattice $\Rightarrow$ measurement of Λ_4
- Alternative method: determine the scalar form factor of the pion, radius $\langle r^2\rangle_s \leftrightarrow \bar{\ell}_4 = \ell n(\Lambda_4^2/M_\pi^2)$

Colangelo, Gasser & L. 2001

Lattice determination of scalar radius

- Scalar form factor can be measured on the lattice
- Most recent lattice determination:

$$\langle r^2 \rangle_s = 0.617 \pm 0.079_{\text{stat}} \pm 0.066_{\text{syst}}\ \text{fm}^2$$

JLQCD/TWQCD collaboration, arXiv:0905.2465

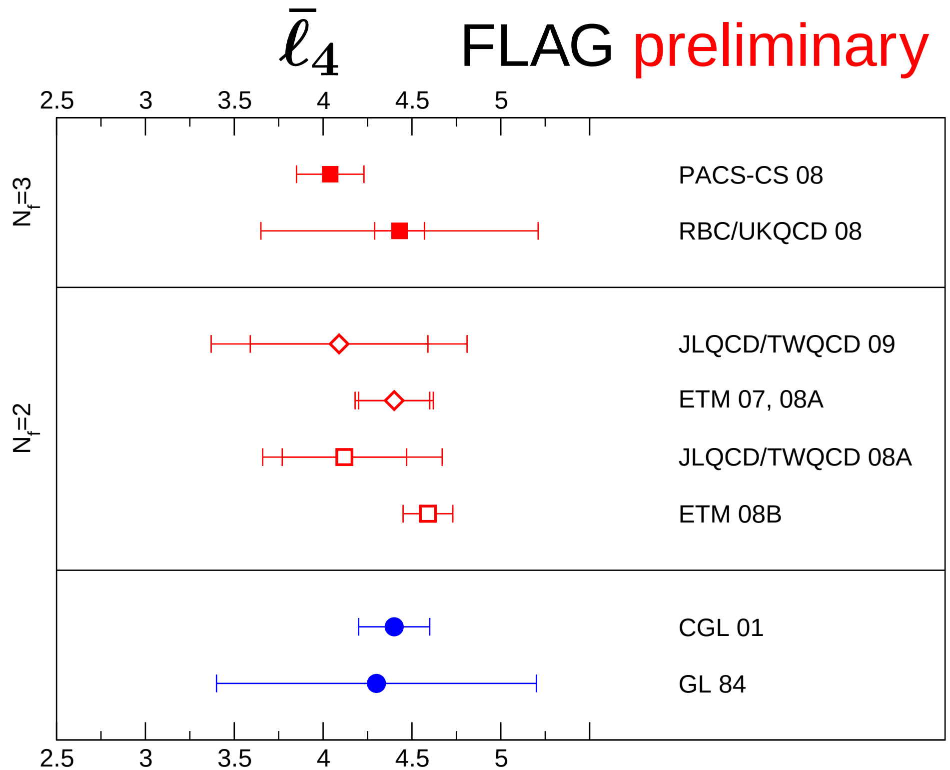
This is consistent with the value obtained on the basis of dispersion theory,

$$\langle r^2 \rangle_s = 0.61 \pm 0.04\ \text{fm}^2$$

Colangelo, Gasser & L. 2001

but the uncertainties in the lattice result are still large

Size of ℓ_4



$$\bar{\ell}_4 = \ell n \frac{\Lambda_4^2}{M_\pi^2}$$

Lattice results are consistent with value obtained from dispersion theory, uncertainties are comparable

$\pi\pi$ interaction

- Symmetry fixes the interaction among the Nambu-Goldstone bosons
- LO formulae for the S-wave scattering lengths:

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\,, \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}$$

Weinberg 1966

- NLO corrections are determined by ℓ_3, ℓ_4 — Gasser + L. 1983
- $\pi\pi$ scattering amplitude known to NNLO

Bijnens, Colangelo, Ecker, Gasser + Sainio 1996

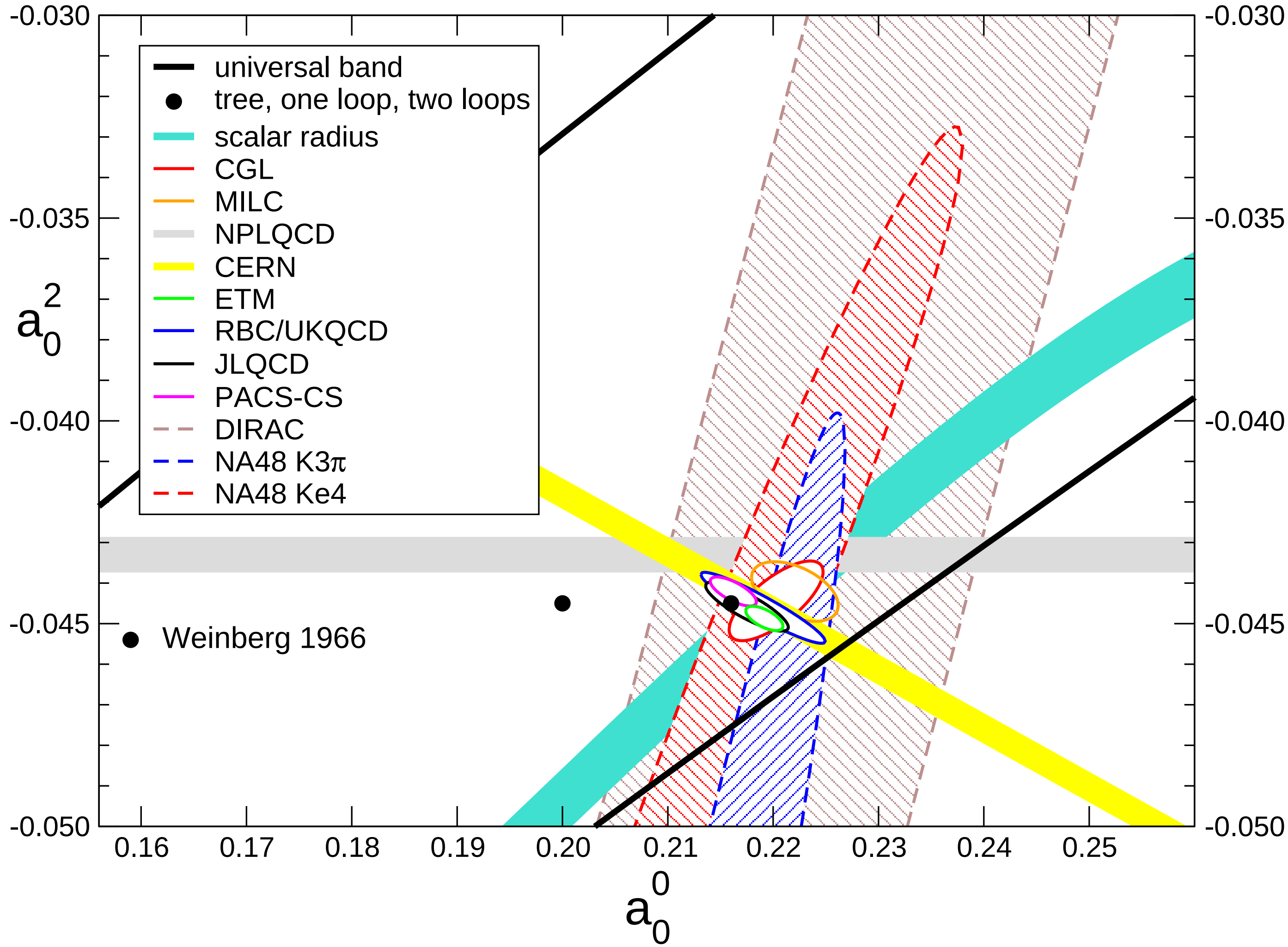
- Uncertainty in predictions for a_0^0, a_0^2 is dominated by the uncertainty in the low energy constants ℓ_3, ℓ_4

$\Rightarrow$ Can make use of the lattice results for these

- Contributions from higher order couplings are tiny

Guo + Sanz-Cillero arXiv:0904.4178

Consequence of lattice results for ℓ_3, ℓ_4



The plot represents beautiful physics: experiment as well as theory

Extension to $\mathrm{SU(3)_L \times SU(3)_R}$

- In the theoretical limiting case $m_\mathrm{u} = m_\mathrm{d} = m_\mathrm{s} = 0$ QCD acquires an exact $\mathrm{SU(3)_L \times SU(3)_R}$ symmetry

Is m_s small enough for this to represent a useful approximate symmetry ?

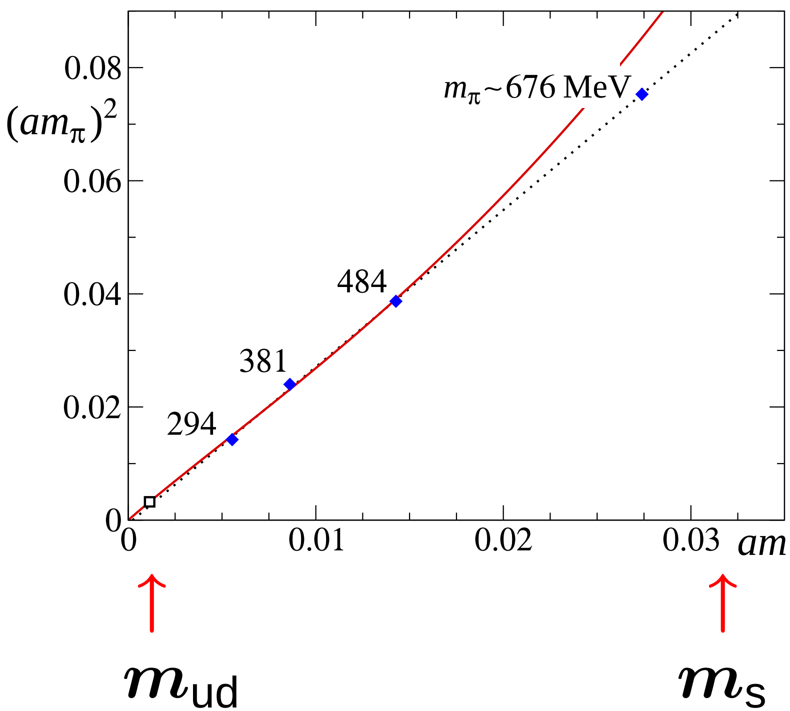
- Theoretical reasoning
  - $\mathrm{SU(3)_{L+R}}$ (eightfold way) is an approximate symmetry
  - Typical size of $\mathrm{SU(3)_{L+R}}$ breaking: $\dfrac{F_K}{F_\pi} = 1.19 \pm 0.01$
  - Only coherent way to understand this in QCD:

    The mass differences $m_\mathrm{s} - m_\mathrm{d}$, $m_\mathrm{d} - m_\mathrm{u}$ must be small, can be treated as perturbations
  - Since $m_\mathrm{u}, m_\mathrm{d} \ll m_\mathrm{s}$

$\Rightarrow$ m_s is small, $\mathrm{SU(3)_L \times SU(3)_R}$ must be an approximate symmetry, breaking not larger than for $\mathrm{SU(3)_{L+R}}$

Expansion in powers of m_u, m_d, m_s

- Expansion in powers of m_u, m_d, m_s ought to work, but expect convergence to be comparatively slow
- Lattice results: $M_\pi^2 \propto m_{ud}$ holds out to $10 \times m_{ud}^{phys}$
- m_s is larger than that: $m_s \simeq 27 \times m_{ud}$



Compare

$$\frac{F_K}{F_\pi} \simeq 1.19$$

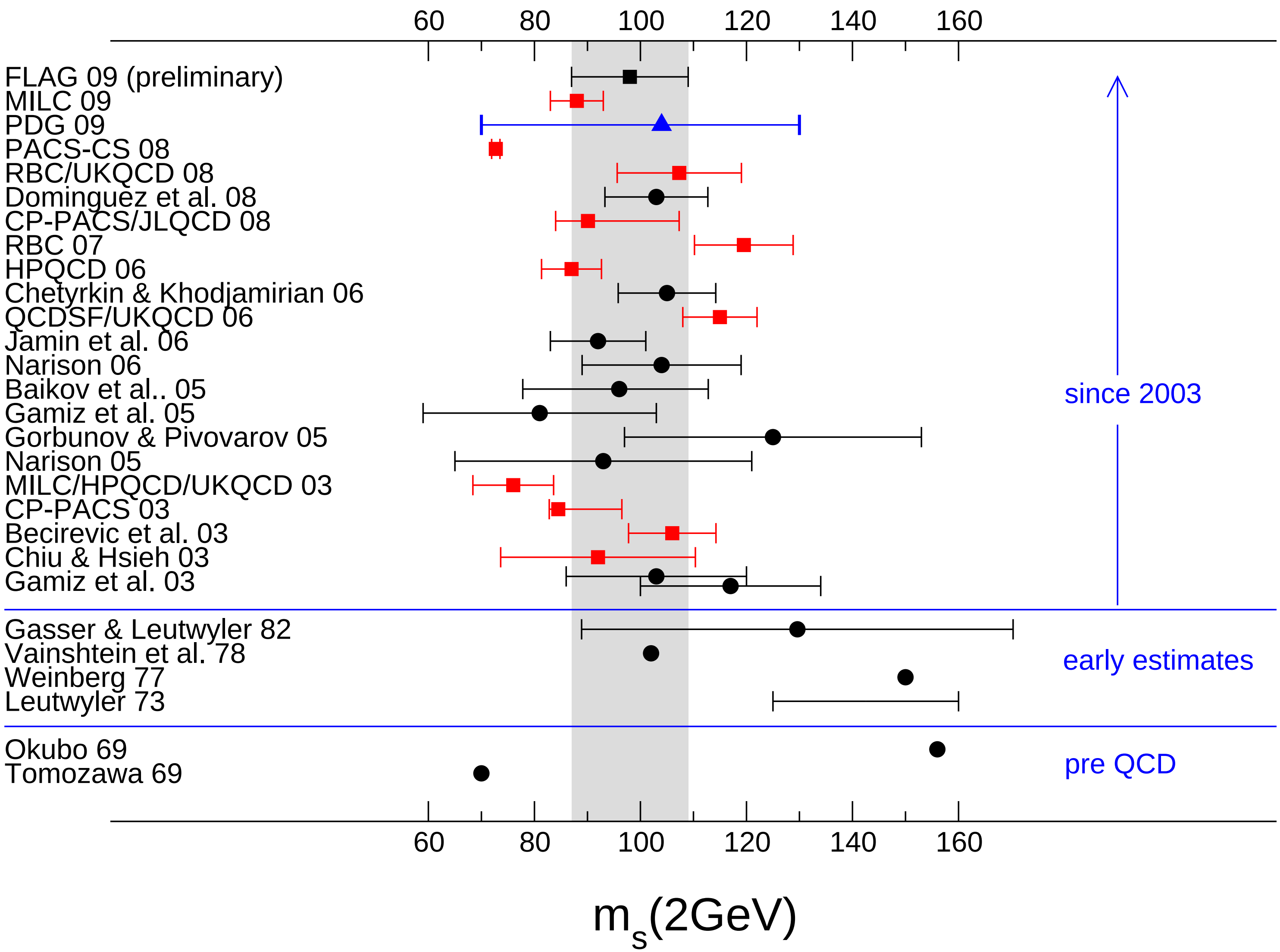
Three light quarks: interface between lattice and χPT

- Steady progress in simulating QCD with light quarks, but the quark masses used are still too large for the NLO formulae of χPT to work
- M_π OK, but M_K too large
- Three options
  - Use smaller quark masses
  - Extrapolate only in $m_{\rm u}, m_{\rm d}$, keep $m_{\rm s}$ fixed
  - Account for NNLO contributions
- Some lattice analyses do allow for NNLO contributions, but the chiral logarithms are accounted for only to NLO
- $\exists$ discrepancies between different lattice results

  In part, these may arise from nonperturbative renormalization effects
  Some of the collaborations still use perturbative renormalization

$\Rightarrow$ Illustrate this with the results for $m_{\rm s}$

Mass of the strange quark



- Lattice and sum rule results agree within errors
- Can expect significant progress in lattice determinations very soon

Relative size of $m_{\rm u}$, $m_{\rm d}$, $m_{\rm s}$

- $$M^2_{\pi^+} = (m_{\rm u} + m_{\rm d})\, B_0 + O(m^2)$$
  $$M^2_{K^+} = (m_{\rm u} + m_{\rm s})\, B_0 + O(m^2)$$
  $$M^2_{K^0} = (m_{\rm d} + m_{\rm s})\, B_0 + O(m^2)$$
- χPT relates B_0 to the quark condensate, but does not predict its size $\Rightarrow$ no prediction for size of quark masses
- Account for e.m. self energies at tree level of χPT and drop effects of second order in isospin breaking
  $$\frac{m_{\rm u}}{m_{\rm d}} = \frac{M^2_{K^+} - M^2_{K^0} + 2M^2_{\pi^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.56$$
  $$\frac{m_{\rm s}}{m_{\rm d}} = \frac{M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 20.2$$
  Weinberg 1977
- Corrections from higher orders ? Could they strongly modify the numerical values ? $m_{\rm u}$ = 0 ?

Higher orders

- $\nexists$ scalar probe analogous to γ, W$^\pm$

$\Rightarrow$ Quark masses cannot be determined from phenomenology alone, not even their ratios

Kaplan & Manohar 1986

- At LO, χPT does determine the quark mass ratios
- At NLO, there is only one relation without unknowns:

$$Q^2 \equiv \frac{m_{\rm s}^2 - m_{\rm ud}^2}{m_{\rm d}^2 - m_{\rm u}^2} = \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2}\,\frac{M_K^2}{M_\pi^2} + \text{NNLO} + \text{e.m.}$$

M_K, M_π: mean masses of the two multiplets Gasser & L. 1985

The relation correlates the two ratios

Value of $Q \rightarrow$ ellipse in the plane $\left(\dfrac{m_{\rm u}}{m_{\rm d}}, \dfrac{m_{\rm s}}{m_{\rm d}}\right)$

- Weinberg's leading order formulae give Q = 24.3.

$\eta \to \pi^+\pi^-\pi^0$

- Critical input for value of Q is the "Dashen theorem": e.m. self energies are accounted for only at tree level
- η decay allows an independent determination of Q

  Gasser & L. 1985
- Dispersive analysis of the decay amplitude

  Kambor, Wiesendanger & Wyler 1996, Anisovich & L. 1996, Walker 1998
- In $\eta \to 3\pi$, the e.m. contributions are suppressed

  Bell & Sutherland 1968

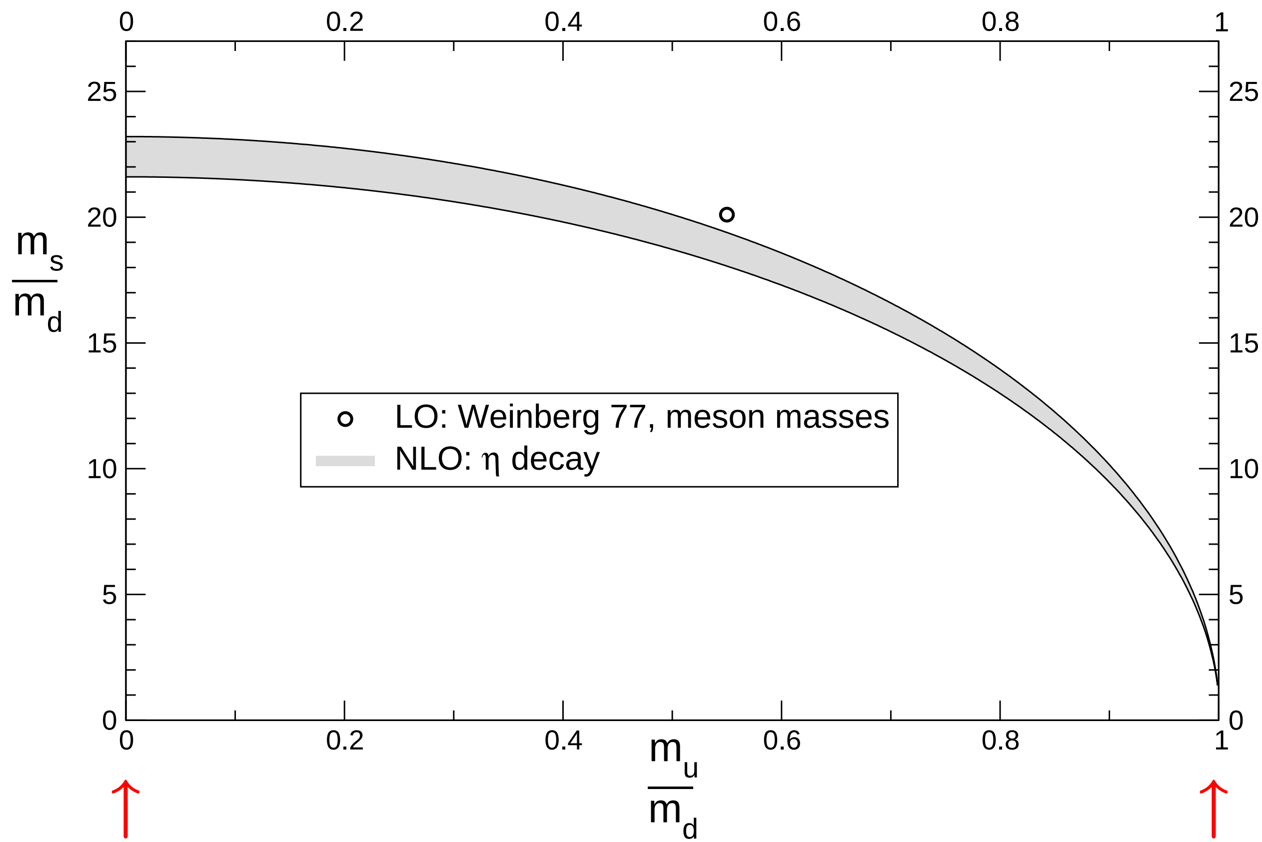
$\Rightarrow$ Uncertainties are smaller

Quantitative analysis of e.m. contributions: Ditsche, Kubis & Meißner 2009

- Update of Walker's calculation with the current experimental information $\Rightarrow$ Q = 22.4 $\pm$ 0.8, to be compared with Q = 24.3 from Dashen theorem
- Comprehensive analysis of $\eta \to 3\pi$ is under way

  PhD thesis of Stefan Lanz, in preparation

χPT at leading and first nonleading order



$m_u = 0$ $\qquad$ $m_u = m_d = m_s$

Where on the ellipse ? $m_{\text{u}} = 0$?

- The vacuum angle θ breaks CP
- Chiral symmetry ensures that θ can enter physical quantities only via det $\mathcal{M} \times e^{i\theta}$
- If m_{u} is zero $\rightarrow$ det $\mathcal{M}$ vanishes $\rightarrow$ θ without physical significance $\rightarrow$ QCD invariant under CP
- Quite a few authors advocated $m_{\text{u}} = 0$ as the solution of the strong CP problem, possibly some still do ...

Nice idea, but amounts to trading one puzzle for the other:

- If m_{u} were zero, the Weinberg formula for $m_{\text{u}}/m_{\text{d}}$ would turn into a prediction for $M_{K^0} - M_{K^+}$:

$$M_{K^0} - M_{K^+} = \frac{2M^2_{\pi^0} - M^2_{\pi^+}}{M_{K^0} + M_{K^+}} + \text{NLO}$$

$\Uparrow$ $\Uparrow$

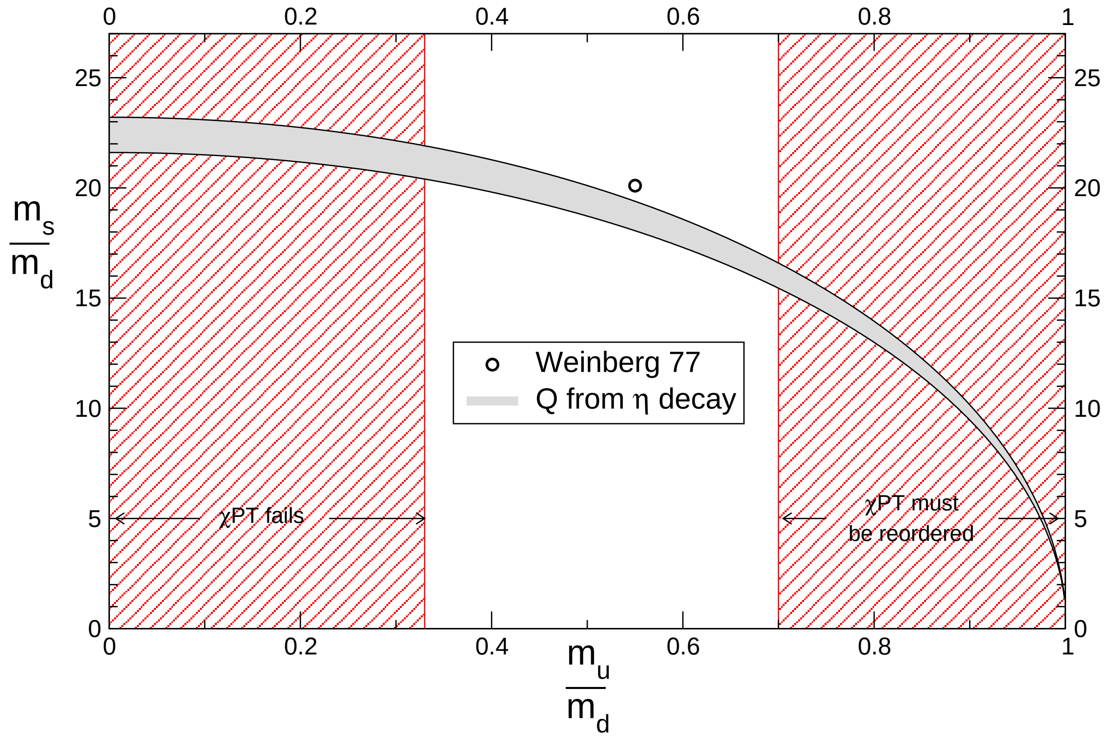
3.9 MeV — experiment

16.9 MeV — theory

$m_u = 0$?

- If m_u were zero, then χPT would be in conflict with the observed mass pattern of the NG bosons
  - chiral series could not be truncated at low orders
  - $SU(3)_L \times SU(3)_R$ not an approximate symmetry
  - Success of Gell-Mann-Okubo formula accidental etc.
- Leading order formula for $M_{K^0} - M_{K^+}$ is off by less than a factor of 2 only if $\boxed{0.7 > m_u/m_d > 0.33}$

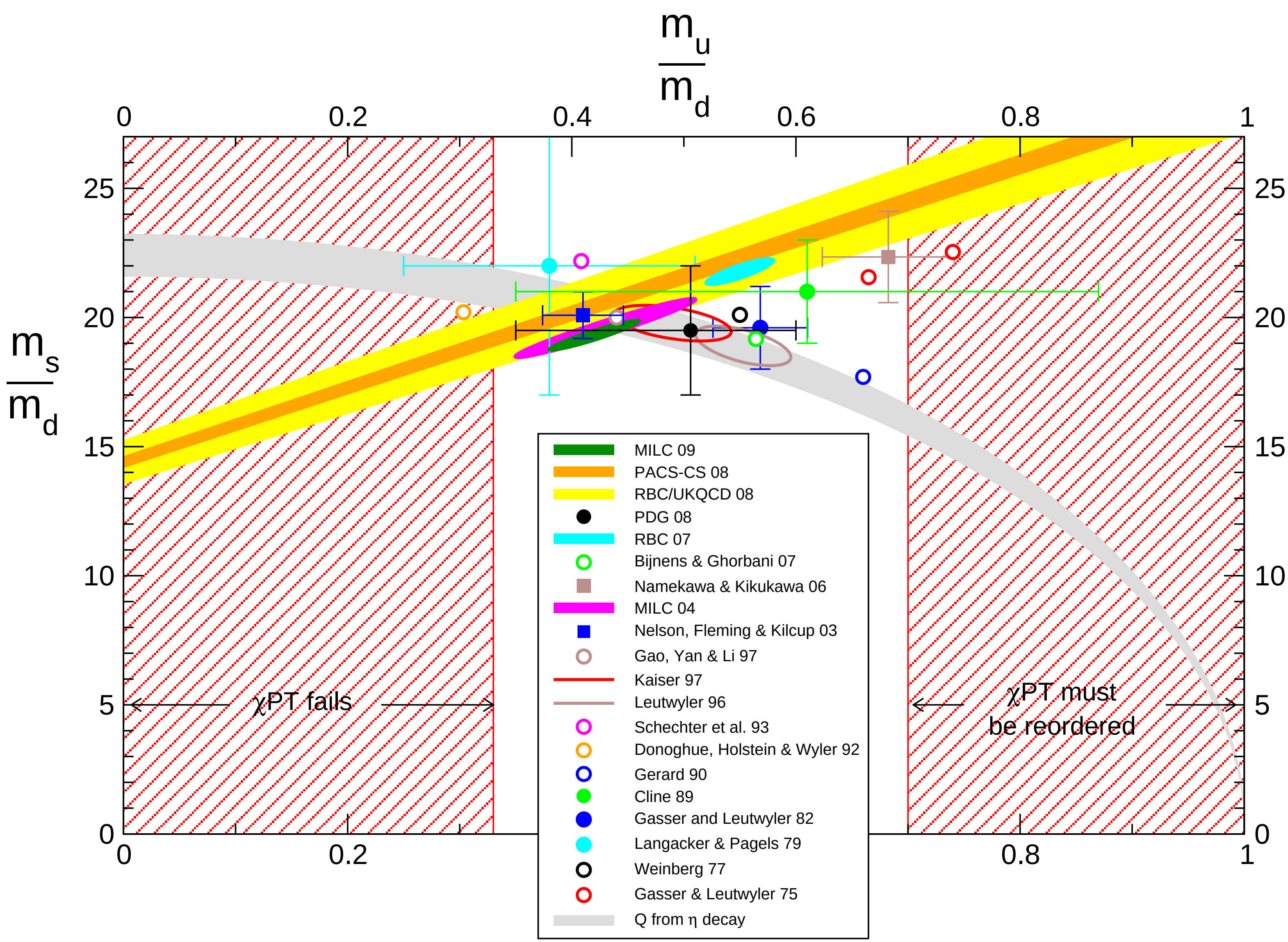
Allowed range of mass ratios



- All of the lattice results are in the range allowed by χPT None is consistent with the solution $m_{\mathrm{u}} = 0$ of the strong CP problem
- The MILC collaboration rules this solution out at 10 σ

$\Rightarrow$ Nature solves the strong CP problem differently

Results for quark mass ratios



Lattice determination of V_{us}, V_{ud}

- Rely on Standard Model, where

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Precision data on K-decays imply

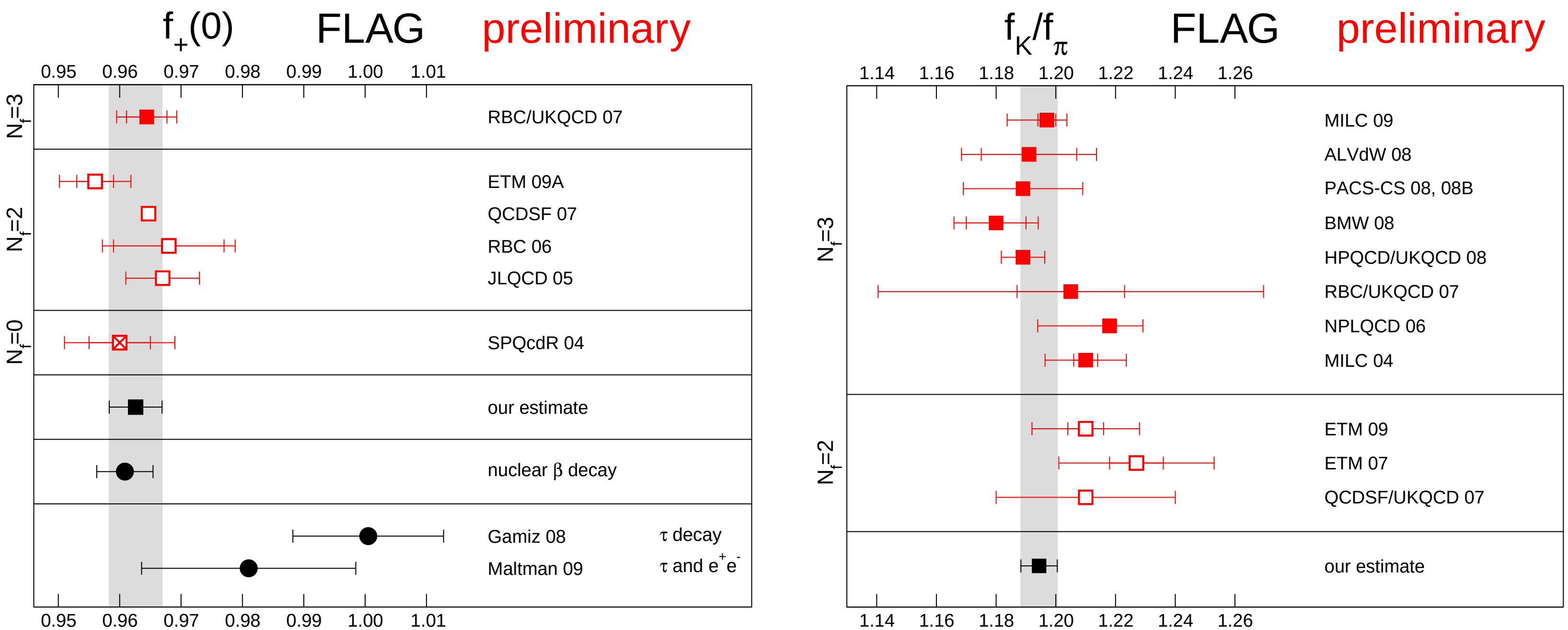
$$|V_{us}|f_+(0) = 0.21661(47)$$

$$\left|\frac{V_{us}F_K}{V_{ud}F_\pi}\right| = 0.27599(59)$$

$\Rightarrow$ Since V_{ub} is tiny and known to good accuracy, $V_{ud}, f_+(0), F_K/F_\pi$ are all determined by V_{us}

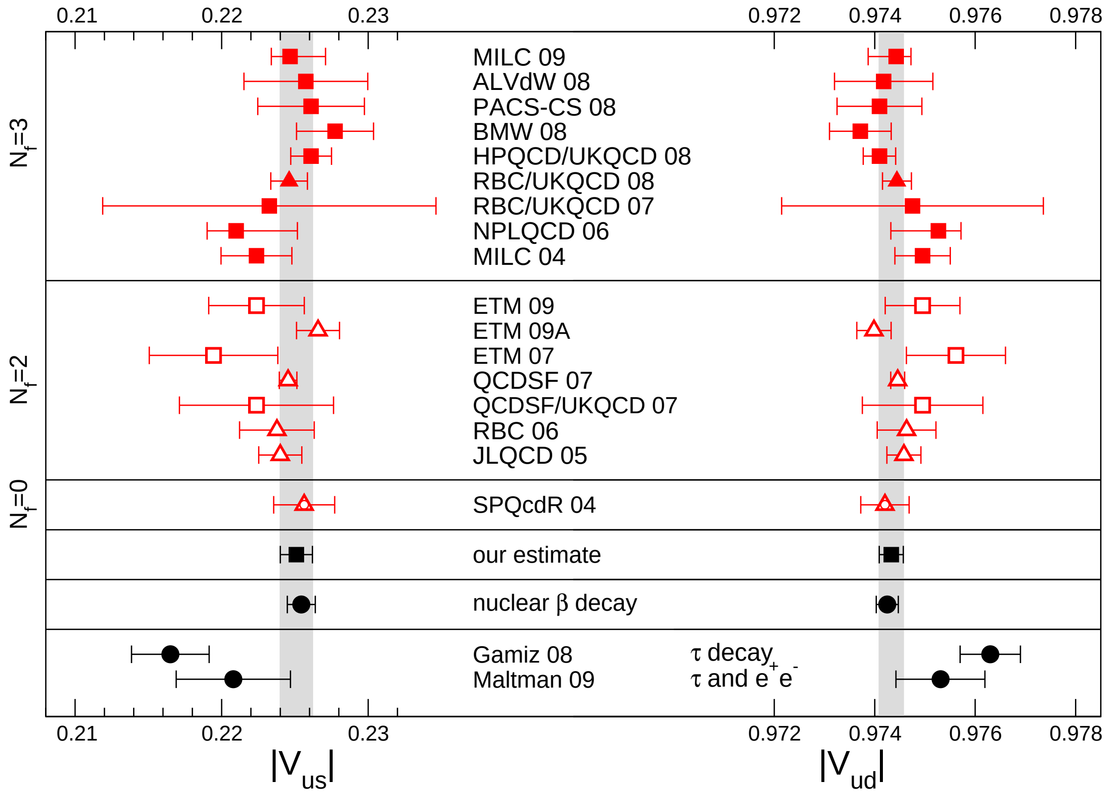
- Lattice allows two independent ways to measure V_{us}: calculate $f_+(0)$ or calculate F_K/F_π

Lattice results for $f_+(0)$ and F_K/F_π



- FLAG estimate combines the lattice data for $f_+(0)$ with those for F_K/F_π

Lattice results for V_{us} and V_{ud}



FLAG preliminary

- Confirms nuclear β decay value for V_{ud} within errors
- τ decay: physics beyond the Standard Model ?

Trying to understand the size of the low energy constants

- $SU(2)_L \times SU(2)_R$: can understand the size of all NLO couplings in terms of resonance exchange Gasser + L. 1984
- Also true for $SU(3)_L \times SU(3)_R$ Ecker, Gasser, Pich, de Rafael 1989
- χPT formulae have been worked out to NNLO for many quantities of physical interest Bijnens and collaborators
- Formulae involve new unknown low energy constants
- "Resonance Chiral Theory": couplings of higher order, effective Lagrangian for e.m. + weak interactions . . .

  Gonzalez-Alonso, Guo, Pich, Portoles, Prades, Rosell, Ruiz-Femenia, Sanz-Cillero . . .
- Comprehensive review of current state of the art:

  Bijnens, arXiv:0904.3713 (Valencia 2009)

Problem with Resonance Chiral Theory in case of $f_+(0)$

- Form factor known to NNLO

  Post + Schilcher 2002, Bijnens + Talavera 2003

- Account for isospin breaking, use RχPT estimates for the low energy constants

$\Rightarrow$ $\boldsymbol{f_+(0) = 0.986(7)}$ Kastner + Neufeld 2008

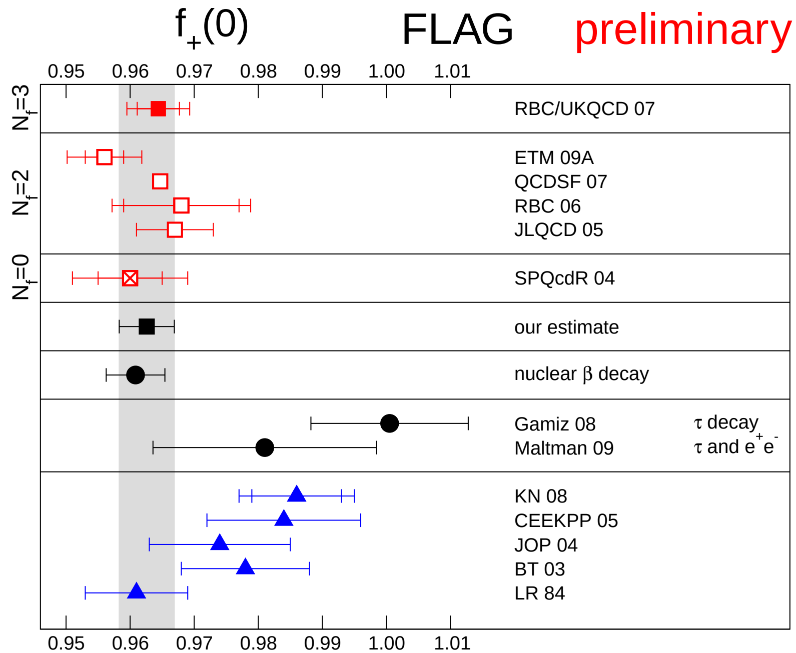
- To be compared with

  $\mathbf{0.961(5)}$ (β decay) or $\mathbf{0.962(5)}$ (lattice)

  (These numbers are obtained by assuming that the CKM matrix is unitary)

  Discrepancy amounts to 2.9 σ and 2.8 σ, respectively

Compare with lattice results



- $1 - f_+(0)$ is a symmetry breaking effect
- No problem at NLO: parameter free prediction

$\Rightarrow$ RχPT does not appear to account properly for the symmetry breaking effects at NNLO

Problems with scalar meson dominance ?

- Quark mass term in $\mathcal{L}_{\text{QCD}}$ is a scalar operator
- Matrix elements dominated by scalar resonances ?

  Can the *dependence on the quark masses* be accounted for with scalar meson dominance ?
- Rapidly rising $\pi\pi$ continuum (large chiral logs), σ makes a broad bump, narrow peak from $f_0(980)$, glueballs, etc.
- Failure of scalar meson dominance may be the origin of the problem

  more detailed discussion in Erice lectures 2007

Conclusions

Conclusions

- Expansion in powers of $m_{\mathrm{u}}, m_{\mathrm{d}}$ yields a very accurate low energy representation of QCD
- Lattice yields remarkably coherent and significant results for pion physics already now

Low energy pion physics is a precision laboratory Theoretical tools: χPT, lattice, dispersion theory

- Limitations:
  - Low energies
  - e.m. interaction must properly be accounted for
  - Calculations cannot be done on back of an envelope

Conclusions

- $\boxed{m_\text{u} \neq 0}$

  Nature solves the strong CP problem differently

- $\boxed{m_\text{s} = 98 \pm 11 \text{ MeV}}$ FLAG 09 (preliminary result) $\overline{\text{MS}}$ scheme, scale 2 GeV

  Lattice results confirm sum rule estimates within errors

- For the physical values of m_u, m_d, m_s, the leading order terms in the chiral perturbation series of M_π, M_K, F_π, F_K do represent a decent approximation

- Summary of current knowledge of quark mass ratios:

$$\boxed{\frac{m_\text{u}}{m_\text{d}} = 0.47 \pm 0.08} \qquad \boxed{\frac{m_\text{s}}{m_\text{d}} = 19.7 \pm 1.5}$$

to be compared with Weinberg's LO formulae, which give 0.56 & 20.2, respectively

Conclusions

- Lattice results indicate that the NLO contributions in M_π, M_K, F_π, F_K do dominate the corrections

$\Rightarrow$ χPT does appear to work for SU(3)$_\mathrm{L}\times$SU(3)$_\mathrm{R}$ as well

- Extension to kaon physics is making progress
  - Except for a few selected quantities, kaon physics is still at an exploratory stage
  - Representations of many quantities of interest are available to NNLO of χPT $\Rightarrow$ Bijnens et al.
  - Main problem at NNLO: the current knowledge of the LECs is rudimentary
  - The RχPT estimates for $f_+(0)$ illustrate the problem
  - There was a problem with the RχPT estimates also for $K \to \pi\pi$, but this puzzle appears to be solved

Cirigliano, Ecker + Pich, Phys. Lett. 2009

Conclusions

- Many open issues:
  - M_K = 600 MeV is beyond reach of χPT
  - Better determination of some of the LECs needed
  - In particular, a meaningful comparison of many of the χPT results in kaon physics with experiment requires better knowledge of those LECs that determine the dependence on the quark masses
  - Size of Okubo-Iizuka-Zweig rule violations ?
  - e.m. self energies, corrections to Dashen Theorem ?

Significant progress at the interface between lattice and effective field theory methods is ante portas

Spares

Phase of final state in $K \to \pi\pi$

- $K \to \pi\pi$ decay: value of $\delta_0^0 - \delta_0^2$ at $s = M_K^2$
  - $\pi\pi$ phase shifts accurately known from dispersion theory $\delta_0^0 - \delta_0^2 = 47.5^\circ \pm 1.5^\circ$ Colangelo, Gasser, L. 2001
  - In the determination from $K \to \pi\pi$ via Watson theorem, isospin breaking is enhanced because of the $\Delta I = \frac{1}{2}$ rule
  - Complete analysis to NLO Cirigliano, Ecker, Neufeld + Pich 2004
  - Recent update of the numerics yields

$$\delta_0^0 - \delta_0^2 = 52.5^\circ \pm 0.8^\circ_{\text{exp}} \pm 2.8^\circ_{\text{th}}$$

Cirigliano, Ecker + Pich, Phys. Lett. 2009

- Remaining difference amounts to 1.5 σ

Large N_c

- In the large N_c limit, the η' also becomes a Nambu-Goldstone boson

$\Rightarrow$ Can extend χPT to include the η', systematic expansion in powers of m_{u}, m_{d}, m_{s} and $1/N_c$

- In this framework, there is no ambiguity at NLO
- Triangle anomaly yields a prediction also for $\Gamma_{\eta' \to \gamma\gamma}$
  Can use this to pin down all unknowns at NLO

Kaiser 1997

η and η' at large N_c

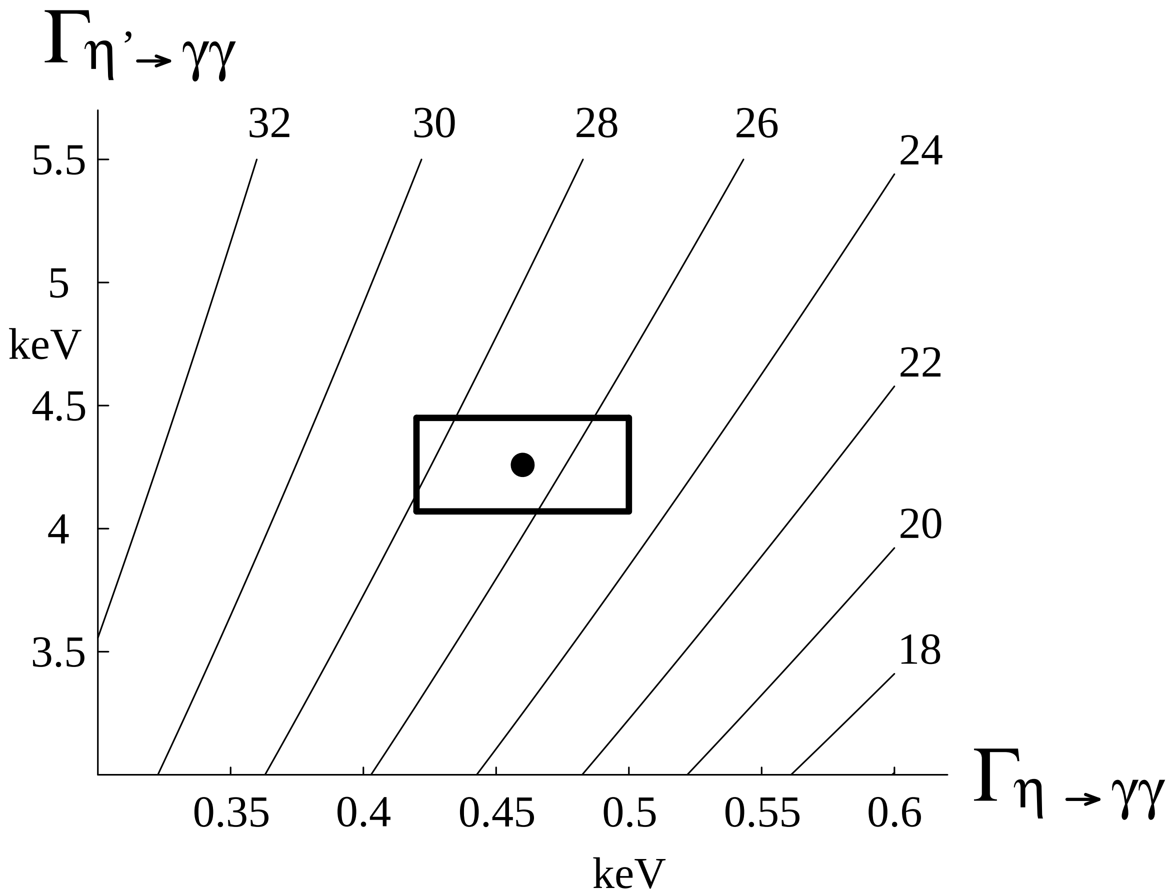


Figure taken from diploma work of Roland Kaiser (1997)

Tilted lines: value of S = $m_{\rm s}/m_{\rm ud}$, rectangle: experiment

Central value found in this determination: S = 26.6

Barely differs from leading order result: S = 25.9